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OPTIMIZATION OF WEAR PARAMETERS FOR DUPLEX-TIALN COATED MDC-K TOOL STEEL USING FUZZY MCDM TECHNIQUES

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Abstract: *The present work evaluates the effects of different tribological process parameters on the measured responses such as hardness, coefficient of friction, surface roughness, wear mass loss and wear depth of duplex-TiAlN coated MDC-K tool steel material. The considered tribological process parameters are load, sliding velocity, and sliding distance. A full factorial design with 27 experimental runs is employed and based on the response values, an optimal combination of the tribological process parameters is subsequently determined. Different multi-objective optimization techniques, like overall evaluation criteria and fuzzy-based multi-criteria decision-making methods (fuzzy evaluation based on distance from the average solution, fuzzy technique for order of preference by similarity to ideal solution, and fuzzy complex proportional assessment) are utilized to identify the optimal intermixes of the considered tribological process parameters. Sensitivity analysis with respect to changing weights of the responses is performed to validate the derived rankings of the trials, whereas the results of analysis of variance revealed the most significant parameters were influencing the responses. In addition to this, two different published problems related to optimization of wear parameters were solved using the proposed method to check its capability.*

Keywords: *MDC-K tool steel, Duplex-TiAlN coating, Fuzzy MCDM, Sensitivity analysis, Optimization.*

1. Introduction

MDC-K hot work tool steel contains a high percentage of chromium along with tungsten, molybdenum, and vanadium, which substantially enhances its mechanical and wear properties required for its application in the manufacturing of extrusion

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dies, die casting dies, hot stamping dies, and forging dies. Untreated tool steel is commercially available with a hardness of ~22 HRC, constraining its application in die manufacturing. Therefore, heat treatment of tool steel becomes mandatory using different hardening processes to attain the desired levels of hardness and toughness. These properties of tool steel mainly depend on its chemical composition, alloying elements, and secondary carbides formation during the hardening processes (Joshy et al. 2019, Kumar et al. 2021a, and Soleimany et al. 2019). The alloying elements can be divided into two classes, i.e., one is responsible for carbide formation and the other is accountable for changing the tempering kinetics during the heat treatment process (Podgornik et al. 2018a and Podgornik et al. 2016b).

Further, the hardened tool steel requires surface modifications, such as nitriding (gas nitriding, salt bath nitriding or plasma nitriding) and deposition of ceramic-based hard coatings. Plasma nitriding has broader advantages over salt bath nitriding and gas nitriding. It allows much closer control of the microstructure during nitriding and is able to provide a surface without the formation of a compound layer. When plasma nitriding is integrated with the physical vapor deposition (PVD) process, it is known as duplex surface treatment. During plasma nitriding, nitrogen diffuses to the surface and forms two different zones, i.e., the compound zone and diffusion zone. The compound zone is made up of Fe_4N and $Fe_{2-3}N$, whereas, the diffusion zone is formed by diffused nitrogen atoms making the surface harder (Aghajani et al. 2017 and Kumar et al. 2020a, 2022a). In addition to the application on nitride surfaces, ceramic coatings, such as TiN, CrN, TiAlN, TiCN, AlCrN, CrAlN, etc. have widely been employed in the manufacturing, tooling, and biomedical industries due to their high resistance to wear, oxidation, corrosion, chemical stability and biocompatibility (Chaliampalias et al. 2017, Prabhu et al. 2018, Kumar et al. 2020b, 2021b, 2022b, 2021c and Patnaik et al. 2021a, 2021b, 2021c, 2021d, 2020a, 2022). Many researchers have observed excellent mechanical, wear, and corrosion properties of TiAlN film coatings (Fu et al. 2019 and Ozkan et al. 2020). Various experimental works have already been conducted to study the tribological, frictional, and wear behaviors of TiAlN coated surfaces under different conditions of normal load, sliding velocity, and sliding distance (Sen et al. 2020, Chowdhury et al. 2017, M'Saoubi et al. 2013, Kumar et al. 2021d, 2022c and Kuo et al. 2018). However, investigations to study the influences of various tribological process parameters on the wear behavior of TiAlN coated surfaces remain unexplored.

In addition to this, Saravanan et al. (2015 and 2016) and Patnaik et al. (2021e and 2021f) adopted the Box-Behnken experimental design plan (L_{15} orthogonal array) and conducted 15 experiments to derive a suitable combination of process parameters for TiN coated SS 316L steel. Out of those 15 experimental runs, one experiment was repeated three times, resulting in performing only 13 actual experiments. Similar studies have been performed by Kumar et al. (2022d & 2022e), where L_{16} orthogonal array was adopted to perform the wear experiment for CrN/TiAlN coating. According to the authors, the use of a small set of experimental runs may not always be sufficient to determine the most suitable parameters for a specific process, and there should be sufficient experimental observations to study the process behavior. Moreover, in the earlier investigations, there has been limited participation of the decision makers and equal weights (relative importance) have usually been assigned to the considered responses. Thus, there is a huge opportunity to adopt different multi-criteria decision-making (MCDM) techniques allowing the involvement of a group of decision makers in deciding the relative importance of various responses under a fuzzy environment. These MCDM techniques are very popular in the material selection for various applications (Maity and Chakraborty 2013 and Prasad et al. 2014). To the best of the authors' knowledge, the application of any of the fuzzy MCDM tools in studying the tribological properties of duplex-TiAlN coated MDC-K tool steel is really limited.

Thus, this paper proposes a simultaneous application of three other fuzzy MCDM techniques, in the form of fuzzy technique for order of preference by similarity to ideal solution (F-TOPSIS), fuzzy evaluation based on distance from the average solution (F-EDAS) and fuzzy complex proportional assessment (F-COPRAS) methods, to investigate effects of different tribological process parameters, like load, sliding velocity and sliding distance on different responses, i.e. hardness, coefficient of friction, surface roughness, wear mass loss and wear depth of duplex-TiAlN coated MDC-K tool steel material. Based on the experimental observations, the most appropriate combination of those tribological process parameters is also singled out using each of the multi-objective optimization methods under consideration. All these fuzzy MCDM techniques are easy to comprehend, robust and mathematically sound. The fuzzy-TOPSIS method endeavors to identify the best alternative based on its minimum distance from the positive ideal solution and maximum distance from the negative ideal solution (Yu and Pan 2021; de Lima Silva et al. 2020 and Petrović et al. 2019). On the other hand, the fuzzy-EDAS method assigns a ranking order to the candidate alternatives based on the positive and negative distances from the average solution (Keshavarz Ghorabaee et al. 2017). The fuzzy-COPRAS method selects the most apposite alternative considering both the positive ideal and negative ideal solutions while taking into account the performance of the alternatives with respect to different criteria and the corresponding criteria weights (Zhan et al. 2020). It adopts a step-wise ranking and evaluating procedure of the alternatives in terms of their significance and utility degree. It is worthwhile to mention here that as the considered multi-objective optimization techniques have different mathematical treatments and have their own advantages and disadvantages, the ranking lists of the alternatives derived using these methods are supposed to vary, and it would be interesting to identify the best performing mathematical tool that would lead to the attainment of the most desired responses for duplex-TiAlN coated MDC-K tool steel.

