Operational Research in Engineering Sciences: Theory and Applications
Vol. 6, Issue 1, 2023, pp. 44-64
ISSN: 2620-1607
eISSN: 2620-1747
DOI: https://doi.org/10.31181/oresta/060103

INVESTIGATION OF THE APPROPRIATE DATA NORMALIZATION METHOD FOR COMBINATION WITH PREFERENCE SELECTION INDEX METHOD IN MCDM

Duc Trung Do [1], Van Dua Tran [1], Van Duc Duong [1], Nhu-Tung Nguyen [2*]

[1] Faculty of Mechanical Engineering, Hanoi University of Industry, Vietnam.
[2] HaUI Institute of Technology, Hanoi University of Industry, Vietnam.

Received: 19 November 2022
Accepted: 11 April 2023
First Online: 30 April 2023

Research Paper

***Abstract**: Preference Selection Index (PSI) that is a Multi-Criteria Decision Making Method (MCDM) does not need to determine the weights for criteria and it has been applied in many different fields. However, using only the data normalization method (DNM) proposed by the inventor of the PSI method may narrow the application scope of this method. This study aims to expand the application range of the PSI method by identifying the appropriate DNMs in combination with the PSI method. Twelve different DNMs were used in combination with the PSI method. These twelve combinations were used in turn to solve several problems in different fields. The ranked results of solutions by these combinations were all compared with the results in the published studies. The sensitivity analysis of the ranked results of the solutions in each case also was performed. In this study, four out of twelve DNMs were found to be appropriate in combination with the PSI method. This discovery has extended the application scope of the PSI method that the previous methods have not met.*

***Keywords**: MCDM, PSI, DNM.*

1. Introduction

Most MCDM methods perform the steps of determining the weights and normalizing the data. Therefore, the ranked results of the solutions depend significantly on the selection of the weighting method and the data normalization method. The research direction to rank solutions using the MCDM method without using the weighting method or without using the data normalization method is being studied by scientists to improve the stability of MCDM.

PSI that is a MCDM method does not need to determine the weights for the criteria. The detailed steps to ranking the solutions according to this method will be presented

[*]Corresponding Author:
doductrung@haui.edu.vn (D.T.Do), duatv@haui.edu.vn (V.D. Tran), duongvanduc@haui.edu.vn (V.D. Duong), tungnn@haui.edu.vn (Nhu-Tung. Nguyen)

in section three of this paper. The application of this method is also considered to be very simple with a small number of calculations (Yadav et al., 2019). This method has been applied to multi-criteria decision making in many cases, in many different fields: to evaluate the performance of machines (SARI, 2019), to propose a method for waste recovery from electrical/electronic products (Sari, 2020), to choose an automated system development method in selecting the students with enough conditions to receive the scholarship (Arifin & Saputro, 2022), for decision-making in the selection of materials for tooth restoration/beautification (Yadav, 2022), to choose the life cycle design solutions of the product system (Attri & Grover, 2015), to select the technological parameters for turning (Prasad et al., 2018), to select the parameters of Electrical Discharge Machining (Phan et al., 2022), to select the technological parameters for the grinding process (Tien et al., 2021), to rank the efficiency of production lines (Akyüz, 2015), to rank the types of materials for engineering (Maniya & Bhatt, 2010), to rank the individuals with enough conditions for credit loans in Indonesia (Sianturi et al., 2020), to choose where to sell used computers (Sahir et al., 2018), to compare the tourism potential of some countries (Stanujkic et al., 2020), to select the machines in the manufacturing companies (Jian et al., 2015), and so on. Thus, it is seen that the PSI method has been successfully applied for MCDM in many different fields.

However, the authors of this study can confirm that all applied PSI studies used linear normalization to normalize the data. Linear normalization is also the method used by the scientists who proposed the PSI method. The formulas for normalizing data in this way as well as many other ways of data normalization will be presented in the second section of this study. However, linear normalization cannot be used if some criterion is equal to zero in some solutions. In these cases, if cannot find other DNMs in combination with the PSI method, the application of the PSI method will not be possible. From this point of view, this study will combine all twelve above-mentioned DNMs with PSI method to identify the appropriate DNMs in combining with PSI method. This is the first study using all twelve DNMs in combination with one MCDM method. Those twelve combinations were used to rank the solutions from different fields. In addition to the linear normalization method, this study identified three other DNMs that were determined to be suitable for combining with the PSI one. This obtained result contributes to extend the application scope of the PSI method.

