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CONTROL OF NON-INSTANTANEOUS DEGRADING INVENTORY UNDER TRADE CREDIT AND PARTIAL BACKLOGGING

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Abstract: *Inventory management is an extremely difficult task. It has become usual practice for a provider during the last few decades to provide a retailer with a credit term. In this article, a non-instantly degradable product inventory system is built with a price-sensitive demand and a Weibull credit term allocation reduction rate. Some backlogged deficiencies are permitted. The aim is to maximize the total profit by taking three cases into account. Numerical examples, graphical representations and sensitivity analysis demonstrate the application of the approach developed in this study.*

Key words: *Inventory control, Weibull deterioration, price-sensitive demand, trade credit, non-instantaneous deterioration.*

1. Introduction

Everyday life is a prevalent phenomenon in the deterioration of commodities. Some examples of these things are vegetables, fruits, dairy products, drugs and blood bank. Therefore, the tendency of the object to deteriorate is important to take into account. In the real world, most products have a shelf life that allows them to maintain their quality or their original condition for a period of time. During that period of time, there was no deterioration in the situation. Examples of such foods include vegetables and fruits as well as meat, fish, and seafood. This is referred to as "non-instantaneous deterioration" in the scientific literature. First and foremost, Ghare and Schrader (1963) took an important stride in this approach. Giri et al. (2003) have presented a mathematical methodology for Weibull decreasing items. Ghosh and Chaudhury (2004), as well as Roy and Chaudhuri (2009), proposed inventory systems for perishable commodities that are in low supply. Das et al. (2010) created A model for

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an item with variable quality that takes into account random machine failure. An inventory model for small lots was developed by Das et al. (2011), with regular and overtime works being combined to produce the production rates. Kawale and Bansode (2012) developed an inventory system for perishable items under the influence of time dependent holding cost using Weibull rate of deterioration. Barik et al. (2013) created a mathematical approach for deteriorating items under the influence of inflation. Confident weights of experts were used by Das et al. (2014) as part of an algorithmic method for MAGDM problems. In this topic, For m secondary warehouses (SWs) and one primary warehouse, Das et al. (2015) created a multi-item multi-warehouse inventory model for degrading goods. Mahata et al. (2018) and Muriana (2020) also offered several models. Ghosh et al. (2021) studied an EOQ model with full backorder for perishable commodities with varied advance and delayed payment conditions. Non-instantaneous models were investigated by Ouyang et al. (2006) and Wu et al. (2009). Soni (2013) used trade credit to overcome the problem of non-instantaneously decaying inventories (Table 1).

Table 1. Literature summary

Authors	Price dependent demand	Deterio ration	Trade Credit	Constant Holding Cost	Non-Instantaneous
Giri et al. (2003)	No	Yes	No	No	No
Ghosh and Chaudhury (2004)	No	Yes	No	Yes	No
Roy and Chaudhuri (2009)	No	Yes	No	Yes	No
Jain and Kumar (2010)	No	Yes	No	Yes	No
Geetha and Udayakumar (2016)	Yes	Yes	No	Yes	Yes
Mahata et al. (2018)	No	Yes	Yes	Yes	No
Singh et al. (2020)	No	Yes	Yes	Yes	No
Halim et al. (2021)	Yes	Yes	No	Yes	No
Present Paper	Yes	Yes	Yes	Yes	Yes

Geetha and Udayakumar (2016) employed advertisement dependent demand, Shaikh and Cárdenas-Barrón (2020), and Udayakumar et al. (2020) developed alternative inventory models for non-instantaneous falling commodities. Models in this direction have also been presented by Ahmad and Benkherouf (2018) and Tripathi and Pandey (2020). Ouyang et al. (2006) introduced a price-dependent inventory system. Goyal and Chang (2009) used stock-based demand to construct inventory policy. Amutha and Chandrasekaran (2013) created an inventory system for perishable items that incorporates the Weibull rate of deterioration and price-based demand. Avinadav et al. (2013), Guchhai et al. (2013), Avinadav et al. (2014), Feng et al. (2017), and Cheng et al. (2020) followed the work. Halim et al. (2021) devised a strategy for resolving an inventory problem involving decaying products. Sana et al. (2008) developed an inventory model with advertising cost and selling price dependent demand using trade credit. Sarkar (2012) also provided an inventory system for deteriorated commodities purchased on trade credit. By assuming two-level trade credits, Shah et al. (2015) pioneered a novel methodology. Several academics, including Aggarwal and Jaggi (2017), Goyal (2017), and Shah et al. (2017), created several approaches to address inventory problems while taking trade credit into account. Tripathi and Chaudhary (2017) and Singh et al. (2020) employed the

Weibull deterioration rate to develop distinct inventory models for perishable products with trade credits. Tripathi et al. (2018) suggested mathematical systems with various trade credits. Sundararajan et al. investigated the impact of trade credit under inflation on an EOQ model (2020). Jain and Kumar (2010) created a strategy for perishable items with Weibull deterioration rates and scarcity. Sarkar and Sarkar (2013) improved a partial backlog solution approach for time-dependent perishable commodities. Mishra (2016) and Gupta et al. (2018) employed partial backlogging to generate multiple systems for Weibull degrading goods. Jamal et al. (2017), San-José et al. (2018), Akbar et al. (2019), Rastogi and Singh (2019), and San-José et al. (2020) all made major contributions in this area.