2. Methodology

2.1. Preparation of the specimen

In this paper, chromium-rich MDC-K tool steel is used as the substrate material and its composition is provided in Table 1. The dimension of the sample (Ø55 mm and thickness 5 mm) is attained using a tool room lathe (Mysore KIRLOSKAR, Model: EP-2215) and high precision hydraulic surface grinding machine (Kingston, Model: KG-CL 3060 AH). The turned substrate is then heat-treated, followed by plasma nitriding. Vacuum hardening is performed at ~1080°C temperature in the absence of oxygen, whereas, quenching is performed in the same chamber in a nitrogen environment under a pressure of ~2 MPa. Application of tempering (at ~0.14 MPa gas pressure and cooled to ~92°C) helps to reduce extra hardness and brittleness while imparting enough toughness to the treated material. Hardness is measured using a Wilson Holbert micro-hardness testing machine, i.e., 460 HV. Furthermore, to increase the corresponding surface hardness, plasma nitriding is performed in presence of hydrogen (75%) and nitrogen (25%) at ~0.8 kV potential.

Table 1. Composition of MDC-K tool steel

Element	Cr	W	V	Mn	C	Si
wt%	4.4	2	1.7	0.5	0.4	0.3

The TiAlN coating is deposited on the plasma nitrided MDC-K tool steel surface using the magnetron sputtering method. Before the deposition process, the substrates are cleaned ultrasonically using an alkaline solution, followed by ethanol for 10-15 minutes. Later, distilled water is used to re-clean the substrate and is dried with ethanol. The substrate surface is then etched using titanium (Ti) ions under a pulse bias of -1000V with an 80% duty cycle for four minutes. The TiAlN film is finally deposited using titanium (Ti) and aluminum (Al) cathode (50:50) under a nitrogen gas pressure of 2.5 Pa. The DC bias is -40V and the temperature is maintained at ~315oC for 30 min to attain a film thickness of 3.5 μm.

2.2. Selection of process parameters

Based on the full-factorial design plan, 27 experiments are conducted using DUCOM TR20LE Tribometer (ASTM: G99 standard) to investigate the effects of various tribological process parameters, like load, sliding velocity, and sliding distance on the considered responses, i.e., hardness, coefficient of friction, surface roughness, wear mass loss and wear depth of duplex-TiAlN coated MDC-K tool steel material. The past literature (Łępicka et al. 2017 & 2019, Ramezani et al. 2018, and Patnaik et al. 2020b, 2021g) suggests that load, sliding velocity, and sliding distance are the most influential parameters influencing the wear properties of TiAlN coated materials. During the experiments, the range of each of these parameters is decided based on pilot experiment runs. When the experiments are conducted at a load less than 10 N load, sliding velocity less than 0.1 m/s, and sliding distance less than 1000 m, no significant effect on the wear properties is noticed due to the lower contact period between the pin and disc surfaces. At 20 N load, 0.3 m/s sliding velocity and 2000 m sliding distance, a wider and deeper wear track is observed on the surface with heavy abrasion and erosion of the coating. High sliding velocity provides sufficient time to repeat the same contact point, and its combined effect with high load increases the interface temperature leading to deformation and erosion of the coating. Based on these results, the corresponding levels and ranges of the considered tribological parameters are determined, as exhibited in Table 2.

Table 2. Experimental conditions

Process parameters and their levels		
Process parameter	Level	Value
Load (L) (in N)	3	10, 15, 20
Sliding velocity (SV) (in m/s)	3	0.1, 0.2, 0.3
Sliding distance (SD) (in m)	3	1000, 1500, 2000
Uncontrollable Parameters		
Parameter	Description	
Disc size	60 mm diameter × 8 mm thickness	
Pin size	8 mm diameter × 30 mm length	
Temperature	Ambient	
Humidity	Ambient	

2.3. Fuzzy-TOPSIS method

Three different fuzzy-based MCDM techniques viz. F-TOPSIS, F-COPRAS, and F-EDAS are also employed for optimization of different tribological parameters to attain the most desired wear properties of duplex-TiAlN coated MDC-K tool steel. The TOPSIS method selects the most apposite alternative which is nearest to the positive-ideal solution and farthest from the negative ideal solution. Based on the negative-ideal solution, non-beneficial attributes get maximized and the beneficial attributes are minimized. On the other hand, based on the positive-ideal solution, beneficial attributes are maximized and non-beneficial attributes get minimized. Furthermore, the integration of fuzzy set theory with TOPSIS helps in dealing with ambiguity and subjectivity in the decision-making process. Usually, in a multi-objective parametric optimization problem involving a single decision maker/process engineer, equal importance is assigned to all the considered responses that also ease out the calculation steps. However, in a real-time machining environment, more than one decision maker participates in assigning importance to the varying responses. The ratings allotted to the responses are usually subjective and vary from one decision to the other. In this paper, in order to assign weight to each of the responses, the triangular linguistic fuzzy numbers of Table 3 is incorporated. In Table 4, the linguistic fuzzy weights allotted to the five responses by a panel of three decision makers are presented, which are finally aggregated in Table 5 to provide the corresponding fuzzy weights for all the responses.

Table 3. Triangular linguistic fuzzy numbers

Lowest	LT	(0, 0, 0.1)
Lower	LR	(0, 0.1, 0.3)
Low	L	(0.1, 0.3, 0.5)
Medium	M	(0.3, 0.5, 0.7)
High	H	(0.5, 0.7, 0.9)
Higher	HR	(0.7, 0.9, 1)
Highest	HT	(0.9, 1, 1)

Table 4. Decision makers' panel

Response	Group of decision makers		
	DM1	DM2	DM3
Ra	L	M	LR
COF	M	L	LR
WML	L	M	L
WD	M	M	L
HV	HR	H	HT

Table 5. Aggregated fuzzy weight

Response	Fuzzy weight
Ra	(0.133, 0.3, 0.5)
COF	(0.133, 0.3, 0.5)
WML	(0.17, 0.37, 0.57)
WD	(0.23, 0.43, 0.63)
HV	(0.7, 0.87, 0.97)

The procedural steps of the F-TOPSIS method are elucidated below (Shivakoti et al. 2017):

Step 1: Based on the experimental dataset consisting of 27 observations and five responses, develop the initial decision/evaluation matrix $U = [u_{ij}]_{27\times5}$, where u_{ij} is the observed value of j^{th} response (j = 1, 2, 3, 4, 5) at i^{th} experimental trial (i = 1, 2...,27).

Step 2: In order to make the performance criteria values of the above decision matrix dimensionless and comparable, normalize all the elements using the vector normalization procedure.

$$x_{ij} = \frac{u_{ij}}{\left(\sum_{i=1}^{27} u_{ij}^2\right)^{0.5}} \quad \text{i =1, 2, ..., 27} \tag{1}$$

where x_{ij} is the normalized value of u_{ij}.

Step 3: Developed the fuzzy weighted normalized decision matrix ($\tilde{N}_{ij}$) while multiplying all the elements of the normalized decision matrix by the corresponding fuzzy weights of the considered responses.

Step 4: The fuzzy positive ideal solution (M^+) and fuzzy negative ideal solution (M^-) is needed to be calculated using Eq. (2) and Eq. (3) respectively.

$$M^+ = \left\{\left[\max\left(m_{ij}\right) \mid j \in J\right] or \left[\min\left(m_{ij}\right) \mid j \in J\right], where \text{ i =1, 2, ..., 27}\right\}$$

$$= \left\{M_1^+, M_2^+, M_3^+, M_4^+, M_5^+\right\} \tag{2}$$

$$M^- = \left\{\left[\min\left(m_{ij}\right) \mid j \in J'\right] or \left[\max\left(m_{ij}\right) \mid j \in J\right], where \text{ i =1, 2, ..., 27}\right\}$$

$$= \left\{M_1^-, M_2^-, M_3^-, M_4^-, M_5^-\right\} \tag{3}$$

Where, $J = \{1, 2, 3, 4, 5\}$ and $J' = \{1, 2, 3, 4, 5\}$ J *and* J' associated with higher the better type and lower the better type respectively. In this paper, Ra, CoF, WML, WD are considered as lower the better and HV was considered as higher the better type.