The structure of the next sections of this study is presented as follows: (1) The literature review presented the importance of determining an appropriate DNM to combine with one of the MCDM methods. This section also presented the formulas for normalizing data by twelve different methods. The suitability of combining some DNMs with some MCDM methods was also confirmed in published studies as the third content in this section; (2) Summary the performed steps according to the PSI method; (3) Perform the calculations in different cases to rank the solutions in different fields using the PSI method; (4) Identify the DNMs (when combined with PSI method) that show the same best solution as in the published studies; (5) Analyze the sensitivity of the ranking results in each case by creating different scenarios to confirm the appropriate DNMs when combined with the *PSI* method; (6) Discuss the obtained results and draw the conclusions from this study as well as propose the research directions in the future (Arato & Kano, 2021).

2. Literature review

Except for some methods such as Collaborative Unbiased Rank List Integration (*CURLI*) and Ranking of the attributes and alternatives (*R*), for most of the remaining MCDM methods, data normalization is the work that needs to be conducted when apply them (Trung, 2022b). Each MCDM method that was proposed often contains at least one DNM. However, because the implementation method in MCDM methods as well as in DNMs is not the same, the ranked results of the solutions when using MCDM methods are also not the same (Zopounidis & Doumpos, 2017). Selection of the DNM has a great influence on the ranking results of the solutions (Aytekin, 2021; Budiman et al., 2021; Souissi & Hafdhi, 2021). When comparing the two methods Vlsekriterijumska optimizacijaI KOmpromisno Resenje (*VIKOR*) and Technique for Order Preference by Similarity to Ideal Solution (*TOPSIS*), the authors have concluded that the ranked results of the solutions are different when using these two methods.

The reason is that these two methods used different DNMs (Opricovic & Tzeng, 2004). Mhlanga and Lall (2022) used the *VIKOR* method to rank ten websites in combination with five different DNMs (Mhlanga & Lall, 2022). This study has shown very different results in those combinations. A solution may rank number one when using one DNM but rank number ten (last rank) when using another DNM. Yazdani et al. (2017) used the COmplex PRoportional ASsessment of alternatives with Grey relations (*COPRAS-G*) method to rank the material types (Yazdani et al., 2017). The authors concluded that the suitability of a DNM when combined with an MCDM method depends on the number of solutions as well as the number of criteria. Sarraf and McGuire (2021) also concluded that with the same DNM but when combined with different MCDM methods, the ranking results can also be different (Sarraf & McGuire, 2021).

The above analysis shows that the determination of the suitable DNM for each MCDM method has a decisive influence on the ranking results of the solutions. It is a very important work to ensure the accuracy of the ranking results of the solutions. Twelve DNMs that listed below are the combined results from two studies of (Aytekin, 2021; Ersoy, 2021a).

Linear normalization (N1)

$$N_{ij} = \frac{y_{ij}}{max\, y_{ij}}, if\ j\ \in B \tag{1}$$

$$N_{ij} = \frac{\min y_{ij}}{y_{ij}}, if\ j\ \in C \tag{2}$$

Weitendorf normalization (N2)

$$N_{ij} = \frac{y_{ij} - \min y_{ij}}{max\, y_{ij} - min\, y_{ij}}, if\ j\ \in B \tag{3}$$

$$N_{ij} = \frac{max y_{ij} - y_{ij}}{max\, y_{ij} - min\, y_{ij}}, if\ j\ \in C \tag{4}$$