Although several researchers have developed inventory models that take the Weibull deterioration rate into account in their work, practitioners have paid less attention to the inclusion of non-instantaneous deterioration. Novelties of present study are as follows:

- We focused our efforts on constructing a mathematical system for non-instantaneous Weibull declining products under trade credit.
- Demand is thought to be price related.
- Shortages are considered partially backlogged are tolerated.
- Impact of different input variables is studied.
- Concavity of profit functions is shown by graphs.

The structure of the paper is in the following format: Segment 2 of this article describes several notations and assumptions. Segment 3 discusses the model formulation. Segment 4 contains the solution technique. Segment 6 demonstrates concavity of profit functions. Segment 7 discusses sensitivity analysis. Segment 8 contains the conclusion.

2. Notations and Assumptions

To create the mathematical model, some notations and assumptions are used.

2.1 Notations

K	The ordering cost /order
Q	The retailer's order quantity
$D(p)$	The demand rate
m	The time in which the item does not decay
M	Permissible delay period
c	Purchasing price /unit
p	Selling price /unit
h	Unit holding price
s	Unit shortage cost/order
c_l	Lost sale cost/ unit

I_p	Rate of interest payable/dollar/unit time
I_e	Rate of interest earned/dollar/unit time
τ	The time at which the inventory level becomes zero
T	Replenishment period
$\theta(t)$	Deterioration rate
$I_1(t)$	Inventory level in period $0 \le t \le m$
$I_2(t)$	Inventory level in period $m \le t \le \tau$
$I_3(t)$	Inventory level in period $\tau \le t \le T$
$Z(\tau)$	Total profit
*	Optimal value

2.2 Assumptions

- The replenishment rate is assumed to be limitless.
- The time stamp begins at zero.
- Shortages that are partially backlogged are allowed. The pace of backlogging is determined by the time required for subsequent replenishing. As a result, during the stock-out period, it is denoted as $B(t) = e^{-\delta(T-t)}$, where $0 \le \delta \le 1$.
- Articles within the cycle period cannot be replaced or repaired in any way.
- The demand D depends on selling price p and $D(p) = \lambda p^{-\mu}$, $\lambda \ge 0, \mu \ge 0$.
- After the interval $[0, m]$ the goods begin to deteriorate with the Weibull deterioration rate, $\theta(t) = \alpha\beta t^{\beta-1}; \alpha \ge 0, \beta \ge 0$.
- For a specified term the supplier gives commercial credit to the retailer.

3. Mathematical Formulation

During the period $[0,\ m]$, there is no deterioration. The inventory level in the period $[m,\ \tau]$ is consumed by both demand and deterioration. In the period $[\tau,\ T]$, shortages occur which are partially backlogged.

The change of inventory level $I(t)$ in different time durations given by

$$\frac{dI_1(t)}{dt} = -\lambda p^{-\mu}, 0 \le t \le m \quad (1)$$

$$\frac{dI_2(t)}{dt} = -\lambda p^{-\mu} - \alpha\beta t^{\beta-1} I_2(t), m \le \tau \quad (2)$$

$$\frac{dI_3(t)}{dt} = -\lambda p^{-\mu} e^{-\delta(T-t)}, \tau \le t \le T \quad (3)$$

The solution of equations (1), (2), and (3) with boundary conditions $I_1(0) = Q, I_2(\tau) = 0 = I_3(\tau)$, are

$$I_1(t) = -\lambda p^{(-\mu)} t + Q. \quad (4)$$

$$I_2(t) = \lambda p^{-\mu}\left[(\tau - t) + \frac{\alpha}{\beta+1}\left(\tau^{\beta+1} - t^{\beta+1}\right) - \alpha t^{\beta}(\tau - t)\right]. \quad (5)$$

$$I_3(t) = \lambda p^{-\mu}(\tau - t)\left[1 - \delta T + \frac{\delta}{2}(\tau + t)\right]. \quad (6)$$

Using boundary conditions, $I_1(m) = I_2(m),\ I_3(T) = -E$ we get

$$Q = \lambda p^{-\mu}\left[\tau + \frac{\alpha}{\beta+1}\left(\tau^{\beta+1} - m^{\beta+1}\right) - \alpha m^{\beta}(\tau - m)\right]. \quad (7)$$

$$E = \lambda p^{-\mu}(T - \tau)\left[1 - \delta T + \frac{\delta}{2}(\tau + T)\right] \quad (8)$$

