Operational Research in Engineering Sciences: Theory and Applications
Vol. 5, Issue 1, 2022, pp. 90-106
ISSN: 2620-1607
eISSN: 2620-1747
DOI: https://doi.org/10.31181/oresta180222016p

ESTIMATING RUBBER COVERED CONVEYOR BELTING CURE TIMES USING MULTIPLE SIMULTANEOUS OPTIMISATIONS ENSEMBLE

Domingo Pavolo*, Delson Chikobvu

Department of Mathematical Statistics and Actuarial Science, Faculty of Natural and Agricultural Sciences, University of the Free State, South Africa

Received: 15 July 2021
Accepted: 08 November 2021
First online: 18 February 2022

Research paper

Abstract: *Multiple response surface methodology (MRSM) has been the favorite method for optimizing multiple response processes though it has two weaknesses which challenge the credibility of its solutions. The first weakness is the use of experimentally generated small sample size datasets, and the second is the selection, using classical model selection criteria, of single best models for each response for use in simultaneous optimization to obtain the optimum or desired solution. Classical model selection criteria do not always agree on the best model resulting in model uncertainty. The selection of single best models for each response for simultaneous optimization loses information in rejected models. This work proposes the use of multiple simultaneous optimizations to estimate multiple solutions that are ensembled in solving a conveyor belting cure time problem. The solution is compared with one obtained by simultaneous optimization of single best models for each response. The two results were different. However, results show that it is possible to obtain a more credible solution through ensembling of solutions from multiple simultaneous optimizations.*

Key words: *Multiresponse surface methodology, ensembling, credibility of results, solution uncertainty, small sample size problems, simultaneous optimisation*

1. Introduction

The mining industry is at the heart of the Southern African Development Community (SADC) region's economic activities and development. Conveyor belts are critical for conveyance of bulk ore over distances and through various stages of processing. The regional product quality standard minimum requirements for general purpose rubber covered conveyor belts for the mining industry were amended. The component adhesion requirement was increased from 5N/mm to 7N/mm. However, key customers were insisting on a minimum of 10N/mm adhesion and 60^0 Shore A rubber compound cover hardness. After redesigning of the specifications of rubber

* Corresponding author.
2017476603@ufs4free.ac.za (D. Pavolo), Chikobvu@ufs.ac.za (D. Chikobvu)

compounds, a client manufacturing company required optimum cure times (T_c), which would ensure a minimum of 12N/mm adhesion and 60^0 Shore A hardness, to be determined for the vulcanisation of different conveyor belt thicknesses (R_t) for use in shop-floor work instructions.

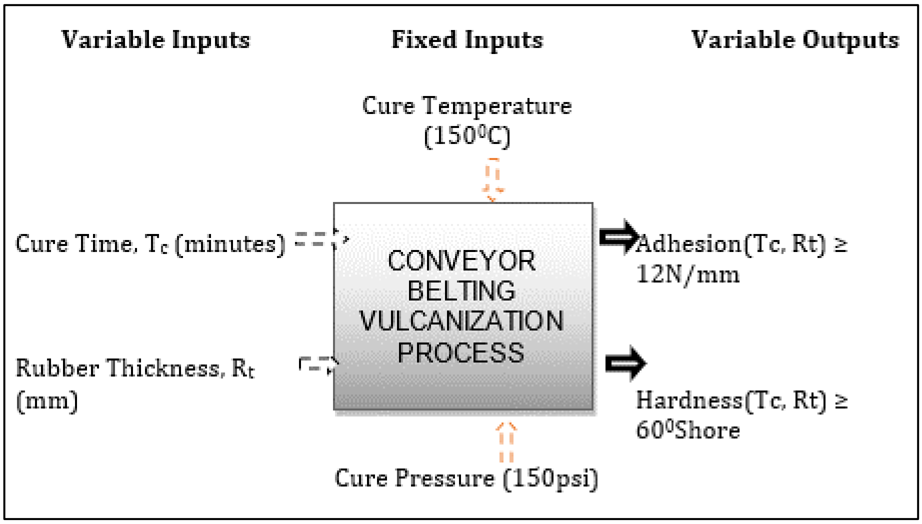

Figure 1. Illustrating the conveyor belting vulcanization process problem

Given the illustrated process in Figure 1, it was thus intended to estimate credible cure times (T_c) for given rubber thicknesses (R_t), as shown in Table 1 below.

Table 1. Showing the expected solution

$\mathbf{R_t}$**(mm)**	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$\mathbf{T_c}$**(min.)**	$\mathbf{T_7}$	$\mathbf{T_8}$	$\mathbf{T_9}$	.	.	.	.	.	.	.	.	.	.	$\mathbf{T_{20}}$

For manufacturers, optimum cure time (T_c) is critical for product quality and production process productivity. Good adhesion between conveyor belt components (covers, skims and reinforcement fabrics) ensures that they do not separate during heavy duty operations in the mines. The top and bottom rubber covers protect the reinforcement fabrics, therefore hardness is essential for wear resistance to the abrasive mining operational environments. The separation of belting components during heavy duty operation and excessive rubber cover wear are the two major failures of conveyor belting during mining operations. Increasing adhesion between belting components and cover hardness ensures more belting life and therefore lower mining operational costs. Beyond just providing a solution to the client company, the study sought to recommend to the conveyor belt manufacturing industry a credible and efficient tool for converting changes in product standard requirements to production process input parameters. Quality and productivity are critical manufacturing industry competitive factors and the speed of successfully implementing change is critical in any industry as it gives first mover advantages. This work is of interest, therefore, to operations researchers, industrial engineers and business management strategists in the conveyor belting manufacturing industry.

The authors could not find anywhere in literature were cure times per given conveyor belt thickness were estimated for the vulcanisation process of general purpose rubber covered conveyor belting. Literature only gives methodologies for estimating the cure times of different rubber compounds. A rubber covered conveyor belt is constructed from a rubber cover compound, a rubber skim compound (which provides the bonding strength between components) and reinforcement fabric. These components individually contribute to the overall vulcanization time due to different heat conductivities.

In this work, the sufficiency of the contemporary multiple response surface methodology (MRSM) framework in estimating a credible solution to the problem was critiqued and two major weaknesses identified. Firstly, it is statistically difficult to extract credible process information from small sample size MRSM datasets. Secondly, the selection of single best models for each response for simultaneous optimization is prone to (1) loss of information in the rejected response models and (2) model uncertainty as model selection criteria do not always agree on the best model. This work proposes the use of multiple simultaneous optimizations to estimate multiple solutions that are then ensembled, to account for the two weaknesses in the MRSM framework, in solving the conveyor belting cure time problem. Results suggest that the proposed ensemble system can provide a credible solution to the problem.