Step 5: The fuzzy Euclidean distance for each experimental result from the fuzzy positive ideal solution $\left(d_i^+\right)$ and fuzzy negative ideal solution $\left(d_i^-\right)$ is needed to be calculated using Eq. (4) and Eq. (5) respectively.

$$d_i^+ = \sum_{i=1}^{5} d\left(m_{ij}, \mathrm{m}_i^+\right) \quad \text{i = 1, 2, ..., 27; j = 1, 2, 3, 4 ,5} \tag{4}$$

$$d_i^- = \sum_{i=1}^{5} d\left(m_{ij}, \mathrm{m}_i^-\right) \quad \text{i = 1, 2, ..., 27; j = 1, 2, 3, 4 ,5} \tag{5}$$

where, *d* is the distance between two fuzzy numbers.

Step 6: Defuzzified the positive ideal solution and negative ideal solution.

Step 7: Calculate the closeness coefficient (CoC_i) for each experimental run as its proximity to the ideal solution.

$$CoC_i = \frac{d_i^-}{d_i^- + d_i^+} \tag{6}$$

Step 8: Rank all the experimental runs based on the descending values of CoC_i. Thus, the experimental run having the maximum CoC_i value would be the best alternative, whereas, the worst alternative should have the minimum CoC_i value.

2.4. Fuzzy-COPRAS method

The COPRAS method usually deals with quantitative information and the candidate alternatives are ranked based on the relative weights of various criteria. However, while solving real-time decision-making problems with incomplete or vague information, this method fails to provide an accurate ranking of the alternatives under consideration. To avoid this deficiency, the COPRAS method is combined with the fuzzy set theory in this paper Use the fuzzy technique to calculate the relative priority of responses/criteria using a fuzzy number rather than the precise number (Sun 2010). In this way, the fuzzy-COPRAs technique was proposed to deal with the insufficiency in the conventional COPRAS method. The weight of the responses/criteria and ranking of the alternatives are evaluated using linguistic terms denoted by a fuzzy number. The following steps are used to perform the fuzzy-COPRAS decision-making Albayrak 2020).

Step 1: Construct the normalized decision matrix using Eq. (1).

Step 2: Construct the fuzzy weighted normalized matrix $\left(\hat{X}\right)$ using Eq. (7) and Eq. (8).

$$\hat{X}_{ij} = w_j \times \overline{x}_{ij} \tag{7}$$

w_j is the fuzzy weight of criteria.

$$\hat{X} = \left[\hat{X}_{ij}\right] = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \cdots & \hat{x}_{mn} \end{bmatrix} i = 1,2,3....,m;\ j = 1,2,3,....,n \tag{8}$$

Step 3: Calculate the sum of fuzzy beneficial and non-fuzzy beneficial responses values using Eq. (9) and Eq. (10) respectively.

$$S_{+i} = \sum_{j=1}^{k} \hat{x}_{-ij} \quad i = 1,2,3....,m; \quad j = 1,2,3,......,n \tag{9}$$

$$S_{-i} = \sum_{j=(k+1)}^{k} \hat{x}_{+ij} \quad i = 1,2,3....,m; \quad j = k+1, k+2, k+3,......,n \tag{10}$$

where, k denotes number of beneficial criteria and (n-k) denotes non-beneficial criteria.

Step 4: Defuzzified the sum of beneficial and non-beneficial responses.

Step 5: Determine the relative significance values (Q_i) for each alternative using Eq. (11).

$$Q_i = S_{+i} + \frac{\sum_{i=1}^{m} S_{-i}}{S_{-i} \times \sum_{i=1}^{m} \frac{1}{S_{-i}}} \qquad i = 1,2,3,\ldots,n \tag{11}$$

Step 6: Determine the performance score of each alternative (P_i) using Eq. (12) and Eq. (13). respectively.

$$Q_{\max} = \max\{Q_i\} \qquad i = 1,2,3,\ldots,\mathrm{m} \tag{12}$$

$$P_i = \frac{Q_i}{Q_{\max}} \times 100\% \tag{13}$$

Based on the performance score (P_i), ranking of alternative was determined. Higher performance score was attributed to best alternative whereas, lowest performance score was attributed to the worst alternative.

2.5. Fuzzy-EDAS method

This method was developed by Ghorabaee et al. (2016), it needs a few computational steps to evaluate the process with good efficiency in comparison with other MCDM methods. Furthermore, it evaluates the alternatives based on the average solution for each response (criterion). In the present study, the EDAS method was integrated with the fuzzy numbers. The EDAS method is elaborated in fuzzy linguistic terms, which are further defined by the triangular fuzzy number (Table 3). In this method, the first step was to determine the average solution of each criterion. From the average solution, the positive and negative distance was calculated. The fuzzy weight of criteria was multiplied with positive and negative distance and then this value was normalized. Finally, an appraisal score was calculated for each alternative, and based on this score, a ranking of alternatives was derived. The following steps were used to determine the ranking using Fuzzy-EDAS (Polat and Bayhan 2020 and Stević et al. 2018; Vukasović et al. 2021).

Step 1: Construct the average decision matrix (X) using following equation:

$$X = \left[\tilde{x}_{ij}\right]_{n\times m} \tag{14}$$

$$\tilde{x}_{ij} = \frac{1}{k} \oplus_{p=1}^{k} \tilde{x}_{ij}^{p} \tag{15}$$

Where, the performance value of alternative $A_i\left(1 \le i \ge n\right)$ is represented by $\tilde{x}_{ij}^{p}$ corresponding to the criteria $c_i\left(1 \le j \ge n\right)$ which assigned by the p^{th} expert $\left(1 \le p \ge k\right)$.

Step 2: Determine the average solutions and form their corresponding matrix.

$$AV = \left[\tilde{av}_j\right]_{1\times m} \tag{16}$$

$$\tilde{av}_j = \frac{1}{n} \oplus_{i=1}^{n} \tilde{x}_{ij} \tag{17}$$

Where, $\tilde{av}_j$ denotes the average solution corresponding to each criterion.

Step 3: Calculate the fuzzy positive and fuzzy negative distances from the average for beneficial and non-beneficial criteria.

$$PDA = \left[\tilde{pda}_{ij} \right]_{n \times m} \tag{18}$$

$$NDA = \left[\tilde{nda}_{ij} \right]_{n \times m} \tag{19}$$

$$\tilde{pda}_{ij} = \begin{cases} \dfrac{\psi\left(\tilde{x}_{ij} - \tilde{av}_j \right)}{k\left(\tilde{av}_j \right)} & if\ \ j \in B \\ \dfrac{\psi\left(\tilde{x}_{ij} - \tilde{av}_j \right)}{k\left(\tilde{av}_j \right)} & if\ \ j \in N \end{cases} \tag{20}$$

$$\tilde{pda}_{ij} = \begin{cases} \dfrac{\psi\left(\tilde{av}_j - \tilde{x}_{ij} \right)}{k\left(\tilde{av}_j \right)} & if\ \ j \in B \\ \dfrac{\psi\left(\tilde{av}_j - \tilde{x}_{ij} \right)}{k\left(\tilde{av}_j \right)} & if\ \ j \in N \end{cases} \tag{21}$$

Where fuzzy positive and fuzzy negative distances are denoted by $\tilde{pda}_{ij}$ and $\tilde{nda}_{ij}$ respectively for i^{th} alternative from the average solution in term of j^{th} criterion.