The total annual profit/cycle is obtained by including the following:

1. Ordering cost (O) = K.
2. Inventory holding cost

(H) $= h\left\{\int_0^m I_1(t)\,dt + \int_m^{\tau} I_2(t)\,dt\right\}$

$$= h\left\{\begin{array}{l} \dfrac{\tau^{\beta+1}(\tau x_6 - x_4)}{x_1} - m^{\beta+1}\dfrac{(m x_6 - x_4)}{x_1} + \dfrac{x_2}{x_5}(\tau - m) \\ -\dfrac{\lambda m^2}{2p^{\mu}} - \dfrac{\lambda(1+\beta)}{x_3}\left(\tau^2 - m^2\right) - \dfrac{\alpha\lambda}{x_1}\left(\tau^{\beta+2} - m^{\beta+2}\right) \\ +\dfrac{m\lambda}{p^{\mu}}\left[\tau - \alpha m^{\beta}(\tau - m) + \dfrac{\alpha\left(\tau^{\beta+1} - m^{\beta+1}\right)}{\beta+1}\right] \end{array}\right\}$$

where

$x_1 = p^y(3b+2+b^2)$,

$x_2 = zx(1+b+az^b)$,

$x_3 = 2p^y(b+1)$,

$x_4 = zax(b+2)$,

$x_5 = p^y(b+1)$,

$x_6 = ax(b+1)$.

3. Purchase cost

$$(\text{P}) = c(Q-E)$$

$$= \frac{c\lambda}{p^\mu}\left[\frac{\alpha}{\beta+1}\left(\tau^{\beta+1}-m^{\beta+1}\right)-(\tau-m)\alpha m^\beta + T\left(\frac{T\delta}{2}-1\right)+\frac{\delta}{2}\tau^2 - T\delta\tau\right].$$

4. Sales revenue

$$® = p\left\{\int_0^\tau D(p)\,dt + \int_\tau^T D(p)e^{-\delta(T-t)}dt\right\} = \frac{p\lambda}{p^\mu}\left[\tau - \frac{e^{-\delta(T-\tau)}-1}{\delta}\right].$$

5. Shortage cost

$$(\text{S}) = s\int_\tau^T -I_3(t)\,dt = \frac{s\lambda}{6p^\mu}(T-\tau)^2\left(2\delta\tau - 2T\delta + 3\right).$$

6. Lost sale cost

$$(\text{L}) = c_l\int_\tau^T D(p)\left(1-e^{-\delta(T-t)}\right)dt = \frac{\lambda c_l}{\delta p^\mu}\left(e^{-\delta(T-\tau)} - \delta\tau + T\delta - 1\right).$$

7. Interest payable

(i) When $0 \le M \le m$

$$IP_1 = cI_p\left\{\int_M^m I_1(t)\,dt + \int_m^\tau I_2(t)\,dt\right\}$$

$$= cIp\left\{\begin{array}{l} \dfrac{\tau^{\beta+1}(\tau x_6 - x_4)}{x_1} - m^{\beta+1}\dfrac{(mx_6-x_4)}{x_1} + \dfrac{x_2}{x_5}(\tau-m) \\ -\dfrac{\lambda(1+b)}{x_3}\left(\tau^2-m^2\right) - \dfrac{\alpha\lambda}{x_1}\left(\tau^{\beta+2}-m^{\beta+2}\right) \\ +\dfrac{\lambda}{2p^\mu}(M-m)\left[\begin{array}{l} M+m-2\tau- \\ 2\alpha m^\beta\tau - \dfrac{2\alpha\left(\tau^{\beta+1}+\beta m^{\beta+1}\right)}{\beta+1}\end{array}\right]\end{array}\right\}.$$

(ii) When $m \le M \le \tau$

$$IP_2 = cI_p\left\{\int_M^{\tau} I_1(t)\,dt\right\} = cIp\left\{\begin{array}{l}\dfrac{\tau^{\beta+1}(\tau x_6 - x_4)}{x_1} - m^{\beta+1}\dfrac{(mx_6 - x_4)}{x_1} - \dfrac{x_2}{x_5}(M-\tau)\\ +\dfrac{\lambda(1+b)}{x_3}\left(M^2-\tau^2\right) + \dfrac{\alpha\lambda}{x_1}\left(M^{\beta+2}-\tau^{\beta+2}\right)\end{array}\right\}.$$

(iii) When $\tau \le M \le T$, $IP_3 = 0$.