2. Literature Review

2.1. Rubber technology perspective

A number of techniques have been proposed in rubber technology literature to estimate the cure time of rubber products such as nuclear magnetic resonance spectroscopy, differential scanning calorimetry, dynamic mechanical analysis, adaptive neuro-fuzzy inference systems, equivalent cure concept, and artificial neural networks and finite element analysis (Gatos and Karger-Kocsis, 2004; Karaagâc et al., 2011; Gough, 2017). The accepted basic tool of cure time estimation is the rheograph (Appendix A) which shows how the shear strength of a sample of rubber changes with time during vulcanisation. The rheograph does not consider the case where there are different layers of constituent rubber compounds and other materials such as conveyor reinforcement fabric (nylon and/or polyester). The conveyor belting case requires a multiple factor and multiple response simultaneous optimisation solution methodology, hence the shift to multiple response surface methodology (MRSM).

2.2 Multiple response surface methodology (MRSM)

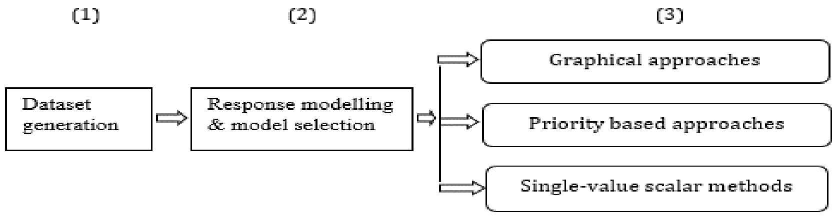

Figure 2. Showing the contemporary MRSM framework

MRSM is an important tool for optimising manufacturing processes in industry. It is a collection of mathematical and statistical techniques that are useful for the modelling and analysis of problems in which multiple responses are influenced by several variables and the objective of the analysis is to optimize the responses by determining the best settings of the input variables (Myers et al., 2016; Hejazi et al., 2017; Khuri, 2017). In Figure 2, the MRSM dataset generation stage, stage (1), involves designing and running screening and MRSM experiments (Myers et al., 2016). The stage (3) are the solution methodologies for estimating the operating conditions that optimise all the responses or at least keep them in desired ranges.

MRSM experimental designs are constructed to eliminate or minimise correlations between chosen variables which allows independent estimation of variable effects and their potential interactions (Myers et al., 2016; Khuri, 2017; Mäkelä, 2017). Examples include central composite designs (CCD), Box-Behnken, Orthogonal Arrays, Placket-Burman, and computer-generated optimal designs (Myers et al., 2016; Khuri, 2017; Alhorn et al., 2019). The strength of MRSM is in efficient experimental designs (Khuri, 2017). However, statistically, it is difficult to extract credible population information from small sample size datasets (Rawlings et al., 1998; Yuan and Yang, 2005; Xu and Goodacre, 2018; Jenkins and Quintana-Ascencio, 2020). This is the first weakness that requires to be accounted for to obtain credible solutions.

Optimisation in MRSM is multi-objective in nature, and is performed after regression modelling and model selection of single "best" models for each response (Myers et al., 2016; Khuri, 2017). MRSM solution methodologies rely heavily on classical model selection criteria for choosing the best model for each response for simultaneous optimisation. This the second weakness of the contemporary MRSM framework. Problems associated with the contemporary MRSM contextual framework are presented in Figure 3 below.

#	WEAKNESS	IMPLICATIONS
1	Small sample size dataset	MS criteria inefficiency (Hurvich & Tsai, 1989) Model over-& underfitting (Burnham & Anderson, 2002) Modelling credibility (Rawlings et al. 1998)
2	Selection of single best model for each response	Model selection bias (Miller, 2002; Lukacs et al. 2010) Model & criteria uncertainty (Schomaker & Heumann, 2018) Dataset uncertainty (Myers et al, 2016) Loss of information (Burnham & Anderson, 2002) Simultaneous optimisation compromise (Myers et al, 2016)

Figure 3. Problems related to the current MRSM contextual framework

In this paper, the authors proposed and utilised a novel solution methodology that accounted for the two weaknesses to obtain a credible solution.

3. Solution Methodology

The MRSM dataset generation for the rubber covered conveyor belting problem is explained in detail in Pavolo and Chikobvu (2020). The dataset was adopted as is and is shown in Table 2.

Table 2. The two-factor CCD experiment MRSM dataset

Run	T (min.)	R_t (mm)	Ave. Hardness (^{0}shore A)	Ave. Adhesion(N/mm)
1	16	7.2	60	10.60
2	30	7.2	63	13.34
3	16	22.8	53	6.20
4	30	22.8	61	12.10
5	23	15	58	11.80
6	23	15	58	12.10
7	13	15	44	6.5
8	33	15	63	13.30
9	23	4	63	13.30
10	23	26	56	3.50
11	23	15	58	12.20
12	23	15	57	12.30
13	23	15	58	12.10

Ensemble-based systems have been recommended for small sample size situations in literature (Kittler, 1998; Burnham and Anderson, 2002; Polikar, 2006; Yang et al., 2016; Ahangi et al., 2019). An ensemble system was considered the best option for accounting for the weaknesses of the contemporary MRSM framework and delivering a credible solution. The solution methodology is summarised in Figure 4.

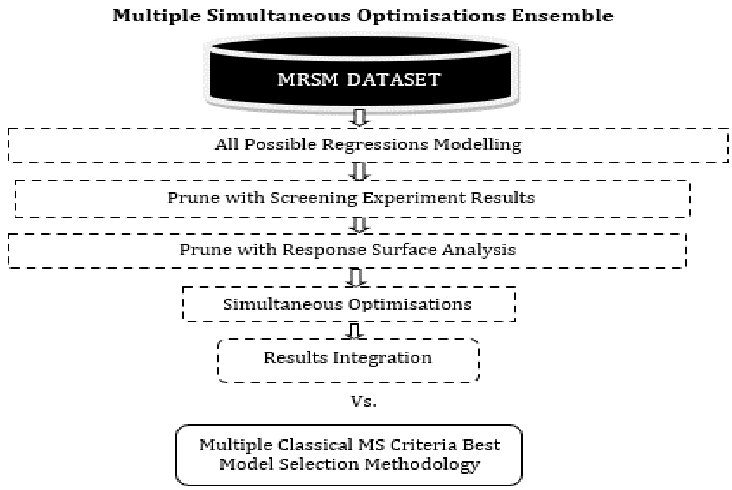

Figure 4. Showing the solution methodology flow diagram

At response surface analysis, the one hardness model with response surface conformity was adopted as is from Pavolo and Chikobvu (2020). However, in this work, all the adhesion response models were assumed to be response surface conforming.