Step 4: Calculate the fuzzy weighted sum of positive and negative distances for each alternative using following equations.

$$\tilde{sp}_i = \oplus_{i=1}^{m} \left(\tilde{w}_j \otimes \tilde{pda}_{ij} \right) \tag{22}$$

$$\tilde{sn}_i = \oplus_{i=1}^{m} \left(\tilde{w}_j \otimes \tilde{nda}_{ij} \right) \tag{23}$$

Step 5: Normalize the value of fuzzy sp_i and fuzzy sn_i for each alternative as follows:

$$\tilde{nsp}_i = \frac{\tilde{sp}_i}{\max_i \left(k\left(\tilde{sp}_i \right) \right)} \tag{24}$$

$$\tilde{nsn}_i = 1 - \frac{\tilde{sn}_i}{\max_i \left(k\left(\tilde{sn}_i \right) \right)} \tag{25}$$

Step 6: Defuzzified the fuzzy normalized value of $\tilde{pda}_{ij}$ and $\tilde{nda}_{ij}$ for each alternative.

Step 7: Determine appraisal score (as_i) for each alternative using Eq. (26)

$$as_i = \frac{1}{2}\left(nsp_i \oplus nsn_i \right) \tag{26}$$

Step 8: Finally, rank the alternatives based on their appraisal score. The highest score corresponds to the best alternative, while the lowest score corresponds to the worst alternatives.

To understand the proposed MCDM methods, a combined procedural flow diagram is presented in Figure 1, where each step is connected to the other denoting process involved in the MCDM methods.

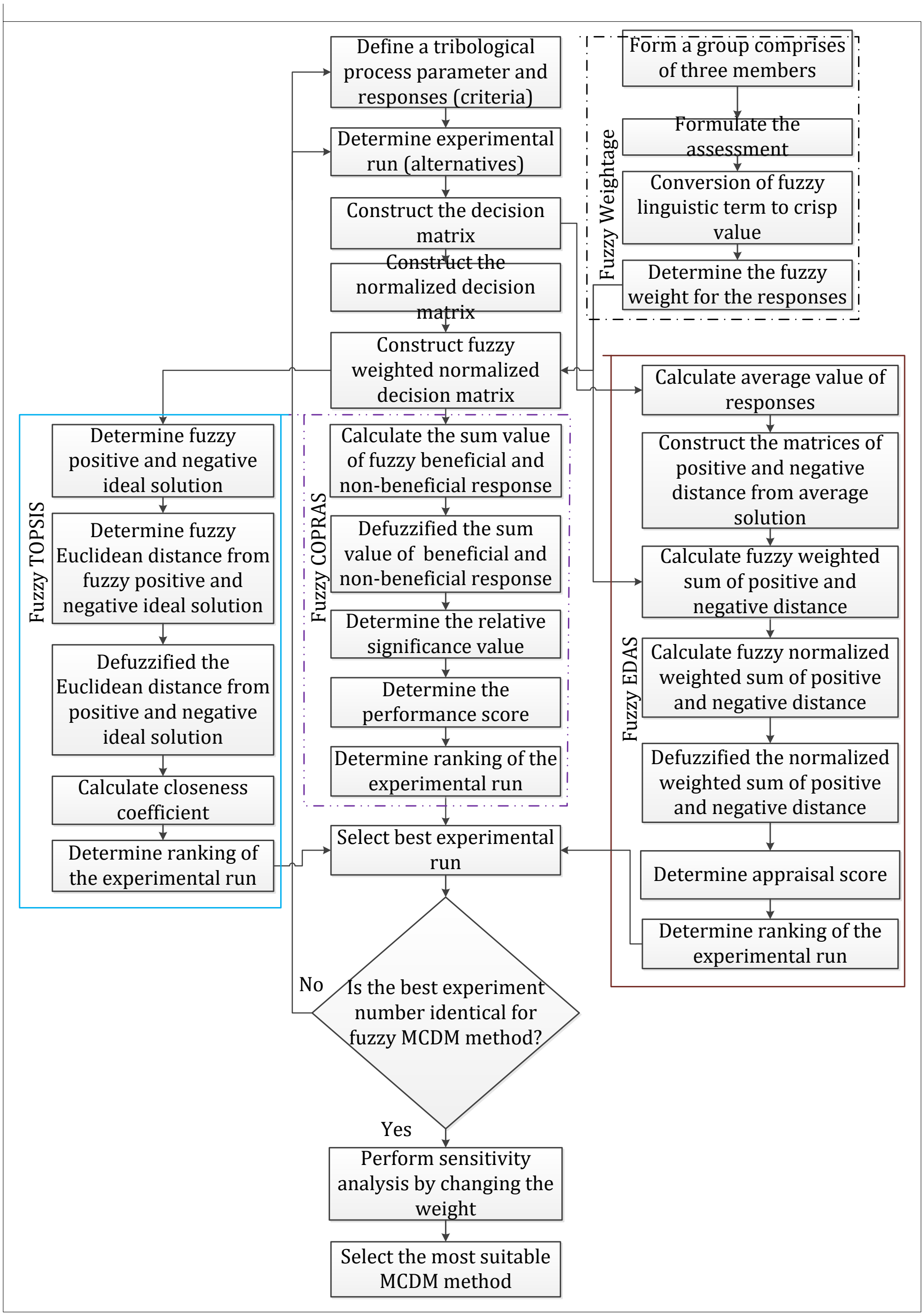


Figure 1. Combined procedural flow diagram for solving multi-objective problems

3. Results and discussion

The tribological experiments were performed according to the full factorial design. Each test was repeated three times to ensure more accuracy in the measured response value. The average value of the responses is tabulated in Table 6. The performance characteristics of the duplex-TiAlN coating were analysed by obtaining Ra, COF, WML, WD, and HV. The experimental data were analysed to understand the effect of the tribological parameters on the measured responses.

Table 6. Experimental design matrix with measured responses

Experiment number (Alternative, EN)	Tribological process parameters			Responses (Criteria, C)				
	L	SV	SD	Ra, C1	COF, C2	WML, C3	WD, C4	HV, C5
EN_1	20	0.1	1000	2.4	0.39	40.28	4.12	1227
EN_2	15	0.1	2000	5.3	0.63	32.38	3.92	1213
EN_3	10	0.3	2000	8.3	0.92	21.08	2.92	1147
EN_4	20	0.2	1500	4.9	0.49	59.58	5.02	954
EN_5	15	0.3	1000	6.2	0.79	54.68	4.42	1201
EN_6	15	0.2	1000	5.5	0.68	30.08	3.82	1137
EN_7	20	0.2	1000	4.6	0.47	51.98	4.52	1126
EN_8	15	0.2	2000	5.9	0.74	27.08	4.12	1130
EN_9	10	0.1	2000	6.7	0.74	16.08	2.42	1798
EN_{10}	10	0.2	1000	6.6	0.75	11.68	1.92	1894
EN_{11}	20	0.3	1500	5.2	0.61	63.68	5.52	798
EN_{12}	10	0.1	1500	6.4	0.7	12.78	2.12	1911
EN_{13}	10	0.3	1500	8.1	0.89	16.08	2.42	1498
EN_{14}	20	0.3	2000	6.1	0.64	74.38	6.72	739
EN_{15}	15	0.1	1000	4.9	0.58	17.98	2.82	1405
EN_{16}	15	0.2	1500	5.7	0.71	41.18	4.22	1171
EN_{17}	10	0.1	1000	6.3	0.75	10.14	1.18	1917
EN_{18}	20	0.1	1500	3.1	0.41	46.68	4.32	1187
EN_{19}	15	0.3	2000	6.9	0.87	49.38	5.42	878
EN_{20}	10	0.3	1000	7.8	0.84	9.58	2.22	1471
EN_{21}	20	0.3	1000	4.3	0.58	57.08	4.82	1031
EN_{22}	10	0.2	2000	7.2	0.81	18.48	2.12	1784
EN_{23}	20	0.1	2000	3.7	0.43	50.58	4.82	992
EN_{24}	20	0.2	2000	5.4	0.52	62.28	5.42	912
EN_{25}	15	0.1	1500	5.1	0.61	22.18	3.42	1415
EN_{26}	15	0.3	1500	6.6	0.84	46.58	5.12	1115
EN_{27}	10	0.2	1500	6.7	0.79	9.67	1.14	1983