8. Interest earned

(i) *When* $0 \le M \le m$

$$IE_1 = pI_e\int_0^M tD(p)\,dt = \frac{p\lambda}{2p^{\mu}} I_e M^2.$$

(ii) When $m \le M \le \tau$

$$IE_2 = pI_e\int_0^M tD(p)\,dt = \frac{p\lambda}{2p^{\mu}} I_e M^2.$$

(iii) When $\tau \le M \le T$

$$IE_3 = pI_e\left\{\int_0^{\tau} tD(p)\,dt + \int_0^{\tau}(M-\tau)D(p)\,dt\right\} = \frac{p\lambda\tau}{p^{\mu}} Ie\left(m - \frac{\tau}{2}\right).$$

The total profit/unit time $Z(\tau)$ is written as

$$Z(\tau) = \begin{cases} Z_1(\tau); 0 \le M \le m \\ Z_2(\tau); m \le M \le \tau \\ Z_3(\tau); \tau \le M \le T \end{cases}$$

$$Z_1(\tau) = \frac{R + IE_1 - O - P - H - S - L - IP_1}{T} = \frac{X_1}{T}. \tag{9}$$

Where

$$X_1 = \left(\frac{p\lambda}{p^{\mu}}\left[\tau - \frac{e^{-\delta(T-\tau)}-1}{\delta}\right]\right) + \left(\frac{p\lambda}{2p^{\mu}} IeM^2\right) - (K)$$
$$-\left(\frac{c\lambda}{p^{\mu}}\left[\frac{\alpha}{\beta+1}\left(\tau^{\beta+1}-m^{\beta+1}\right) - (\tau-m)\alpha m^{\beta} + T\left(\frac{T\delta}{2}-1\right) + \frac{\delta}{2}\tau^2 - T\delta\tau\right]\right)$$
$$-\left(h\left\{\begin{array}{l}\dfrac{\tau^{\beta+1}(\tau x_6 - x_4)}{x_1} - m^{\beta+1}\dfrac{(mx_6 - x_4)}{x_1} + \dfrac{x_2}{x_5}(\tau - m) - \dfrac{\lambda m^2}{2p^{\mu}}\\ -\dfrac{\lambda(1+\beta)}{x_3}\left(\tau^2 - m^2\right) - \dfrac{\alpha\lambda}{x_1}\left(\tau^{\beta+2} - m^{\beta+2}\right)\\ +\dfrac{m\lambda}{p^{\mu}}\left[\tau - \alpha m^{\beta}(\tau - m) + \dfrac{\alpha\left(\tau^{\beta+1}-m^{\beta+1}\right)}{\beta+1}\right]\end{array}\right\}\right)$$

$$-\left(\frac{s\lambda}{6p^{\mu}}(T-\tau)^2(2\delta\tau-2T\delta+3)\right)-\left(\frac{\lambda c_l}{\delta p^{\mu}}\left(e^{-\delta(T-\tau)}-\delta\tau+T\delta-1\right)\right)$$

$$-\left(cIp\left\{\begin{array}{l}\dfrac{\tau^{\beta+1}(\tau x_6-x_4)}{x_1}-m^{\beta+1}\dfrac{(mx_6-x_4)}{x_1}+\dfrac{x_2}{x_5}(\tau-m)\\ -\dfrac{\lambda(1+b)}{x_3}\left(\tau^2-m^2\right)-\dfrac{\alpha\lambda}{x_1}\left(\tau^{\beta+2}-m^{\beta+2}\right)\\ +\dfrac{\lambda}{2p^{\mu}}(M-m)\left[\begin{array}{l}M+m-2\tau-2\alpha m^{\beta}\tau\\ -\dfrac{2\alpha\left(\tau^{\beta+1}+\beta m^{\beta+1}\right)}{\beta+1}\end{array}\right]\end{array}\right\}\right)$$

$$Z_2(\tau)=\frac{R+IE_2-O-P-H-S-L-IP_2}{T}=\frac{X_2}{T}. \quad (10)$$

Where

$$X_2=\left(\frac{p\lambda}{p^{\mu}}\left[\tau-\frac{e^{-\delta(T-\tau)}-1}{\delta}\right]\right)+\left(\frac{p\lambda}{2p^{\mu}}IeM^2\right)$$

$$-(K)-\left(\frac{c\lambda}{p^{\mu}}\left[\begin{array}{l}\dfrac{\alpha}{\beta+1}\left(\tau^{\beta+1}-m^{\beta+1}\right)-(\tau-m)\alpha m^{\beta}\\ +T\left(\dfrac{T\delta}{2}-1\right)+\dfrac{\delta}{2}\tau^2-T\delta\tau\end{array}\right]\right)$$