The estimated cure time solution was compared with one from a methodology structured after the contemporary MRSM contextual framework. Figure 5 shows the strategies used in the solution methodology to deal with each problem listed in Figure 3.

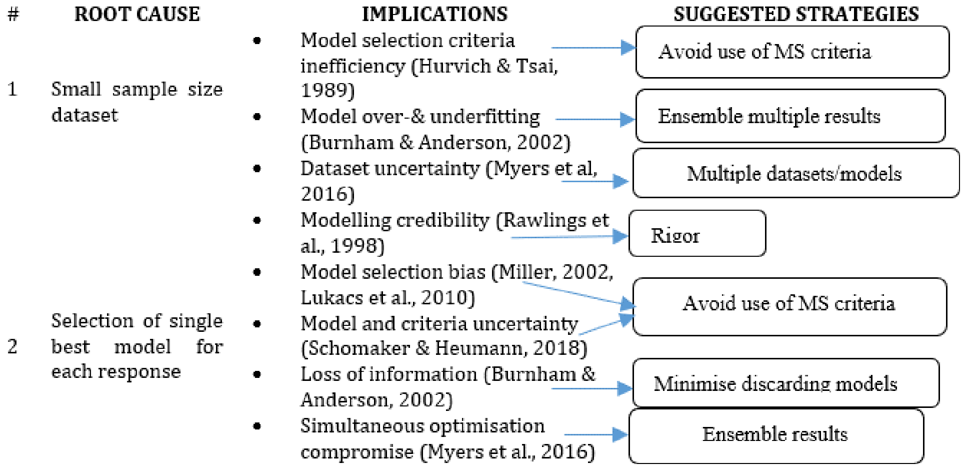

Figure 5. Showing the strategies employed to deal with problems

Figure 6 summarises the problems of the contemporary solution methodologies and presents the advantages of the ensembling methodology from literature.

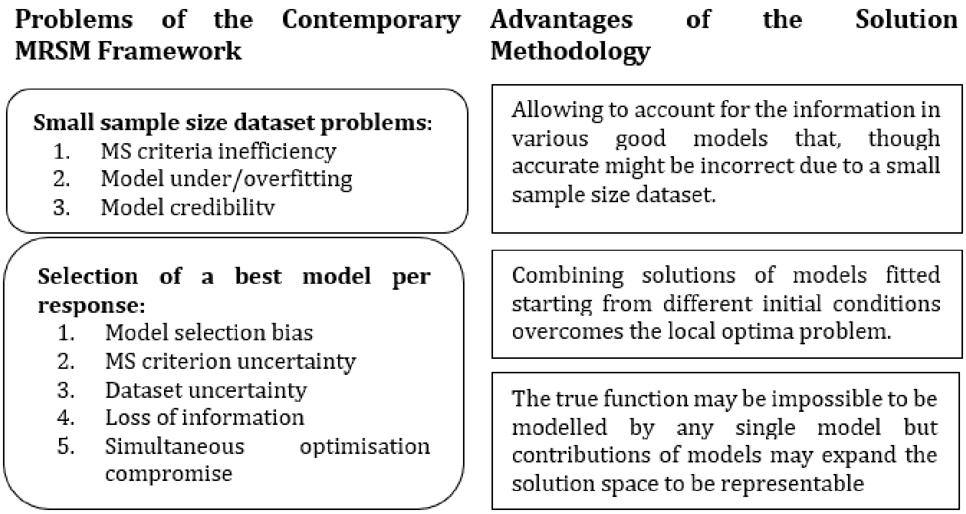

Figure 6. The advantages of the solution methodology vs. problems of contemporary MRSM

The formulae used for computation of theoretical accuracy are shown below. Validation was computed against the minimum targeted response values as the sample size was too small to be split into a fitting set and a validation set.

$MSPE_{val(min.)}$: The validation minimum mean squared prediction error ($MSPE_{val(min.)}$) of a response model measures the minimum squared deviation of the model predictions from the targeted. $MSPE_{val(min.)}$ is given below for a sample size *n*.

$$MSPE_{val(min.)} = \frac{\sum_{i=1}^{n}(Y_i - Y_T)^2}{n}, \quad (1)$$

where Y_i is the i^{th} estimated response, Y_T is the a response value.

$MSPE_{simul}$: The mean squared prediction error at simultaneous optimisation ($MSPE_{simul}$) of a response model in an adhesion – hardness model pair indicates the mean squared deviation of the model predictions from the targeted and is given below for a sample size *n* as:

$$MSPE_{simul} = \frac{\sum_{i=1}^{n}(Y_i - Y_T)^2}{n}, \quad (2)$$

where Y_i is the i^{th} estimated response value at simultaneous optimisation.

The $MSPE_{simul}$ bias-variance decomposition estimates were integrated by arithmetic averaging to estimate the bias-variance-covariance decomposition of the $MSPE_{simul}$ of the ensembled results (Geman et al., 1992; Ueda and Nakano, 1996).

$$\mathrm{MSE}(f) = \mathrm{bias}(f)^2 + \mathrm{var}(f) \quad (3)$$

And the expected ensemble MSE is,

$$E\{\mathrm{MSE}(f ens.)\} = \mathrm{Bias}^2 + (\frac{1}{k}) \times \mathrm{Variance} + (1 - \frac{1}{k}) \times \mathrm{Covariance} \quad (4)$$

Where *k* is the number of base models in the ensemble.

Prediction Accuracy Compromise: Define Prediction Accuracy Compromise (PAC) as the difference between $MSPE_{simul}$ and the $MSPE_{val(min.)}$ of a response model. PAC gives a picture of how models compromise their accuracy in the process of simultaneous optimisation. Then % PAC will be the percentage change in $MSPE_{val(min.)}$ to achieve simultaneous optimisation.

$$\% \mathrm{PAC} = 100\% \times (MSPE_{simul} - MSPE_{val(min.)}) / MSPE_{val(min.)} \quad (5)$$

Relative accuracy: The Relative Accuracy is computed for each base model in the ensemble relative to the ensemble result and is given by:

$$\text{Relative Accuracy} = \frac{\textit{Number of Base Model or Ensemble Correct Predictions}}{\textit{Total Number of Instances}} \quad (6)$$

4. Results

4.1 All possible regression modelling results

Tables 3 and 4 below show the all possible ordinary least squares (OLS) regression models for both responses after removal of response models that did not conform to the recommendations of the screening experiment at dataset generation.