3.1. Ranking of the alternatives using fuzzy MCDM methods

The selection of the optimum conditions of the tribological process parameters was considered to reveal the applicability of fuzzy-TOPSIS, fuzzy-COPRAS, and fuzzy-EDAS method. Previously, the applicable steps of the techniques were discussed. After obtaining the weightage of the responses in accordance with the decision of the decision-maker, different MCDM techniques were used to rank the alternatives.

3.1.1. Ranking of the alternatives using fuzzy-TOPSIS method

Value of each response was normalized using Eq. 1 to obtain the normalized matrix (Supplementary Table 1) and this value was further multiplied with fuzzy weight of responses (Table 5) to construct the fuzzy normalized weighted matrix (Supplementary Table 2). With the help of positive and negative ideal solutions closeness coefficient value was determined for each alternative (Table 7) and based on this coefficient value ranking of the alternative was obtained. Experiment number EN_{27} (L = 10 N, SV = 0.2 m/s, and SD = 1500 m) secured first rank with highest closeness coefficient value (0.843) whereas experiment number EN_{14} (L = 20 N, SV = 0.3 m/s, and SD = 2000 m) secured last rank with lowest closeness coefficient value (0.217) among all 27 number of experiments.

Table 7. Coefficient of closeness and ranking of the alternatives

Experiment number	Positive ideal solution (d_i^+)	Negative ideal solution (d_i^-)	Closeness coefficient (CoC_i)	Rank
EN_1	0.178	0.300	0.628	11
EN_2	0.215	0.310	0.591	13
EN_3	0.237	0.285	0.546	16
EN_4	0.291	0.224	0.435	22
EN_5	0.295	0.229	0.438	21
EN_6	0.220	0.301	0.578	14
EN_7	0.248	0.273	0.524	17
EN_8	0.232	0.289	0.555	15
EN_9	0.137	0.415	0.752	5
EN_{10}	0.111	0.446	0.801	4
EN_{11}	0.335	0.176	0.344	25
EN_{12}	0.109	0.449	0.804	3
EN_{13}	0.187	0.350	0.651	10
EN_{14}	0.399	0.111	0.217	27
EN_{15}	0.140	0.393	0.737	6
EN_{16}	0.253	0.270	0.516	19
EN_{17}	0.088	0.473	0.843	2
EN_{18}	0.207	0.317	0.605	12
EN_{19}	0.344	0.168	0.328	26
EN_{20}	0.164	0.372	0.694	8
EN_{21}	0.278	0.239	0.462	20
EN_{22}	0.148	0.403	0.731	7
EN_{23}	0.247	0.269	0.521	18
EN_{24}	0.316	0.197	0.385	24
EN_{25}	0.166	0.368	0.689	9
EN_{26}	0.310	0.211	0.405	23
EN_{27}	0.088	0.473	0.843	1*

*Most preferable setting of tribological process parameters

3.1.2. Ranking of the alternatives using fuzzy-COPRAS method

In this method normalization of response value was similar to the fuzzy TOPSIS method. Hence, the same normalized decision matrix (Supplementary Table 1) and fuzzy normalized weighted matrix (Supplementary Table 2) were used for the fuzzy COPRAS method. The next step was to calculate the relative significance value for each alternative using Eq. 11 and the calculated value tabulated in Table 8. The relative significance value performance score was obtained using Eq. 13 and with the help of this value ranking of alternatives was determined (Table 10). The highest performance score (100) was determined for experiment number EN_{27} (L = 10 N, SV = 0.2 m/s, and SD = 1500 m) and lowest performance score (41.359) was determined for the experiment number EN_{14} (L = 20 N, SV = 0.3 m/s, and SD = 2000 m).

Table 8. Performance score and ranking of the alternatives

Experiment number	Relative significance value (Qi)	Performance score (Ui)	Rank
EN_1	0.089	72.557	11
EN_2	0.081	66.523	13
EN_3	0.077	63.328	16
EN_4	0.065	53.273	23
EN_5	0.069	56.402	20
EN_6	0.080	65.073	14
EN_7	0.074	60.319	18
EN_8	0.078	63.344	15
EN_9	0.105	86.033	5
EN_{10}	0.114	92.962	4
EN_{11}	0.058	47.014	26
EN_{12}	0.114	93.270	3
EN_{13}	0.091	74.459	10
EN_{14}	0.051	41.359	27
EN_{15}	0.102	83.410	7
EN_{16}	0.074	60.617	17
EN_{17}	0.122	99.546	2
EN_{18}	0.082	66.837	12
EN_{19}	0.058	47.734	25
EN_{20}	0.097	78.944	8
EN_{21}	0.068	55.616	21
EN_{22}	0.102	83.656	6
EN_{23}	0.072	58.895	19
EN_{24}	0.061	50.247	24
EN_{25}	0.094	77.123	9
EN_{26}	0.066	53.966	22
EN_{27}	0.122	100.000	1*

*Most preferable setting of tribological process parameters

3.1.3. Ranking of the alternatives using the fuzzy-EDAS method

In this method, initially, the average value of each response was calculated (Table 6). In the next step, positive (PDA_{ij}) and negative (NDA_{ij}) distances from the average solution were calculated (Supplementary Table 3 and Supplementary Table 4 respectively). Further, the fuzzy weight of the criterion was multiple with the value of positive and negative distances respectively, to obtain the fuzzy weighted sum of positive ($\widetilde{sp}_i$) and negative distance ($\widetilde{sn}_i$) from the average solution (Supplementary Table 5 and Supplementary Table 6 respectively). The next step is to calculate the normalized weighted sum of positive ($\widetilde{nsp}_i$) and negative ($\widetilde{nsn}_i$) distance from the average solution (Table 9). Finally, the appraisal score was calculated using Eq. 21 for each alternative, and based on the appraisal score, a ranking of alternatives was derived (Table 9). Experiment number EN_{27} (L = 10 N, SV = 0.2 m/s, and SD = 1500 m) was obtained first rank with the highest appraisal value (0.549) whereas experiment number EN_{14} (L = 20 N, SV = 0.3 m/s, and SD = 2000 m) was obtained last rank with the lowest appraisal value (0.070) among all 27 number of experiments.