Sum linear normalization (N3)

$$N_{ij} = \frac{y_{ij}}{\Sigma_{i=1}^{m} y_{ij}}, if\ j \ \in B \tag{5}$$

$$N_{ij} = \frac{1/y_{ij}}{\Sigma_{i=1}^{m} 1/y_{ij}}, if\ j \ \in C \tag{6}$$

Vector normalization (N4)

$$N_{ij} = \frac{y_{ij}}{\sqrt{\Sigma_{i=1}^{m}(y_{ij})^2}}, if\ j \ \in B \tag{7}$$

$$N_{ij} = 1 - \frac{y_{ij}}{\sqrt{\Sigma_{i=1}^{m}(y_{ij})^2}}, if\ j \ \in C \tag{8}$$

Logarithmic normalization (N5)

$$N_{ij} = \frac{ln y_{ij}}{ln(\prod_{i=1}^{m} y_{ij})}, if\ j \ \in B \tag{9}$$

$$N_{ij} = 1 - \frac{ln y_{ij}}{ln(\prod_{i=1}^{m} y_{ij})}, if\ j \ \in C \tag{10}$$

Max linear normalization (N6)

$$N_{ij} = \frac{y_{ij}}{max\, y_{ij}}, if\ j \ \in B \tag{11}$$

$$N_{ij} = 1 - \frac{y_{ij}}{max\, y_{ij}}, if\ j \ \in B \tag{12}$$

Min linear normalization (N7)

$$N_{ij} = 1 - \frac{min y_{ij}}{y_{ij}}, if\ j \ \in B \tag{13}$$

$$N_{ij} = \frac{min y_{ij}}{y_{ij}}, if\ j \ \in C \tag{14}$$

Jüttler-Körth normalization (N8)

$$N_{ij} = 1 - \left|\frac{max y_{ij} - y_{ij}}{max y_{ij}}\right|, if\ j \ \in B \tag{15}$$

$$N_{ij} = 1 - \left|\frac{min y_{ij} - y_{ij}}{max y_{ij}}\right|, if\ j \ \in C \tag{16}$$

Peldschus normalization (N9)

$$N_{ij} = \left(\frac{y_{ij}}{max\, y_{ij}}\right)^2, if\ j \ \in B \tag{17}$$

$$N_{ij} = \left(\frac{y_{ij}}{max\, y_{ij}}\right)^3, if\ j \ \in C \tag{18}$$

Stop normalization (N10)

$$N_{ij} = \frac{100y_{ij}}{max\, y_{ij}}, if\ j\ \in B \quad (19)$$

$$N_{ij} = \frac{100miny_{ij}}{max\, y_{ij}}, if\ j\ \in C \quad (20)$$

Z-score normalization (N11)

$$N_{ij} = \frac{y_{ij} - \frac{\Sigma_{i=1}^{m} y_{ij}}{m}}{\sqrt{\frac{\Sigma_{i=1}^{m}(y_{ij}-\mu_j)^2}{m}}}, if\ j\ \in B \quad (21)$$

$$N_{ij} = -\frac{y_{ij} - \frac{\Sigma_{i=1}^{m} y_{ij}}{m}}{\sqrt{\frac{\Sigma_{i=1}^{m}(y_{ij}-\mu_j)^2}{m}}}, if\ j\ \in C \quad (22)$$

Enhanced accuracy normalization (N12)

$$N_{ij} = 1 - \frac{maxy_{ij} - y_{ij}}{\Sigma_{i=1}^{m}(maxy_{ij} - y_{ij})}, if\ j\ \in B \quad (23)$$

$$N_{ij} = 1 - \frac{y_{ij} - miny_{ij}}{\Sigma_{i=1}^{m}(y_{ij} - miny_{ij})}, if\ j\ \in c \quad (24)$$

In the equations from Eq. (1) to Eq. (24), *yij* is the value of criterion *j* at the solution *i*; N_{ij} is the normalized value of criterion *j* in solution *i; B* describes the larger the better criterion; *C* describes the smaller the better criterion; *m* is the number of solutions; μ_j is the mean value of the solutions of the criterion *j*. In addition to have to determine the appropriate DNM in combining with each MCDM method as mentioned above, even if a suitable DNM has been identified, but if only one DNM in combining with a MCDM method may narrow the application scope of that MCDM method.