Table 3. The twenty-five OLS adhesion all possible regression response models

MODEL	β_0	β_1	β_2	β_{12}	β_{11}	β_{22}
$T_c.R_t$	12.2600			-0.0039		
T_c, R_t	7.9500	0.3244	-0.3127			
T_c, $T_c.R_t$	3.2600	0.5100		-0.0124		
T_c, Rt^2	6.1800	0.3244				-0.0111
R, $T_c.R_t$	15.4100		-0.7910	0.0208		
R_t, T_c^2	11.6700		-0.3127		0.0067	
$T_c.R_t$, T_c^2	8.9600			-0.0119	0.0105	
$T_c.R_t$, R_t^2	10.4970			0.0203		-0.0258
T_c^2, R_t^2	9.9100				0.0066	-0.0111
T_c, R_t, $T_c.R_t$	12.9400	0.1070	-0.6460	0.0145		
T_c, R_t, T_c^2	2.4100	0.8350	-0.3127		-0.0111	
T_c, R_t, R_t^2	3.6100	0.3244	0.3800			-0.0231
T_c, $T_c.R_t$, T_c^2	-2.2800	1.0200		-0.0124	-0.0111	
T_c, $T_c.R_t$, R_t^2	9.1400	0.0910		0.0156		-0.0224
T_c, T_c^2, R_t^2	-0.2500	0.9190			-0.0129	-0.0112
R_t, $T_c.R_t$, T_c^2	15.2400		-0.7710	0.0199	0.0003	
R_t, $T_c.R_t$, R_t^2	11.0800		-0.0980	0.0208		-0.0231
R_t, T_c^2, R_t^2	7.5200		0.3580		0.0066	-0.0224
$T_c.R_t$, T_c^2, R_t^2	10.3900			0.0189	0.0005	-0.0249
T_c, R_t, $T_c.R_t$, T_c^2	7.4000	0.6180	-0.6460	0.0145	-0.0111	
T_c, R_t, $T_c.R_t$, R_t^2	8.6100	0.1070	0.0470	0.0145		-0.0231
T_c, R_t, T_c^2, R_t^2	-4.2500	1.0210	0.4300		-0.0151	-0.0248
T_c, $T_c.R_t$, T_c^2, R_t^2	1.9500	0.7590		0.0168	-0.0149	-0.0234
R_t, $T_c.R_t$, T_c^2, R_t^2	11.2100		-0.1130	0.0215	-0.0003	-0.0232
T_c, R_t, $T_c.R_t$, T_c^2, R_t^2	0.7400	0.8040	0.0970	0.0145	-0.0151	-0.0248

Table 4. The twenty-five OLS hardness all possible regression response models

MODEL	β_0	β_1	β_2	β_{12}	β_{11}	β_{22}
$T_c.R_t$	56.1800			0.0040		
T_c, R_t	48.4600	0.5130	-0.1800			
T_c, $T_c.R_t$	45.7500	0.6040		0.0061		
T_c, Rt^2	46.5300	0.5130				-0.0030
R, $T_c.R_t$	60.2500		-0.9610	0.0339		
R_t, T_c^2	54.8400		-0.1800		-0.0097	
$T_c.R_t$, T_c^2	52.8900			0.0045	-0.0111	
$T_c.R_t$, R_t^2	55.0800			0.0209		-0.0181
T_c^2, R_t^2	52.9000				0.0097	-0.0030
T_c, R_t, $T_c.R_t$	57.5000	0.0320	-0.7180	0.0321		
T_c, R_t, T_c^2	18.0000	3.3200	-0.1800		-0.0610	
T_c, R_t, R_t^2	57.5100	0.5130	-1.6290			0.0483
T_c, $T_c.R_t$, T_c^2	15.3000	3.4100		-0.0061	-0.0610	
T_c, $T_c.R_t$, R_t^2	41.9300	0.8760		-0.0242		0.0146
T_c, T_c^2, R_t^2	15.9000	3.3400			-0.06160	-0.0034
R_t, $T_c.R_t$, T_c^2	65.0600		-1.4960	0.0572	-0.00860	

R_t, $T_c.R_t$, $R_t{}^2$	69.3100		-2.4090	0.0339		0.0483
R_t, $T_c{}^2$, $R_t{}^2$	64.0100		-1.6610		0.0098	0.0494
$T_c.R_t$, $T_c{}^2$, $R_t{}^2$	52.6600			-0.0094	0.0127	0.0039
T_c, R_t, $T_c.R_t$, $T_c{}^2$	29.1000	2.8400	-0.9180	0.0321	-0.0610	
T_c, R_t, $T_c.R_t$, $R_t{}^2$	68.6000	0.0320	-2.3660	0.0321		0.0483
T_c, R_t, $T_c{}^2$, $R_t{}^2$	13.4000	3.5300		-0.0196	-0.0592	0.0108
T_c, $T_c.R_t$, $T_c{}^2$, $R_t{}^2$	29.4200	3.0020	-1.4500		-0.0541	0.0423
R_t, $T_c.R_t$, $T_c{}^2$, $R_t{}^2$	73.31000		-2.8470	0.0540	-0.0074	0.0048
T_c, R_t, $T_c.R_t$, $T_c{}^2$, $R_t{}^2$	40.5000	2.5210	-2.1870	0.0321	-0.0541	0.0423

Hardness response model [T_c, R_t, $T_c.R_t$, $T_c{}^2$] was the only hardness model with a conforming response surface.

4.2. Simultaneous optimisation results

Table 5 shows the simultaneous optimisation of the adhesion-hardness model pair [$T_c.R_t$, $R_t{}^2$] - [T_c, R_t, $T_c.R_t$, $T_c{}^2$] using an Excel spreadsheet tool. The rest of the adhesion response models were similarly optimised with the same hardness model.