$$-\left(h\left\{\begin{array}{l}\dfrac{\tau^{\beta+1}(\tau x_6-x_4)}{x_1}-m^{\beta+1}\dfrac{(mx_6-x_4)}{x_1}+\dfrac{x_2}{x_5}(\tau-m)\\ -\dfrac{\lambda m^2}{2p^{\mu}}-\dfrac{\lambda(1+\beta)}{x_1}\left(\tau^2-m^2\right)-\dfrac{\alpha\lambda}{x_1}\left(\tau^{\beta+2}-m^{\beta+2}\right)\\ +\dfrac{m\lambda}{p^{\mu}}\left[\tau-\alpha m^{\beta}(\tau-m)+\dfrac{\alpha\left(\tau^{\beta+1}-m^{\beta+1}\right)}{\beta+1}\right]\end{array}\right\}\right)$$

$$-\left(\frac{s\lambda}{6p^{\mu}}(T-\tau)^2(2\delta\tau-2T\delta+3)\right)$$

$$-\left(\frac{\lambda c_l}{\delta p^{\mu}}\left(e^{-\delta(T-\tau)}-\delta\tau+T\delta-1\right)\right)$$

$$-\left(cIp\left\{ \begin{array}{l} \dfrac{\tau^{\beta+1}\left(\tau x_6 - x_4\right)}{x_1} - m^{\beta+1}\dfrac{\left(mx_6 - x_4\right)}{x_1} - \dfrac{x_2}{x_5}\left(M-\tau\right) \\ +\dfrac{\lambda\left(1+b\right)}{x_3}\left(M^2-\tau^2\right) + \dfrac{\alpha\lambda}{x_1}\left(M^{\beta+2}-\tau^{\beta+2}\right) \end{array} \right\} \right)$$

$$Z_3\left(\tau\right) = \frac{R + IE_3 - O - P - H - S - L - IP_3}{T} = \frac{X_3}{T}. \tag{11}$$

Where

$$X_3 = \left(\frac{p\lambda}{p^{\mu}}\left[\tau - \frac{e^{-\delta(T-\tau)}-1}{\delta}\right]\right) + \left(\frac{p\lambda\tau}{p^{\mu}} Ie\left(m - \frac{\tau}{2}\right)\right)$$

$$-\left(K\right) - \left(\frac{c\lambda}{p^{\mu}}\left[\begin{array}{l} \dfrac{\alpha}{\beta+1}\left(\tau^{\beta+1}-m^{\beta+1}\right) - \left(\tau - m\right)\alpha m^{\beta} \\ +T\left(\dfrac{T\delta}{2}-1\right) + \dfrac{\delta}{2}\tau^2 - T\delta\tau \end{array} \right]\right)$$

$$-\left(h\left\{ \begin{array}{l} \dfrac{\tau^{\beta+1}\left(\tau x_6 - x_4\right)}{x_1} - m^{\beta+1}\dfrac{\left(mx_6 - x_4\right)}{x_1} + \dfrac{x_2}{x_5}\left(\tau - m\right) \\ -\dfrac{\lambda m^2}{2p^{\mu}} - \dfrac{\lambda\left(1+\beta\right)}{x_3}\left(\tau^2 - m^2\right) - \dfrac{\alpha\lambda}{x_1}\left(\tau^{\beta+2}-m^{\beta+2}\right) \\ +\dfrac{m\lambda}{p^{\mu}}\left[\tau - \alpha m^{\beta}\left(\tau - m\right) + \dfrac{\alpha\left(\tau^{\beta+1}-m^{\beta+1}\right)}{\beta+1}\right] \end{array} \right\}\right)$$

$$-\left(\frac{s\lambda}{6p^{\mu}}\left(T-\tau\right)^2\left(2\delta\tau - 2T\delta + 3\right)\right)$$

$$-\left(\frac{\lambda c_l}{\delta p^{\mu}}\left(e^{-\delta(T-\tau)} - \delta\tau + T\delta - 1\right)\right)$$

4. Solution Procedure

The aim of this article is to maximize total profit. The following condition must be fulfilled by $Z_i\left(\tau\right)$ for maximization:

$$\frac{dZ_i\left(\tau\right)}{d\tau} = 0,\ i = 1,2,3 \tag{12}$$

Solving the equation (12) for τ, we get optimal value τ^* of τ, for which $\frac{d^2 Z_i(\tau)}{d\tau}|_{\tau=\tau^*} < 0 \quad ; i = 1,2,3$

5. Numerical Examples

Example 5.1

When $0 \le M \le m$

$K = 10000, h = 10, c = 20, p = 100, m = 0.08, M = 0.05, \alpha = 0.125,$
$\beta = 2, \lambda = 6000, \mu = 0.9, I_e = 0.20, I_p = 0.25, T = 1, \delta = .7, s = 60, c_l = 70$