Table 9. Normalized weighted sum of positive and negative distance, appraisal score and ranking of the alternatives

Experiment Number	Normalized weighted sum of nsp_i	Normalized weighted sum of nsn_i	Appraisal value (as_i)	Rank
EN_1	0.266	0.126	0.196	21
EN_2	0.932	0.016	0.474	3
EN_3	0.212	0.319	0.265	16
EN_4	0.136	0.579	0.358	10
EN_5	0.000	0.390	0.195	22
EN_6	0.065	0.129	0.097	26
EN_7	0.157	0.344	0.250	17
EN_8	0.090	0.190	0.140	24
EN_9	0.692	0.063	0.377	9
EN_{10}	0.849	0.066	0.458	4
EN_{11}	0.053	0.766	0.410	6
EN_{12}	0.837	0.032	0.435	5
EN_{13}	0.456	0.184	0.320	14
EN_{14}	0.070	0.073	0.072	27
EN_{15}	0.416	0.000	0.208	20
EN_{16}	0.002	0.205	0.104	25
EN_{17}	0.019	1.000	0.509	2
EN_{18}	0.235	0.233	0.234	18
EN_{19}	0.000	0.699	0.349	11
EN_{20}	0.520	0.148	0.334	12
EN_{21}	0.096	0.484	0.290	15
EN_{22}	0.675	0.116	0.395	7
EN_{23}	0.206	0.447	0.326	13
EN_{24}	0.104	0.662	0.383	8
EN_{25}	0.314	0.000	0.157	23
EN_{26}	0.000	0.452	0.226	19
EN_{27}	1.000	0.091	0.546	1*

*Most preferable setting of tribological process parameters

Thus, according to all the proposed MCDM methods, experiment number EN_{27} (L = 10 N, SV = 0.2 m/s, and SD = 1500 m) was the most suitable parametric setting for the tribological test of duplex TiAlN coating. With this parametric setting, the desirable value of wear responses was obtained whereas, the undesirable value was obtained with the parametric setting of L = 20 N, SV = 0.3 m/s, and SD = 2000 m (experiment number EN_{14}) and this parametric setting was the worst parametric seating suggested by all the proposed MCDM methods.

3.2. Sensitivity analysis

Sensitivity analysis was conducted to understand the stability of the rankings under different sets of response weights (Table 10). Based on these weights, a ranking of alternatives was obtained using all the proposed MCDM methods (Fig. 2). There are four scenarios of a group of three decision makers (Table 10 (a-d)), and based on their opinion criteria weights were calculated (Table 10(a'-d')).

Table 10. Group of decision makers and fuzzy criteria weights

(a) Opinion of the decision maker for scenario 1

Responses	Scenario 1		
	DM1	DM2	DM3
Ra	M	M	LR
COF	L	LR	L
WML	L	M	M
WD	M	M	LR
HV	HT	HT	H

(a') Fuzzy criteria weight of scenario 1

Responses	Fuzzy criteria weight
Ra	(0.200, 0.367, 0.567)
COF	(0.067, 0.233, 0.433)
WML	(0.233, 0.433, 0.633)
WD	(0.200, 0.367, 0.567)
HV	(0.767, 0.900, 0.967)

(b) Opinion of the decision maker for scenario 2

Responses	Scenario 2		
	DM1	DM2	DM3
Ra	LR	L	M
COF	M	M	L
WML	LT	L	M
WD	L	L	LR
HV	H	HR	HR

(b') Fuzzy criteria weight of scenario 2

Responses	Fuzzy criteria weight
Ra	(0.033, 0.167, 0.367)
COF	(0.233, 0.433, 0.633)
WML	(0.133, 0.267, 0.433)
WD	(0.067, 0.233, 0.433)
HV	(0.567, 0.767, 0.933)

(c) Opinion of the decision maker for scenario 3

Responses	Scenario 3		
	DM1	DM2	DM3
Ra	L	LT	L
COF	LR	M	L
WML	M	LR	M
WD	M	L	M
HV	HT	H	HR

(c') Fuzzy criteria weight of scenario 3

Responses	Fuzzy criteria weight
Ra	(0.067, 0.200, 0.367)
COF	(0.133, 0.300, 0.500)
WML	(0.200, 0.367, 0.567)
WD	(0.233, 0.433, 0.633)
HV	(0.700, 0.867, 0.967)

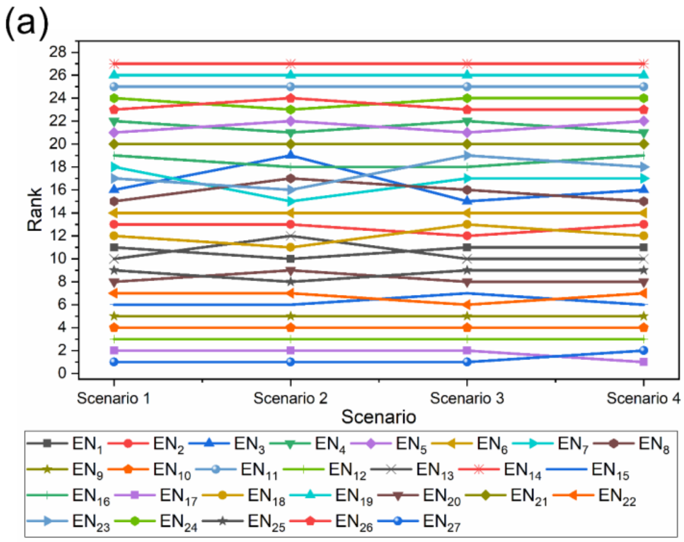
(d) Opinion of the decision maker for scenario 4

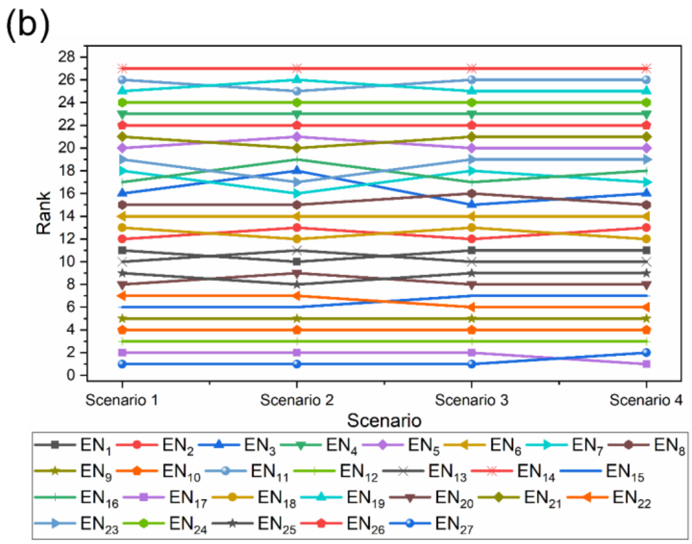
Responses	Scenario 4		
	DM1	DM2	DM3
Ra	LT	L	LR
COF	M	LR	M
WML	LT	M	M
WD	LR	L	L
HV	HR	HT	H

(d') Fuzzy criteria weight of scenario 4

Responses	Fuzzy criteria weight
Ra	(0.033, 0.133, 0.300)
COF	(0.200, 0.367, 0.567)
WML	(0.200, 0.333, 0.500)
WD	(0.067, 0.233, 0.433)
HV	(0.700, 0.867, 0.967)

The finding of sensitivity analysis for the F-TOPSIS method is represented in Figure 2(a). There are no changes observed in the ranking of experiment numbers EN_6, EN_9, EN_{10}, EN_{11}, EN_{12}, EN_{14}, EN_{19}, and EN_{21} when the value of fuzzy weight was changed. But there were few changes observed in the ranking of experiment numbers EN_1, EN_2, EN_{13}, EN_{15}, EN_{17}, EN_{20}, EN_{22}, EN_{24}, EN_{25}, EN_{26}, and EN_{27}. The ranking of the remaining experiment numbers was changed frequently and it was not stable at all. The sensitivity results of the F-COPRAS method (Figure 2(b)) showed that there was no effect of criteria weight change observed on the ranking of experiment numbers EN_4, EN_6, EN_9, EN_{10}, EN_{12}, EN_{14}, EN_{24}, and EN_{26}. Unlike the remaining experiment, numbers could not hold their actual ranking and there were changes observed with criteria weight change. The F-EDAS method (Figure 2(c)) shows more consistent in their ranking of the experiment numbers against criteria weight change and the experiments are EN$_2$, EN$_3$, EN$_6$, EN$_8$, EN$_{10}$, EN$_{11}$, EN$_{12}$, EN$_{14}$, EN$_{16}$, EN$_{17}$, EN$_{21}$, EN$_{22}$, EN$_{25}$, and EN$_{27}$. But there were few experiments (EN$_5$, EN$_7$, EN$_9$, and EN$_{15}$) whose ranking slightly changed with criteria weight change. The rest of the experiment number changes its ranking frequently against criteria weight change.