Table 5. Showing simultaneous optimisation on an Excel spreadsheet

T_c	R_t		Adhesion			Hardness	
(min.)	(mm)		[T*R_t, $R_t{}^2$]	e^2		[T, R_t, T* R_t, T^2]	e^2
21	7	12	12.2169	0.0470	60	60.1317	0.0173
22	8	12	12.4186	0.1752	60	60.3616	0.1308
22	9	12	12.4266	0.1820	60	60.1498	0.0224
23	10	12	12.5860	0.3434	60	60.3540	0.1253
23	11	12	12.5111	0.2612	60	60.1743	0.0304
24	12	12	12.6282	0.3946	60	60.3528	0.1245
24	13	12	12.4704	0.2213	60	60.2052	0.0421
24	14	12	12.2610	0.0681	60	60.0576	0.0033
25	15	12	12.3045	0.0927	60	60.2425	0.0588
25	16	12	12.0122	0.0001	60	60.1270	0.0161
26	17	12	12.0134	0.0002	60	60.2862	0.0819
27	18	12	12.0036	0.0000	60	60.3876	0.1502
29	19	12	12.3685	0.1358	60	60.4041	0.1633
30	20	12	12.3570	0.1274	60	60.3000	0.0900
			Ave. 12.3270	**MPSE**:0.1464		**Ave.**: 60.2525	**MPSE**: 0.0755
			Bias: 0.3270	**Var.**: 0.0394		**Bias**: 0.2525	**Var.**: 0.0117

Table 6 shows cure time estimates which each adhesion response model gave at simultaneous optimisation.

Table 6. Showing the cure time estimate results for each adhesion response model

$R_{t\,(mm)}$	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$T_c.R_t$	21	22	22	23	24	25	26	26	27	28	29	30		
T_c, $T_c.R_t$	21	22	22	23	24	25	26	27	28	29	30			
T_c, $R_t{}^2$	21	22	22	23	23	24	24	25	26	27	28	30		
R_t ,$T_c.R_t$,	21	22	22	23	24	25	26	27	28	28	29	29	30	30
T_c, $R_t{}^2$	21	22	22	23	24	25	26	27	28	29	30	30		
$T_c.R_t$, $T_c{}^2$	22	23	23	24	25	26	26	27	28	29	30			
$T_c.R_t$, $R_t{}^2$	21	22	22	23	23	24	24	24	25	25	26	27	29	30
$T_c{}^2$, $R_t{}^2$	21	22	22	23	24	25	26	27	28	29	30	30		
T_c, R_t, $T_c.R_t$	21	22	22	23	24	25	26	27	27	28	29	30	30	31
Tc ,R_t, $T_c{}^2$	21	22	22	23	24	25	26	27	28	29	30			
T_c, R_t, $R_t{}^2$	22	22	22	23	23	24	24	24	25	26	27	28	30	31
T_c, $T_c.R_t$, $T_c{}^2$,	21	22	22	23	24	25	26	27	28	29	30			
T_c, $T_c.R_t$, $R_t{}^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30

T_c, T_c^2, R_t^2	21	22	22	23	23	23	24	24	25	26	28	30		
$R_t, T_c.R_t, T_c^2$	21	22	22	23	24	25	26	27	28	28	29	29	30	30
$R_t, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	26	28	29	30
R_t, T_c^2, R_t^2	22	22	22	22	23	23	24	25	26	27	28	29	30	31
$T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$T_c, R_t, T_c.R_t, T_c^2$	21	22	22	23	23	24	25	26	27	28	29	30		
$T_c, R_t, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$T_c, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	25	26	27	29	30
$T_c, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	23	24	24	25	25	26	27	30	
$R_t, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$T_c, R_t, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	25	25	27	28	31

Table 7 shows the remaining thirteen adhesion response models with their cure time estimates after dropping those results that did not give estimates for the full rubber thickness range.

Table 7. Showing the adhesion response models with simultaneous optimisation cure time estimates for the full rubber thickness range

Rt(mm)	7	8	9	10	11	12	13	14	15	16	17	18	19	20
MODEL														
$R_t, T_c.R_t,$	21	22	22	23	24	25	26	27	28	28	29	29	30	30
$T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	25	26	27	29	30
$T_c, R_t, T_c.R_t$	21	22	22	23	24	25	26	27	27	28	29	30	30	31
T_c, R_t, R_t^2	22	22	22	23	23	24	24	24	25	26	27	28	30	31
$T_c, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$R_t, T_c.R_t, T_c^2$	21	22	22	23	24	25	26	27	28	28	29	29	30	30
$R_t, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	26	28	29	30
R_t, T_c^2, R_t^2	22	22	22	22	23	23	24	25	26	27	28	29	30	31
$T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$T_c, R_t, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$T_c, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	25	26	27	29	30
$R_t, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30
$T_c, R_t, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	25	25	27	28	31

A frequency analysis of the occurrence of the different cure time results is given in Table 8. There were only seven possible cure time estimate solutions in Table 7. The solution with the highest occurrence had a frequency of five.

Table 8. Showing frequency of occurrence of cure time estimates results

R_t	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Frequency
1	21	22	22	23	24	25	26	27	27	28	29	30	30	31	1
2	22	22	22	23	23	24	24	24	25	26	27	28	30	31	1
3	22	22	22	22	23	23	24	25	26	27	28	29	30	31	1
4	21	22	22	23	23	24	24	24	25	25	25	27	28	31	1
5	21	22	22	23	24	25	26	27	28	28	29	29	30	30	2
6	21	22	22	23	23	24	24	24	25	25	26	27	29	30	2
7	21	22	22	23	23	24	24	24	25	26	27	28	29	30	5

4.3 Integration results

Table 9 shows the results of integrating the cure time estimates of Table 6 using arithmetic averaging (A. Ave.) and majority vote (M. Vote).