Putting these values in equation (9), we obtain the optimal solutions

$\tau_1^* = 0.6924$

$Z_1^* = 2371$

$Q_1^* = 67.1094$

Example 5.2

When $m \le M \le \tau$

$K = 10000, h = 10, c = 20, p = 100, m = 0.05, M = 0.07, \alpha = 0.125,$
$\beta = 2, \lambda = 6000, \mu = 0.9, I_e = 0.20, I_p = 0.25, T = 1, \delta = .7, s = 60, c_l = 70$

Putting these values in equation (10), we obtain the optimal solutions

$\tau_2^* = 0.6930$

$Z_2^* = 2363.2$

$Q_2^* = 67.1989$

Example 5.3

When $\tau \le M \le T$

$K = 10000, h = 10, c = 20, p = 100, m = 0.05, M = 0.9, \alpha = 0.125,$
$\beta = 2, \lambda = 6000, \mu = 0.9, I_e = 0.20, I_p = 0.25, T = 1, \delta = .7, s = 60, c_l = 70$

Putting these values in equation (11), we obtain the optimal solutions

$\tau_3^* = 0.7335$

$Z_3^* = 1530$

$Q_3^* = 71.2940$

To solve the above examples, MATLAB software is used.

6. Concavity of Profit Functions

Figures 1, 2, and 3 depict the concavity of the profit functions, and this finding corresponds to the theoretical idea of a profit functions with concavity.

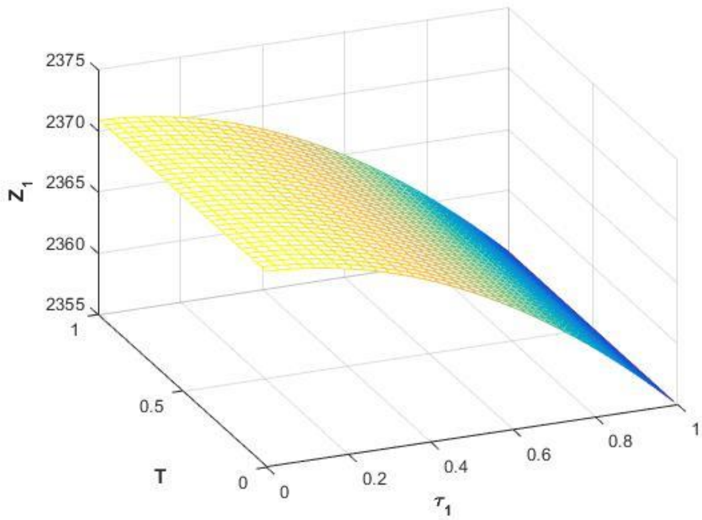

Figure 1. Concavity of profit function $Z_1(\tau)$

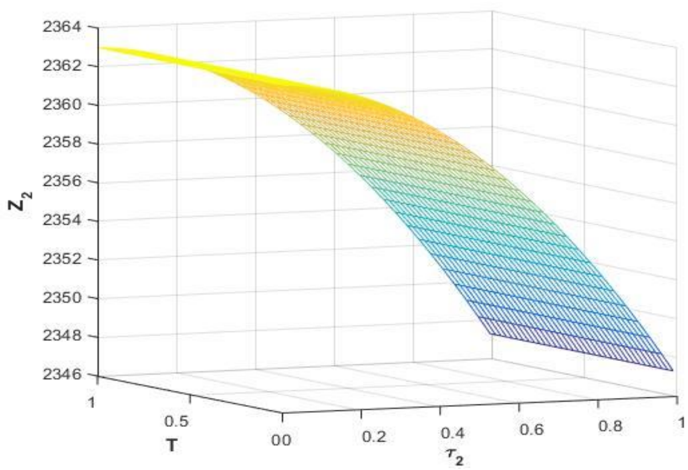

Figure 2. Concavity of profit function $Z_2(\tau)$

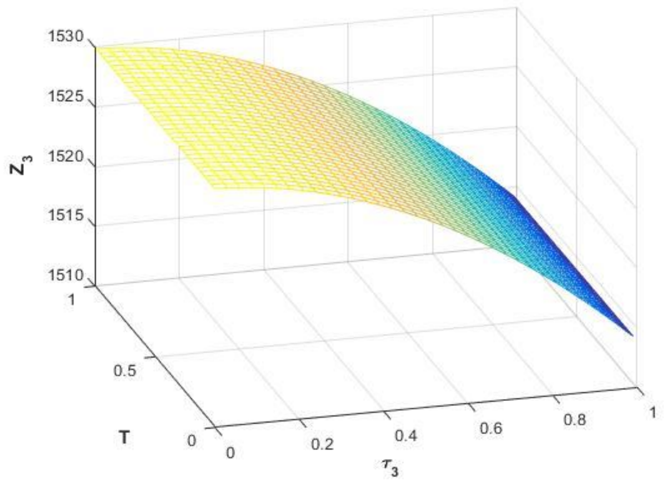

Figure 3. Concavity of profit function $Z_3(\tau)$

7. Sensitivity Analysis and Observations

For sensitivity analysis, we have used the preceding cases. The impacts of parameter modifications on optimal values of $Z*, \tau*$ and $Q*$ in this segment have been studied. The results are summarized in Tables 2, 3 and 4.