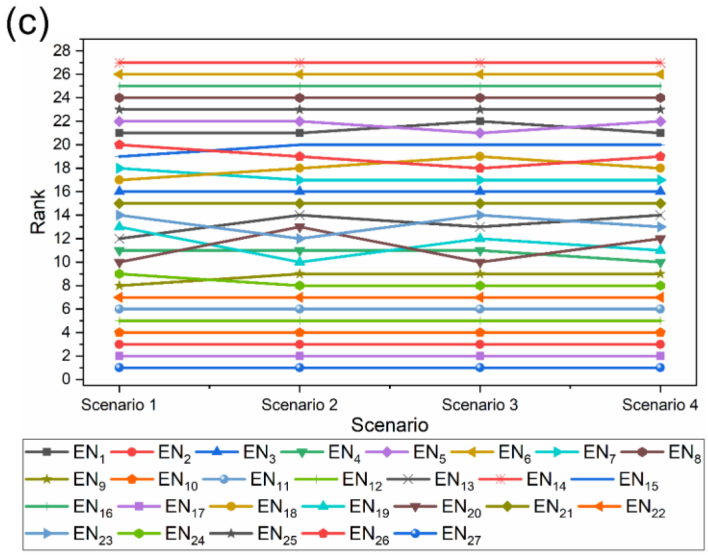


Figure 2. Result of the sensitivity analysis for different ranking methods viz; (a) F-TOPSIS, (b) F-COPRAS, and (c) F-EDAS.

From sensitivity analysis, it was noted that the F-EDAS method was less sensitive to criteria weight change compared to F-TOPSIS and F-COPRAS methods. Moreover was, it noticed that ranking of the best alternative (experiment number EN_{27}) was changed with criteria weight change in F-TOPSIS and F-COPRAS methods. Thus, it can be said that the stability of the ranking given by the F-EDAS was the highest compared to F-TOPSIS and F-COPRAS methods. Thus, F-EDAS was the more robust method to solve this kind of multi-attributed problem. These obtained results were further validated by a comparative study, where Spearman's rank correlation coefficient was calculated for each scenario of MCDM methods.

3.2.1. Comparison of MCDM methods

Spearman's rank correlation coefficient for F-TOPSIS methods is shown in Table 11(a). The correlation coefficient value of each scenario shows that there is a lack of inconsistency in the ranking of the F-TOPSIS method according to different fuzzy criteria weights. From Table 11(a), it can be seen that the correlation coefficient value for scenario-(1-2), scenario-(1-3), scenario-(1-4), scenario-(2-3), scenario-(2-4) and scenario-(3-4) are 0.989, 0.996, 0.998, 0.985, 0.989 and 0.996 respectively. It can be said that coefficient values are varying from 0.985 to 0.996. Similarly, for F-COPRAS method (Table 11(b)) the correlation coefficient is obtained for scenario-(1-2), scenario-(1-3), scenario-(1-4), scenario-(2-3), scenario-(2-4) and scenario-(3-4) are 0.992, 0.998, 0.997, 0.989, 0.993 and 1.000 respectively. Here the coefficient values are varying from 0.989 to 1.000 and this range is higher than the F-TOPSIS range of spearman coefficient value. For the F-EDAS method (Table 11(c)), the value of correlation coefficient value for all the scenarios is higher than 0.990. In other words, it can be said that the Spearman correlation coefficient for the scenario the of F-EDAS method is higher than the F-TOPSIS and F-COPRAS methods. Based on the overall results of sensitivity analysis and correlation coefficient the F-EDAS method is the most robust method to solve the multi-attribute decision-making problem.

Table 11. Spearman's rank correlation coefficient

(a) Coefficient values for F-TOPSIS

Scenarios	S_2	S_3	S_4
S_1	0.989	0.996	0.998
S_2	-	0.985	0.989
S_3	-	-	0.996

(b) Coefficient values for F-COPRAS

Scenarios	S_2	S_3	S_4
S_1	0.992	0.998	0.997
S_2	-	0.989	0.993
S_3	-	-	1.000

(c) Coefficient values for F-EDAS

Scenarios	S_2	S_3	S_4
S_1	0.990	0.995	0.994
S_2	-	0.993	0.999
S_3	-	-	0.996

3.3. Other wear parameter optimization problems solved by the proposed methodology

In this section, the proposed methodology solves two wear optimization problems, which have already been solved and published elsewhere. The first problem is the optimization of wear parameters for composite coating, while the second problem is to optimize the wear parameters for heat-insulated ceramic coating.

3.3.1. Optimization of wear parameter for composite coating

This optimization problem was solved using the gray relation analysis (GRA) method (Raghavendra et al. 2021). Table 12 presents the alternatives for wear parameters and their criteria, based on which alternatives were ranked. Each criterion presented in Table 12 was identified as non-beneficial criteria, and the criteria weight (Table 14) was derived using the opinion of decision-makers as mentioned in Table 13.

Table 12. List of alternatives and their criteria (Initial decision matrix) method (Raghavendra et al. 2021)

Alternative	(Specific wear rate, Ws) C_1	(Pin Temperature, P_T) C_2	(Friction Coefficient, CoF) C_3
EN_1	0.3330	91.990	0.123
EN_2	0.3470	92.140	0.038
EN_3	0.8750	98.340	0.144
EN_4	0.2520	90.760	0.089
EN_5	1.1900	94.840	0.153
EN_6	0.4000	73.960	0.089
EN_7	1.5550	105.990	0.116
EN_8	0.4770	78.660	0.011

EN_9	0.4750	88.760	0.065
EN_{10}	0.8530	87.810	0.627
EN_{11}	0.2920	86.910	0.046
EN_{12}	0.6411	80.120	0.103
EN_{13}	0.7020	93.690	0.112
EN_{14}	1.2400	90.160	0.035
EN_{15}	1.1710	111.780	0.119
EN_{16}	0.7840	91.920	0.459
EN_{17}	2.1320	110.380	0.104
EN_{18}	1.4450	95.910	0.099
EN_{19}	1.5500	88.480	0.119
EN_{20}	1.3700	117.190	0.016

Table 13. Opinion of the decision maker for problem 1

Response	DM1	DM2	DM3
W_S	LR	LR	L
P_T	L	M	M
CoF	L	L	M

Table 14. Fuzzy criteria weight for problem 1

Responses	Fuzzy criteria weight
W_S	(0.033, 0.100, 0.233)
P_T	(0.233, 0.433, 0.633)
CoF	(0.167, 0.367, 0.567)

One by one, each MCDM method (F-TOPSIS, F-COPRAS, and F-EDAS) was employed to derive the ranking of alternatives (Table 15). From the obtained results (Table 15), it was noticed that the ranking of the best alternative (EN_6) remains similar to it obtained in the past study method (Raghavendra et al. 2021) [36]. Further, the correlation between rankings was studied by calculating Spearman's rank correlation coefficient. It found these rankings have a good correlation as their coefficient value lies above 0.767, in the acceptable range.