The analysis results from mentioned above about twelve DNMs show that, if there exists a certain criterion whose maximum value is zero, then the methods N1, N3, N5, N6, N7, N8, N9, and N10 will not be available. Or when there exists at least one value of a certain criterion is negative, the N5 method cannot be used. At that time, if an alternative DNM cannot be identified, the decision-making will be difficult, even impossible. However, even if a different DNM is chosen to instead, will the ranked results of the solutions be accurate? Because the ranked results of the solutions are heavily influenced by the used DNMs (Aytekin, 2021; Dragiša et al., 2013; Kaplinski & Tamošaitienė, 2015; Trung, 2022b).

From this aspect, many studies that have been performed to combine each MCDM method with several different DNMs. The aim of these studies is determination of the suitable DNMs when combining with each MCDM method. Sanjib and Dragan (2021) simultaneously used two methods N1 and N5 to combine with COmbinative Distance-based Assessment (*CODAS*) method when ranking the smartphones (Sanjib & Dragan, 2021). They found that in determining the best solution, N1 was equivalent to N5, but in terms of rank inversion, N5 was better than N1. Trung (2022b) combined the *CODAS* method with six methods including N1, N2, N3, N4, N5, and N6 to make a decision in choosing a robot, assessing the air quality in the working room, and evaluating the machining in lathe machine (Trung, 2022b). The author showed that if only in terms of finding the best solution, the five methods including N1, N2, N3, N4, and N5 are all suitable to combine with *CODAS* method except for N6 method. Vafaei et al. (2022) combined the Simple Additive Weighting (*SAW*) method with four methods including N2, N3, N4, and N6 to make decisions in the evaluation of the PhD candidates (Vafaei et al., 2022). They showed that only N2 is suitable for combination with the *SAW* method. Ersoy (2021a) combined the Proximity Indexed Value (*PIV*) method with N2, N11, and N12 to rank the financial position of forty-five companies (Ersoy, 2021a). He showed that only N2 is suitable to combine with the *PIV* method. Ersoy (2021b) combined the Range Of Value (*ROV*) method with eight methods including N1, N2, N3, N4, N6, N7, N9, and N12 to rank the financial performance of ten companies (ERSOY, 2021b). He concluded that only N9 was suitable for combining with the *ROV* method. Vafaei et al. (2016) combined the Analytic Hierarchy Process (*AHP*) method with 5 methods including N2, N3, N4, N5, and N6 to rank smart parking locations (Vafaei et al., 2016). They concluded that N6 was the most suitable method to combine with *AHP*, whereas the combination of *AHP* and N3 was the worst method. Martin (2021) combined two methods Weighted Aggregates Sum Product Assessment (*WASPAS*) and *TOPSIS* with four DNMs including N1, N2, N3, and N4 to select the food processing methods (Martin, 2021). This research showed an amazing result that all those combinations determine the best solution. Mic & Antmen (2021) used simultaneously three methods including the *WASPAS*, *TOPSIS*, and Multiobjective Optimization On the basis of Ratio Analysis (*MOORA*) to select the location of universities in Turkey (Miç & Antmen, 2021). Although the DNMs that were used in combination with the MCDM methods were different, all three cases gave a similar ranked result in all solutions. Zavadskas et al. (2022) combined the Simple Weighted Sum Product (*WISP-S*) method with three methods including N1, N3, and N4 to rank the solutions for a set of random numbers (Zavadskas et al., 2022). The authors have confirmed that the *WISP-S* method is really powerful when combined with all three DNMs. All these combinations gave the same ranking results. Vafaei et al. (2018) combined the *TOPSIS* method with six methods including N1, N2, N3, N4, N5, and membership function to rank the drone landing solutions (Vafaei et al., 2018). They confirmed that only N3 is suitable for combination with the *TOPSIS* method. In another study, Vafaei et al. (2021) also combines the *TOPSIS* method with six methods including N1, N2, N3, N4, N5, and membership function to select the cars (Vafaei et al., 2021). In this case, the authors point out that the membership function is the best method when combined with the *TOPSIS* method. Baghla and Bansal (2014) combined the *VIKOR* method with three methods including N1, N2, and N4 to rank the wireless internet systems (Baghla & Bansal, 2014). They showed that combining N2 with the *VIKOR* method gives the best results. Alrababah and Atyeh (2019) combined the *VIKOR* method with four methods including N1, N2, N3, and N4 to rank the products through the customer feedback (Alrababah & Atyeh, 2019). They showed that the combination of *VIKOR* and N4 gives the best results. Mathew et al. (2017) combined the *WASPAS* method with six methods including N2, N3, N4, N5, N6, and N12 to rank the robots (Mathew et al., 2017). The authors found that the combination of *WASPAS* with N2 gave the best results. Even, in several studies, when applying a certain MCDM method, people did not even use the DNMs available by itself but use other DNMs. Zolfani et al. (2020) combined simultaneously N5 with *TOPSIS* and *VIKOR* methods to rank the solutions in two cases, case one is the ranking of the apartments in Madrid (Spain) and the other is the ranking of the solutions with a set of random data (Zolfani et al., 2020). It should be noted that N5 is not the DNM proposed by the authors of both *TOPSIS* and *VIKOR* methods. However, an unexpected result occurred, the ranked results when combining *TOPSIS* with N5 completely coincided with the case when combining *VIKOR* with N5.