Table 9. Showing the integration of the thirteen cure time estimate results

MODEL	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Rel. Acc.
R_t ,$T_c.R_t$,	21	22	22	23	24	25	26	27	28	28	29	29	30	30	36%
$T_c.R_t$, R_t^2	21	22	22	23	23	24	24	24	25	25	26	27	29	30	79%
T_c, R_t, $T_c.R_t$	21	22	22	23	24	25	26	27	27	28	29	30	30	31	29%
T_c, R_t, R_t^2	22	22	22	23	23	24	24	24	25	26	27	28	30	31	86%
T_c, $T_c.R_t$, R_t^2	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
R_t ,$T_c.R_t$, T_c^2	21	22	22	23	24	25	26	27	28	28	29	29	30	30	36%
R_t ,$T_c.R_t$, R_t^2	21	22	22	23	23	24	24	24	25	26	26	28	29	30	100%
R_t, T_c^2, R_t^2	22	22	22	22	23	23	24	25	26	27	28	29	30	31	29%
$T_c.R_t$, T_c^2, R_t^2	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
T_c, R_t, $T_c.R_t$, R_t^2	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
T_c, $T_c.R_t$, T_c^2, R_t^2	21	22	22	23	23	24	24	24	25	25	26	27	29	30	79%
R_t, $T_c.R_t$, T_c^2, R_t^2	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
T_c, R_t, $T_c.R_t$, T_c^2, R_t^2	21	22	22	23	23	24	24	24	25	25	25	27	28	31	64%
AVE	**21**	**22**	**22**	**23**	**23**	**24**	**24**	**25**	**26**	**26**	**27**	**28**	**29**	**30**	
M. Vote	**21**	**22**	**22**	**23**	**23**	**24**	**24**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	

Three observations to note: (1)The two integration methods did not agree on two cure time estimates for rubber thicknesses 14 and 15 mm; (2) Some adhesion-hardness pairs had relative accuracy less than 50%; and (3) The majority vote result was equivalent to the result with the highest frequency in Table 8.

Table 10 shows the bias-variance-covariance decomposition of the $MSPE_{simul}$ of the ensemble of results.

Table 10. Showing the bias-variance-covariance decomposition of the $MSPE_{simul}$

	MSPE	Bias	Var.	Covar	MSPE	Bias	Var.	Covar.
MODEL								
R_t ,$T_c.R_t$,	0.1430	0.3849	0.0609		0.1841	0.4015	0.0229	
$T_c.R_t$, R_t^2	0.1464	0.3270	0.0394		0.0755	0.2500	0.0117	
T_c, R_t, $T_c.R_t$	0.1298	0.2980	0.0410		0.1726	0.3738	0.0329	
T_c, R_t, R_t^2	0.1121	0.2878	0.0292		0.1048	0.2890	0.0202	
T_c, $T_c.R_t$, R_t^2	0.1417	0.3543	0.0162		0.0957	0.2852	0.0144	
R_t ,$T_c.R_t$, T_c^2	0.1297	0.2636	0.0602		0.1841	0.4015	0.0229	
R_t ,$T_c.R_t$, R_t^2	0.1513	0.3699	0.0144		0.0957	0.2852	0.0144	
R_t, T_c^2, R_t^2	0.5463	0.3323	0.0161		0.0957	0.2852	0.0144	
$T_c.R_t$, T_c^2, R_t^2	0.1051	0.3006	0.0147		0.0957	0.2852	0.0144	
T_c, R_t, $T_c.R_t$, R_t^2	0.0992	0.2774	0.0223		0.0957	0.2852	0.0144	
T_c, $T_c.R_t$, T_c^2, R_t^2	0.4951	0.6186	0.1125		0.0755	0.2500	0.0117	
R_t, $T_c.R_t$, T_c^2, R_t^2	0.1265	0.3323	0.0161		0.0957	0.2852	0.0144	
T_c, R_t, $T_c.R_t$, T_c^2, R_t^2	0.1561	0.6155	0.1373		0.0651	0.2200	0.0117	
AVE	**0.21864**	**0.3586**	**0.0446**	**0.0937**	**0.1105**	**0.2998**	**0.0174**	**0.0209**

There were six adhesion-hardness model pairs that have the same accuracy values on the hardness side. Generally, for a high adhesion side $MSPE_{simul}$, there was a low $MSPE_{simul}$ on the hardness side. This pattern, however, did not seem to have any significant relationship with the accuracy of the cure time estimates.

Table 11 gives the percentage accuracy compromise of the base model pairs of the ensemble at simultaneous optimisation.

Table 11. Showing the PAC results at simultaneous optimisation

R_t Adhesion Model	$MSPE_{valmin}$	MSE	PAC(H) %	PAC(A) %
R_t, $T_c.R_t$	0.0242	2.3855	310	487
$T_c.R_t$, R_t^2	0.0321	1.3382	69	357
T_c, R_t, $T_c.R_t$	0.0405	2.3441	285	221
T_c, R_t, R_t^2	0.0458	1.4710	145	134
T_c, $T_c.R_t$, R_t^2	0.0456	1.2815	114	211
R_t, $T_c.R_t$, T_c^2	0.0194	2.5300	114	569
R_t, $T_c.R_t$, R_t^2	0.0226	1.3200	114	220
R_t, T_c^2, R_t^2	0.0445	1.8069	114	1128
$T_c.R_t$, T_c^2, R_t^2	0.0474	1.3377	114	219
T_c, R_t, $T_c.R_t$, R_t^2	0.0407	1.2789	113	144
T_c, $T_c.R_t$, T_c^2, R_t^2	0.0348	0.9842	33	1323
R_t, $T_c.R_t$, T_c^2, R_t^2	0.0345	1.3689	114	267
T_c, R_t, $T_c.R_t$, T_c^2, R_t^2	0.0366	0.9762	69	1323

Table 11 shows that it's very difficult, were simultaneous optimisation is concerned, to find a model pair that has all the model accuracy criteria aligning.

- The response model pair with best $MSPE_{valmin}$ (0.0194) had the worst MSE (2.53).
- The adhesion response model with the best MSE (0.9762) compromised the worst (% accuracy compromise, PAC(A) = 1323%) to achieve simultaneous optimisation.
- The response model pair with the best PAC on the hardness side (33%), had the worst PAC on the adhesion side (1323%).
- There were six adhesion-hardness model response pairs with the same PAC(H) value. These six were not necessarily the same ones with similar $MSPE_{simul}$ values on the hardness side.

The average PAC(H) was lower compared to the PAC(A). This suggests that response models do not necessary compromise the same to achieve simultaneous optimisation.

4.4 Ensemble review results

Elimination of model pairs with relative accuracy less than 50% left nine adhesion-hardness pairs in the ensemble. The arithmetic average and majority vote results of the reviewed ensemble were equal throughout the whole rubber thickness range as shown in Table 12.

Table 12. Showing the result of eliminating response models with relative accuracy<50%

Rt (mm) Model	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Rel. Acc.
$T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	25	26	27	29	30	79%
T_c, R_t, R_t^2	22	22	22	23	23	24	24	24	25	26	27	28	30	31	86%
$T_c, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
$R_t, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
$T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
$T_c, R_t, T_c.R_t, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	29	30	100%
$T_c, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	25	26	27	29	30	79%
$R_t, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	26	27	28	28	30	100%
$T_c, R_t, T_c.R_t, T_c^2, R_t^2$	21	22	22	23	23	24	24	24	25	25	25	27	28	31	64%
AVE	21	22	22	23	23	24	24	24	25	26	27	28	29	30	
M. Vote	21	22	22	23	23	24	24	24	25	26	27	28	29	30	

Table 13 shows that the accuracy results at simultaneous optimisation significantly improved, but more on the hardness side than the adhesion side.