Observations and Managerial Implications

From Table 2, Table 3 and Table 4, we observe that

- As we increase the parameter h by 10% and 20% we observe that the total profit Z^* remains almost constant and the optimal values of τ^* and $Q*$ decrease. We can therefore suggest to the firm that they are free to accept any type of lot as long as the profit remains constant in accordance with the above parameters.
- Increasing the value of K by 10% and 20% increases the value of Z^* very rapidly but the optimal values of τ^* and $Q*$ decreases. In order to increase profits, a company will increase the value of K parameter.
- Enlarge of c by 10% and 20% results increase in Z^* and decrease in $Q*$ while it drops the value of τ^* very sharply. In order to increase profits, a company will increase the value of c parameter.
- When p increases by 10% and 20%, it makes a decrease in Z^* and an increase in τ^* but it causes the value of total inventory $Q*$ to drop very rapidly. In order to increase profits, a company will decrease the value of p parameter.

- When we increase the parameters s and c_l by 10% and 20% we see that the optimal values of τ^* and $Q*$ show the same behavior (they increase) while the total profit remains unchanged. We can therefore suggest to the firm that they are free to accept any type of lot as long as the profit remains constant in accordance with the above parameters. Figures 4, 5, and 6 demonstrate the effect of various factors on profit functions.

Table 2. Change in $\tau*, Q_1^*$ and Z_1^* with respect to parameters

Parameters	%change in parameters	$\tau*$	Q_1^*	Z_1^*
h	-20	0.7013	68.0065	2323.9
	-10	0.6968	67.5528	2347.6
	+10	0.6881	66.6765	2394.1
	+20	0.6837	66.2338	2416.9
K	-20	0.6924	67.1094	371.041
	-10	0.6924	67.1094	1371.0
	+10	0.6924	67.1094	3371.0
	+20	0.6924	67.1094	4371.0
c	-20	0.7415	72.0749	2168.0
	-10	0.7169	69.5820	2274.3
	+10	0.6680	64.6568	2458.4
	+20	0.6437	62.2236	2536.6
p	-20	0.6668	78.8904	2922.2
	-10	0.6801	72.4239	2624.5
	+10	0.7037	62.6383	2151.2
	+20	0.7142	58.8202	1957.5
s	-20	0.6739	65.2490	2322.3
	-10	0.6834	66.2037	2347.3
	+10	0.7010	67.9762	2393.6
	+20	0.7092	68.8038	2415.0
c_l	-20	0.6749	65.3494	2327.7
	-10	0.6839	66.2539	2350.0
	+10	0.7004	67.9157	2391.1
	+20	0.7080	68.6826	2410.1

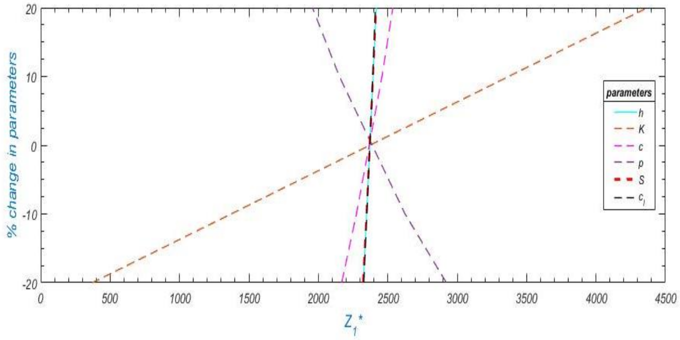

Figure 4. Changes in Z_1^* with respect to parameters

Table 3. Change in $\tau*$, Q_2^* and Z_2^* with respect to parameters

Parameters	%change In parameters	$\tau*$	Q_2^*	Z_2^*
h	-20	0.7018	68.0864	2316.0
	-10	0.6974	67.6425	2339.8
	+10	0.6886	66.7557	2386.4
	+20	0.6843	66.3228	2409.2
K	-20	0.6930	67.1989	363.2157
	-10	0.6930	67.1989	1363.2
	+10	0.6930	67.1989	3363.2
	+20	0.6930	67.1989	4363.2
c	-20	0.7420	72.1570	2161.0
	-10	0.7175	69.6729	2266.8
	+10	0.6686	64.7449	2450.3
	+20	0.6444	62.3204	2528.2
p	-20	0.6674	78.9980	2913.5
	-10	0.6807	72.5215	2616.3
	+10	0.7043	62.7211	2143.7
	+20	0.7148	58.8972	1950.3
s	-20	0.6745	65.3374	2314.6
	-10	0.6840	66.2926	2339.6
	+10	0.7016	68.0662	2385.7
	+20	0.7097	68.8842	2407.0
c_l	-20	0.6755	65.4379	2320.0
	-10	0.6845	66.3430	2342.2
	+10	0.7010	68.0056	2383.2
	+20	0.7086	68.7730	2402.1