Thus, it is seen that finding the appropriate DNMs for each MCDM method has been carried out by many scientists and has also been applied in many different fields. In addition, any study that has done in this direction has attracted a lot of interest. Based on the characteristics of the *PSI* method as discussed in the introduction, this study was selected the *PSI* method to perform the research mission follow the proposed research direction.

3. PSI Method

The order of the performing the ranking of solutions according to the *PSI* method is presented as follows (Maniya & Bhatt, 2010).

- Build a decision matrix including the solutions and the criteria.

- Standardized the data.

+ For the larger the better criterion.

$$N_{ij} = \frac{y_{ij}}{max\, y_{ij}} \tag{25}$$

+ For the smaller the better criterion.

$$N_{ij} = \frac{min\, y_{ij}}{y_{ij}} \tag{26}$$

Eq. (25) and (26) that are data normalization formulas used by the proponent of the *PSI* method (method N1). The application cases in the next sections of this paper will fully apply all twelve DNMs as presented in section 2.

- Calculate the mean values of the standardized data (N).

$$N = \frac{1}{n}\sum_{i=1}^{n} N_{ij} \tag{27}$$

- Determine the preference values from the mean values (φ_j).

$$\varphi_j = \sum_{i=1}^{n}[N_{ij} - N_{ij}]^2 \tag{28}$$

- Determine the deviation in the preference values (θ_j).

$$\theta_j = [1 - \varphi_j] \tag{29}$$

- Determine the overall preference value (β_j) for the criteria.

$$\beta_j = \frac{\theta_j}{\sum_{j=1}^{m} \theta_j} \tag{30}$$

- Calculate the PSI_j of each solution, with $i = 1 \div m$

$$PSI_J = \sum_{j=1}^{m} N_{ij}.\beta_j \tag{31}$$

where n is the number of criteria.

- Rank the solutions according to the principle that the solution with the largest PSI_J is the best one.

To identify the appropriate DNMs when combined with the *PSI* method, this study performed ranking in several cases from the different fields. In each case, the number of criteria and the number of solutions is also different. Selecting the cases from different fields will lead to draw the most general conclusions. The selected cases were all referenced from published studies. The reason for this is: in those studies, the solutions were also ranked either by *PSI* method combined with N1 or by another MCDM method. The ranking results of the solutions in the published studies will be used to compare with the obtained ranking results in this study. Specific contents when ranking the solutions in each case are presented in the section 4 of this paper.

4. Results and Discussion

4.1 Application Cases

In this section, a combination of the *PSI* method and the twelve data normalization methods as described above will be used to rank the solutions in four different cases. The data of all four cases were referenced from published studies. In those studies, the ranking of the solutions was also performed by different *MCDM* methods. The ranked results of the solutions when using different *MCDM* methods will be used to compare with those ones when using *PSI* method.