Table 13. Showing the accuracy results of the reviewed ensemble

MODEL	MSPE	Bias	Var.	Covar	MSPE	Bias	Var.	Covar.
$T_c.R_t, R_t^2$	0.1464	0.3270	0.0394		0.0755	0.2500	0.0117	
T_c, R_t, R_t^2	0.1121	0.2878	0.0292		0.1048	0.2890	0.0202	
$T_c, T_c.R_t, R_t^2$	0.1417	0.3543	0.0162		0.0957	0.2852	0.0144	
$R_t, T_c.R_t, R_t^2$	0.1513	0.3699	0.0144		0.0957	0.2852	0.0144	
$T_c.R_t, T_c^2, R_t^2$	0.1051	0.3006	0.0147		0.0957	0.2852	0.0144	
$T_c, R_t, T_c.R_t, R_t^2$	0.0992	0.2774	0.0223		0.0957	0.2852	0.0144	
$T_c, T_c.R_t, T_c^2, R_t^2$	0.4951	0.6186	0.1125		0.0755	0.2500	0.0117	
$R_t, T_c.R_t, T_c^2, R_t^2$	0.1265	0.3323	0.0161		0.0957	0.2852	0.0144	
$T_c, R_t, T_c.R_t, T_c^2, R_t^2$	0.1561	0.6155	0.1373		0.0651	0.2200	0.0117	
Arithmetic Ave	0.2104	0.3870	0.0447	0.0619	0.0888	0.2706	0.0148	0.0157

If the base models were the five adhesion response models with the same cure time estimates the theoretical accuracy of the ensemble would be as shown in Table 14. It appeared, for this problem, that when the cure time estimates for different adhesion-hardness pairs were the same, the theoretical accuracy on the hardness response side was the same.

Table 14. Accuracy results of the ensemble with five base models with similar T_c estimates

MODEL	MSPE	Bias	Var.	Covar	MSPE	Bias	Var.	Covar.
T_c, $T_c.R_t$, $R_t{}^2$	0.1417	0.3543	0.0162		0.0957	0.2852	0.0144	
R_t, $T_c.R_t$, $R_t{}^2$	0.1513	0.3699	0.0144		0.0957	0.2852	0.0144	
$T_c.R_t$, $T_c{}^2$, $R_t{}^2$	0.1051	0.3006	0.0147		0.0957	0.2852	0.0144	
T_c, R_t ,$T_c.R_t$, $R_t{}^2$	0.0992	0.2774	0.0223		0.0957	0.2852	0.0144	
R_t, $T_c.R_t$, $T_c{}^2$, $R_t{}^2$	0.1265	0.3323	0.0161		0.0957	0.2852	0.0144	
Arithmetic Ave	0.1248	0.3269	0.0167	0.0182	0.0957	0.2852	0.0144	0.0144

4.5 Multiple MS criteria best model selection methodology results

The first result was for the selection of the best model using majority vote of fifteen different model selection criteria. The formulae used to compute the criteria values are shown in the Appendix. Table 15 shows the model selection criteria values and their votes.

Adhesion response model [$T_c.R_t$, R_t^2] is the obvious best with a vote of 10. This minimises uncertainty. Model [T_c, $T_c.R_t$, T_c^2, R_t^2] follows behind with a vote of 6.

Table 15. Showing multiple MS criteria selections

MODEL	**T_c, R_t, $T_c.R_t$, $T_c{}^2$, $R_t{}^2$**	**T_c, $T_c.R_t$, $T_c{}^2$, $R_t{}^2$**	**R_t, $T_c.R_t$, $R_t{}^2$**	**$T_c.R_t$, $R_t{}^2$**	**T_c, R_t, $R_t{}^2$**	**T_c, $T_c.R_t$,**
R^2 (pr.)	26.5	51.5	49.9	65.4	49.4	52
Adeq. pr.	10.4	4.1	5.7	1.8	11.9	5.4
Cp-k	1.0	0.1	0	0	2.1	0
PRESS	88.1	59.3	81.3	42.1	62.2	58.6
AIC	11.7	9.8	11.6	9.8	13	17.3
BIC	15.1	12.6	13.9	11.5	15.3	19
AICc	20.2	14.8	14.3	11	15.7	18.5
APCp	2.9	2.4	2.6	2.9	2.6	4.0
SBC	1.9	1.7	4.9	4.8	6.1	11.9
HQc	1.1	1.1	4.3	4.5	5.6	11.5
KICc	87.3	64.4	51.2	38.1	52.6	45.6
HQ	0.5	0.5	4.2	4.2	5.6	11.7
KIC	20.7	17.8	18.6	15.8	20	23.3
MKIC	18.2	12	9.1	5.4	9.8	10.7
TIC	13.7	11.8	13.6	11.8	15	19.3
VOTE	**2**	**6**	**1**	**10**	**1**	**1**

The simultaneous optimisation results of both response models [$T_c.R_t$, R_t^2] and [T_c, $T_c.R_t$, T_c^2, R_t^2] are shown in Tables 12 and 13. The two have the same cure time estimate and theoretical accuracy results on the hardness side. The cure time estimate result, however, was different from the multiple simultaneous optimisations ensemble one.

5. Conclusion

The dilemma of choosing from two different solutions, see Table 16, both of which standing on strong positions makes the problem at hand challenging. However, an objective analysis and critic of each position helps in separating the two.

Table 16. Multiple MS criteria solution (S_6) vs. Ensemble solution (S_7)

Rt	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Vote
S_6	21	22	22	23	23	24	24	24	25	25	26	27	29	30	2
S_7	21	22	22	23	23	24	24	24	25	26	27	28	29	30	5

The multiple simultaneous optimisations ensemble cure time estimate solution (S_7) shows credibility in that (i) it is the most frequent solution from the adhesion-hardness model pairs, (ii) there is agreement between the two integration methodologies used, and (iii) by design, it fairly accounts for all the listed problems of the contemporary MRSM contextual framework. It accounts for dataset uncertainty, loss of information, model over-/underfitting, and model parameter bias by utilising multiple models and minimising discarded models. It minimises model uncertainty and small sample size inefficiency by totally avoiding the use of classical model selection criteria.