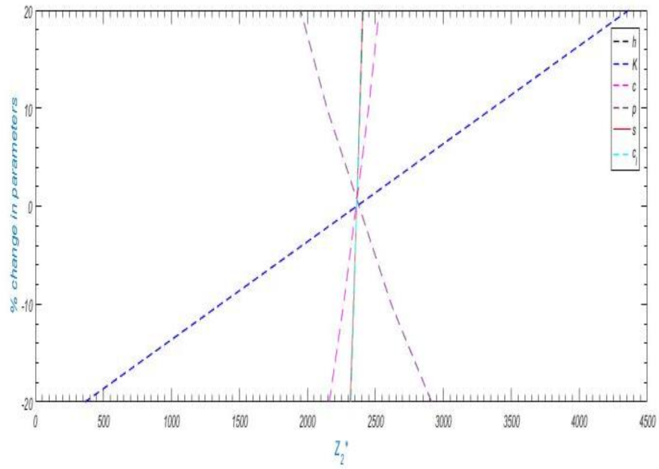

Figure 5. Changes in Z_2^* with respect to parameters

Table 4. Change in τ^*, Q_3^* and Z_3^* with respect to parameters

Parameters	%change In parameters	τ^*	Q_3^*	Z_3^*
h	-20	0.7418	72.1367	1477.1
	-10	0.7376	71.7101	1503.7
	+10	0.7294	70.8782	1556.0
	+20	0.7253	70.4626	1581.7
K	-20	0.7335	71.2940	-470.0094
	-10	0.7335	71.2940	529.9906
	+10	0.7335	71.2940	2530.0
	+20	0.7335	71.2940	3530.0
c	-20	0.7741	75.4279	1322.1
	-10	0.7538	73.3572	1429.7
	+10	0.7132	69.2379	1623.0
	+20	0.6930	67.1989	1708.8
p	-20	0.7092	84.1433	2092.8
	-10	0.7220	77.1037	1789.2
	+10	0.7440	66.4120	1304.7
	+20	0.7536	62.2385	1105.9
s	-20	0.7188	69.8044	1492.7
	-10	0.7263	70.5639	1511.8
	+10	0.7403	71.9843	1547.3
	+20	0.7469	72.6551	1563.8
c_l	-20	0.7200	69.9259	1497.3
	-10	0.7269	70.6247	1514.1
	+10	0.7398	71.9335	1545.2
	+20	0.7457	72.5331	1559.7

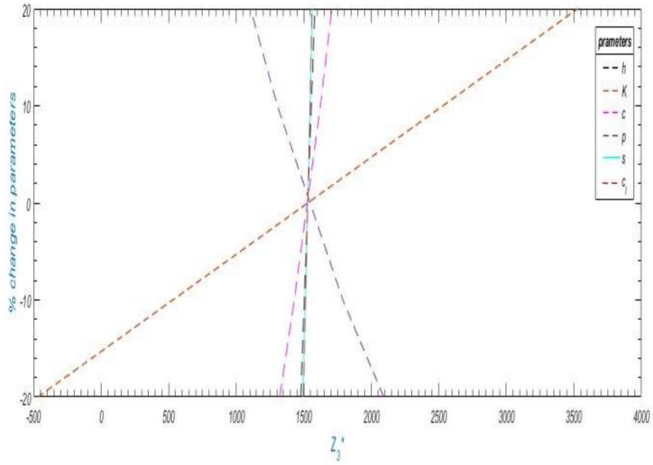

Figure 6. Changes in Z_3^* with respect to parameters

8. Conclusion and Future Scope

In present article Weibull rate of deterioration is influenced by trade credit and demand depends on selling price. The model is assessed with the variable time and optimized. Stagnant shortages are acceptable with backlogged allowances.

Numerical examples and sensitivity analysis illustrate the constructed model. Our findings demonstrate:

- Increasing holding cost increases the overall profit.
- The optimal overall profit grows as the shortage costs increase.

The supplied model can be used to keep inventory of things that do not perish quickly, such as electronic products, and fashion items. In retail trading, the approach is useful for optimizing unit time profit when partial backlogging occurs. Future work in this area will examine freight charges and other factors. This is also applicable when all the parameters are clear and precise, but if there is any uncertainty in the future, we can use fuzzy mathematics to deal with the situation.

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