Table 15. Coefficient of closeness, performance score, appraisal score of alternatives, and its ranking

Alternative	F-TOPSIS		F-COPRAS		F-EDAS		Rank method (Raghavendra et al. 2021)
	Closeness coefficient (CoC_i)	Rank	Performance score (U_i)	Rank	Appraisal value (as_i)	Rank	
EN_1	0.643	10	8.878	10	0.623	10	8
EN_2	0.640	11	8.854	11	0.871	3	4
EN_3	0.499	16	7.762	16	0.485	17	12
EN_4	0.670	9	9.123	9	0.740	8	5
EN_5	0.579	14	8.338	14	0.463	18	15
EN_6	1.000	1	13.734	1	1.000	1	1
EN_7	0.313	17	6.676	17	0.499	14	18
EN_8	0.915	2	12.147	2	0.804	4	2
EN_9	0.713	7	9.539	7	0.789	6	7
EN_{10}	0.718	6	9.553	6	0.036	20	19
EN_{11}	0.752	4	9.952	4	0.883	2	3
EN_{12}	0.886	3	11.698	3	0.695	9	6
EN_{13}	0.605	13	8.556	13	0.593	11	10

EN_{14}	0.683	8	9.232	8	0.792	5	9
EN_{15}	0.162	19	6.009	19	0.497	16	16
EN_{16}	0.637	12	8.807	12	0.196	19	17
EN_{17}	0.199	18	6.147	18	0.498	15	20
EN_{18}	0.555	15	8.153	15	0.571	12	14
EN_{19}	0.718	5	9.569	5	0.534	13	13
EN_{20}	0.018	20	5.468	20	0.786	7	11

3.3.1. Optimization of wear parameter for heat-insulated ceramic coating

The WASPAS method was used to solve this optimization problem by Sahoo et al. in the past study (Sahoo et al. 2021). The evaluating criteria and alternative wear parameters are listed in Table 16. There are two criteria, namely weight loss, and friction coefficient, which are identified as non-beneficial criteria. The weights (Table 18) of these criteria were obtained based on the decision of the expert panel (Table 17).

Table 16. List of criteria and alternatives (Sahoo et al. 2021)

Alternative	(Weight loss (W_l), mg) C_1	(Friction coefficient (CoF), μ) C_2
EN_1	0.19	0.077
EN_2	0.60	0.084
EN_3	4.70	0.026
EN_4	5.10	0.040
EN_5	3.50	0.079
EN_6	9.20	0.064
EN_7	14.20	0.080
EN_8	9.30	0.080
EN_9	9.90	0.067
EN_{10}	20.20	0.090
EN_{11}	11.20	0.087
EN_{12}	17.00	0.057
EN_{13}	19.20	0.078
EN_{14}	13.50	0.070
EN_{15}	9.20	0.170
EN_{16}	9.20	0.063

Table 17. Opinion of the decision maker for problem 2

Response	DM1	DM2	DM3
W_l	LT	LR	L
CoF	L	L	M

Table 18. Fuzzy criteria weight for problem 2

Response	Fuzzy criteria weight
W_l	(0.033, 0.133, 0.300)
CoF	(0.167, 0.367, 0.567)

The obtained criteria weights were integrated with MCDM methods as described in sections 2.4, 2.5, and 2.6 to derive the ranking of alternatives. The derived rankings are listed in Table 19, and a minor deviation can be observed in the ranking of alternatives. But this deviation does not affect the overall results. The ranking of the best alternative remains the same for each MCDM method, which exactly matches the past result (Sahoo et al. 2021). Although, these rankings have an excellent correlation among them as Spearman's rank correlation coefficient values are equal and more than 0.85.

Table 19. Final preference values of alternatives and its ranking

Alternative	F-TOPSIS		F-COPRAS		F-EDAS		Rank (Sahoo et al. 2021)
	Closeness coefficient (CoC_i)	Rank	Performance score (U_i)	Rank	Appraisal value (as_i)	Rank	
EN_1	0.988	1	246.310	1	0.731	1	1
EN_2	0.985	2	199.395	2	0.683	4	3
EN_3	0.948	4	75.490	4	1.000	3	2
EN_4	0.938	5	61.938	5	0.885	2	4
EN_5	0.962	3	88.167	3	0.638	5	5
EN_6	0.797	7	19.312	7	0.602	7	7
EN_7	0.520	13	8.259	13	0.400	13	12
EN_8	0.789	8	18.375	8	0.489	10	11
EN_9	0.765	9	16.726	9	0.563	8	9
EN_{10}	0.049	16	4.137	16	0.242	15	15
EN_{11}	0.696	11	12.859	11	0.415	12	13
EN_{12}	0.321	14	5.900	14	0.515	9	8
EN_{13}	0.131	15	4.597	15	0.323	14	14
EN_{14}	0.567	12	9.189	12	0.478	11	10
EN_{15}	0.755	10	14.560	10	0.014	16	16
EN_{16}	0.797	6	19.344	6	0.610	6	6

3. Conclusions

This study focuses on the optimization of the wear parameters for duplex-TiAlN coated MDC-K tool steel. Three different fuzzy MCDM methods were proposed to solve this optimization problem. A total of five wear responses, namely surface roughness, friction coefficient, wear mass loss, wear depth, and hardness, were identified as the criteria to evaluate the alternatives, which consist of different combinations of wear parameters such as applied load, sliding velocity, and sliding distance. The criteria weight was determined using triangular fuzzy numbers that are integrated into fuzzy MCDM methods to solve the problem. The following conclusions are drawn from the results:

- The obtained results showed that alternative EN_{27} (L = 10 N, SV = 0.2 m/s, and SD = 1500 m) to be the best alternative whereas EN_{14} (L = 20 N, SV = 0.3 m/s, and SD = 2000 m) as the worst alternative parameters for duplex-TiAlN coated MDC-K tool steel.
- These results were tested and validated by performing a comprehensive sensitivity analysis. Additionally, two sets of wear parameters from the literature were also solved using the proposed methodology to substantiate its capability. The result obtained from the proposed methodology was found similar to the result obtained in the literature.
- The validation result proved that the F-EDAS method is more robust and less sensitive to the criteria weight change. Hence, it can be further used to solve

this type of multi-decision-making problem with some modifications (either addition or removal of new alternatives or criteria).

The proposed methodology is designed to solve the multi-criteria such as the selection of optimal parameters for duplex-TiAlN coating, where three wear parameters (load, sliding velocity, and sliding distance) and five wear responses (Ra, COF, WML, WD, and Hv) were considered to solve the above problem. Further, It was noticed that if some new evaluating criteria were introduced, the calculation process becomes lengthy which grows exponentially for high-dimensional decision-making problems.

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Appendix

Nomenclature			
L	Load (N)	FT	Fuzzy-TOPSIS
SV	Sliding velocity (m/s)	FC	Fuzzy-COPRAS
SD	Sliding distance (m)	FE	Fuzzy-EDAS
Ra	Average surface roughness (μm)	TOPSIS	Technique for order of preference by similarity to ideal solution
CoF	Coefficient of friction	COPRAS	Complex proportional Assessment
WML	Wear mass loss (mg)	EDAS	Evaluation based on distance from the average solution
WD	Wear depth (μm)	S_1	Scenario 1
HV	Vickers hardness	S_2	Scenario 2
EN	Experiment number	S_3	Scenario 3
MCDM	Multi-criteria decision making	S_4	Scenario 4