On the other hand, seven of the ten model selection criteria that voted for the best single adhesion response model [$T_c.R_t$, R_t^2] are information criteria and three are prediction model selection criteria. This implies that the response model has the best parsimonious fit to the MRSM dataset, of all the 25 OLS adhesion response models, and has good prediction capability. However, the cure time estimate solution (S_6) is not considered the best in credibility because (i) model selection criteria have a small sample size inefficiency problem, (ii) they do not deal with the problem of model parameter bias, (iii) dataset uncertainty and (iv) since the methodology is structured as the contemporary MRSM framework, it loses information in discarded models by the selection and use of one model per response in simultaneous optimisation. It should be emphasised that where the model with the best parsimonious fit to the dataset is required, response model [$T_c.R_t$, R_t^2] is the model.

The arguments above clearly separate the most credible cure time solution (S_7) from the model with the best parsimonious fit to the MRSM dataset (S_6]. The multiple simultaneous optimisations ensemble, therefore, is both logically and empirically a better way of obtaining credible results compared to the current MRSM contextual framework which must first select a best model for each response before simultaneous optimisation. The multiple simultaneous optimisations ensemble is thus recommended to the rubber covered conveyor belting manufacturing industry for use in reviewing cure times when adhesion and cover hardness minimum quality standard requirements change.

The use of targeted values in validation is worth mentioning here, as well, since the size of the MRSM dataset is small and it would be senseless to split it. The other option would have been to use cross validation which would have taken back the solution methodology to the weakness of the contemporary MRSM framework. In itself, the practice is worth considering where targeted quality values have to be effectively converted to production process parameters.

Noting the fact that the multiple simultaneous optimisations ensemble worked well on a two factors and two responses problem, it then makes it imperative to investigate its generalisability to other more complex MRSM problems. As the number of factors and responses increase, the number of models to deal with quickly multiplies. This will definitely require software and intelligent algorithms to manage complexity and still achieve credible results.

Acknowledgements: We would like to thank the editor and anonymous referees for their valuable contributions that helped us to improve the quality and presentation of this paper.

References

Ahangi, A., Langroudi, A. F., Yazdanpanah, F., & Mirroshandel, S. A. (2019). A novel fusion mixture of active experts algorithm for traffic signs recognition. Multimedia Tools and Applications, 78(14), 20217-20237. https://doi.org/10.1007/s11042-019-7391-0

Alhorn, K., Schorning, K., & Dette, H. (2019). Optimal designs for frequentist model averaging. Biometrika, 106(3), 665-682. https://doi.org/10.1093/biomet/asz036

Burnham, K., Anderson, D. (2002). Model selection and multi-model inference. A practical information-theoretic approach, Springer, New York [Chapter 1; Chapter 4]

Geman, S., Bienenstock, E., & Doursat, R. (1992). Neural networks and the bias/variance dilemma. Neural computation, 4(1), 1-58. https://doi.org/10.1162/neco.1992.4.1.1

Gatos, K. G., & Karger-Kocsis, J. (2004). Estimation of the vulcanization time for rubber by considering their linear viscoelastic response assessed by a plate-plate rheometer. Kautschuk Gummi Kunststoffe, 57(7-8), 350-354.

Gough, J. (2017). Calculation of times and temperatures for press vulcanization of thick rubber pads. Rubber Chemistry and Technology, 90(1), 89-107. https://doi.org/10.5254/rct.16.83774

Hejazi, T.H., Seyyed-Esfahani, M., Antony, J. (2017) “A New Methodology based on Multistage Stochastic Programming for Quality Chain Design Problem”, International Journal of Industrial Engineering: Theory, Applications and Practice, 24(1), 12-31.

Jenkins D.G., Quintana-Ascencio P.F. (2020). A solution to minimum sample size for regressions. PLoS ONE 15(2): e0229345. https://doi.org/10.1371/journal.pone.0229345

Karaağaç, B., İnal, M., & Deniz, V. (2012). Predicting optimum cure time of rubber compounds by means of ANFIS. Materials & Design, 35, 833-838. https://doi.org/10.1016/j.matdes.2011.03.062

Khuri, A. I. (2017). A general overview of response surface methodology. Biometrics & Biostatistics International Journal, 5(3), 87-93. DOI: 10.15406/bbij.2017.05.00133

Kittler, J., (1998). “On combining classifiers,” IEEE Transactions on Pattern Analysis and Machine Intelligence, 20(3), 226–239.

Mäkelä, M. (2017). Experimental design and response surface methodology in energy applications: A tutorial review. Energy Conversion and Management, 151, 630-640. https://doi.org/10.1016/j.enconman.2017.09.021

Myers, R.H., Montgomery, D.C., Anderson-Cook, C.M, (2016). Response Surface Methodology: Process and Product Optimisation Using Designed Experiments, *4th Edition, ISBN*: 978-1-118-91601-8. [Chapter 6]

Domingo Pavolo, D. C. (2020). Determining Cure Time of Rubber-Covered Mining Conveyor Belts using Multi response Surface Methodology. International journal of operations and quantitative management, 26(1), 29-48.

Polikar, R., & Polikar, R. (2006). Ensemble based systems in decision making. IEEE Circuit Syst. Mag. 6, 21–45 (2006).

Rawlings, J.O., Pantula, G.S., Dickey A.D. 1998, Applied Regression Analysis: A Research Tool, Second Edition, Springer-Verlag New York Inc.

Ueda, N., & Nakano, R. (1996, June). Generalization error of ensemble estimators. In Proceedings of International Conference on Neural Networks (ICNN'96) (Vol. 1, pp. 90-95). IEEE. https://doi.org/10.1109/ICNN.1996.548872

Xu, Y., & Goodacre, R. (2018). On splitting training and validation set: a comparative study of cross-validation, bootstrap and systematic sampling for estimating the generalization performance of supervised learning. Journal of analysis and testing, 2(3), 249-262. https://doi.org/10.1007/s41664-018-0068-2

Yang, P., Hwa Yang, Y., B Zhou, B., & Y Zomaya, A. (2010). A review of ensemble methods in bioinformatics. Current Bioinformatics, 5(4), 296-308.

Yuan, Z., & Yang, Y. (2005). Combining linear regression models: When and how?. Journal of the American Statistical Association, 100(472), 1202-1214. https://doi.org/10.1198/016214505000000088