Case 1

The data on the personnel selection solutions for a textile company in Denizli (Turkey) were used in this example (Tus & Adalı, 2018). Selection of a marketing assistant from seven candidates was performed.

Table 1. The data of case 1 (Tus & Adalı, 2018)

No.	C1	C2	C3	C4	C5
A1	2	110	3	2	3
A2	5	100	5	3	3
A3	3	90	4	5	2
A4	10	80	3	4	4
A5	4	85	2	4	5
A6	8	80	3	4	4
A7	5	95	2	4	3

Five criteria to evaluate the candidates include work experience (C1), foreign language ability (C2), problem-solving ability (C3), communication ability (C4), and group management ability (C5). The scores for each criterion for each candidate are presented in Table 1. In which, all five criteria are in the form of the larger the better criteria. In this study, the ranking of solutions was conducted by two methods: one is the *PSI* method combined with N1 and the other one is the *CODAS* method. The ranked results from two above methods will be used for comparison with the ranked results from this study.

And next, the ranking of solutions according to the *PSI* method combined with different DNMs will be performed. First of all, the data normalization by the N2 method will be applied. Eq. (3) and Eq. (4) were used to normalize the data according to the N2 method, the normalized data are presented in Table 2.

Table 2. The data normalization values in case 1 according to the N2 method

No.	C1	C2	C3	C4	C5
A1	0.0000	1.0000	0.3333	0.0000	0.3333
A2	0.3750	0.6667	1.0000	0.3333	0.3333
A3	0.1250	0.3333	0.6667	1.0000	0.0000
A4	1.0000	0.0000	0.3333	0.6667	0.6667
A5	0.2500	0.1667	0.0000	0.6667	1.0000
A6	0.7500	0.0000	0.3333	0.6667	0.6667
A7	0.3750	0.5000	0.0000	0.6667	0.3333

Eq. (27) and Eq. (28) were used to determine the preference values from the mean (φ_j). The calculated results are presented in Table 3.

Table 3. Values of φ_j in case 1 when data normalization according to the N2 method

	C1	C2	C3	C4	C5
φ_j	0.7411	0.8175	0.7619	0.6032	0.6349

The deviation in the preference value (β_j) is calculated by Eq. (29), the overall preference value (θ_j) is determined by Eq. (30), and the calculated results are presented in Table 4.

Table 4. Values of β_j and θ_j in case 1 when data normalization according to N2 method

	C1	C2	C3	C4	C5
β_j	0.2589	0.1825	0.2381	0.3968	0.3651
θ_j	0.1796	0.1266	0.1652	0.2753	0.2533

The PSI_i is calculated according to Eq. (31), the calculated results are presented in Table 5. The ranked results of the solutions according to the values of the *PSI* were also stored in this table.

Table 5. PSI_i values in case 1 when data normalization according to the N2 method and ranked results of the solutions

No.	PSI_i	Rank
A1	0.2661	7
A2	0.4931	4
A3	0.4501	5
A4	0.5871	1
A5	0.5028	3
A6	0.5422	2
A7	0.3986	6

Thus, the ranking of the solutions for case 1 when normalizing data by the N2 method was completed. The ranking of solutions using other DNMs (from N3 to N12) was also performed. Table 6 presents the ranking results of the solutions when using all DNMs. The ranked results of the solutions according to the *CODAS* method and *PSI* method combined with N1 by Tus and Adalı (2018) were also included in this table.

Table 6. The ranked results of solutions in case 1

No.	*CODAS*	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12
A1	7	7	7	7	1	7	7	7	7	7	7	7	7
A2	3	3	4	3	2	1	3	1	3	1	3	2	2
A3	5	5	5	5	4	4	5	6	5	3	5	5	5
A4	1	1	1	1	7	2	1	2	1	2	1	1	1
A5	4	4	3	4	5	5	4	4	4	4	4	4	4
A6	2	2	2	2	6	3	2	3	2	5	2	3	3
A7	6	6	6	6	3	6	6	5	6	6	6	6	6

From the results in Table 6.

-When using eleven DNMs to combine with the *PSI* method, all confirmed A1 as the worst solution (except for N4). Solution A1 was also confirmed to be the worst one when using the *CODAS* method (Tus & Adalı, 2018). From these results, a solid conclusion can be drawn that A1 is the worst solution.

-Solution A4 was determined to be the best solution when using *CODAS* method (Tus & Adalı, 2018). When using the *PSI* method in combination with eight DNMs including N1, N2, N3, N6, N8, N10, N11, and N12, A4 was also determined to be the best solution. However, it would be a subjective statement if only considering the results in case 1 to conclude that all eight methods including N1, N2, N3, N6, N8, N10, N11, and N12 are all suitable to be combined with the *PSI* method. To draw the generalized conclusions, it is necessary to perform more applications with many cases in many different fields. Furthermore, sensitivity analysis in different situations is also required to ensure the accuracy of the conclusions.

Case 2

The investigated data on robots were used in this case (Keshavarz-Ghorabaee et al., 2016; Trung, 2022b). Seven types of robots were given for the ranking process. Five criteria were selected to evaluate the robots including Load capacity (C1), Maximum tip speed (C2), Memory capacity (C3), Manipulator reach (C4), and Repeatability (C5).

In which C1, C2, C3, and C4 are the larger the better criteria, whereas C5 is the smaller the better criterion. The investigated data is presented in Table 7.

Similar to case 1, for this case, the ranking results of the solution when applying the *PSI* method with twelve different DNMs (N1 to N12) are presented in Table 8. The ranking results of the solutions using the *CODAS* method (Keshavarz-Ghorabaee et al., 2016) and the two methods R and CURLI (Trung, 2022a) are also presented in this table.

Table 7. The data of case 2 (Keshavarz Ghorabaee et al., 2016; Trung, 2022b)

No.	C1	C2	C3	C4	C5
A1	60	0.4	500	990	2540
A2	6.35	0.15	3000	1041	1016
A3	6.8	0.1	1500	1676	1727.2
A4	10	0.2	2000	965	1000
A5	2.5	0.1	500	915	560
A6	4.5	0.08	350	508	1016
A7	3	0.1	1000	920	1778

The obtained results in Table 8 show that A2 is the best solution when ranking by the *CODAS* method (Keshavarz Ghorabaee et al., 2016) and when ranking by two methods R and CURLI (Trung, 2022a). A2 was also identified as the best solution when combining the *PSI* method with six DNMs including N1, N4, N5, N6, N8, and N11.

Table 8. The ranked results of solutions in case 2

No.	*CODAS*	R	CURLI	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12
A1	3	2	2	2	3	1	3	1	3	3	3	1	1	2	3
A2	1	1	1`	1	2	2	1	5	1	2	1	3	2	1	2
A3	2	4	4	4	1	4	4	2	4	4	4	2	4	4	7
A4	5	3	3	3	4	3	2	3	2	1	2	4	3	3	1
A5	7	5	5	5	5	5	5	7	5	5	5	6	5	5	4
A6	6	7	7	7	7	7	7	4	7	7	7	7	7	7	6
A7	4	6	6	6	6	6	6	6	6	6	6	5	6	6	5

Thus, if we only consider the results in this case, it is seen that five methods N1, N4, N6, N8, and N11 are suitable methods to combine with the *PSI* method. However, to draw general conclusions, further applications of the ranking of these processes in other fields are still needed to perform.

Case 3

The experimental data about the turning processes were used in this case (Prasad et al., 2018). In this study, nine different solutions to a turning process were implemented. Each solution is evaluated through three criteria including arithmetic average roughness height (C1), Ten-point mean roughness (C2), and material removal rate (C3). In which, C1 and C2 are the smaller the better criteria, whereas C3 is the larger the better criterion. The calculated results are presented in Table 9.

Table 9. The data of case 3 (Prasad et al., 2018)

No.	C1	C2	C3
A1	2.11	9.04	9.21
A2	5.023	22.68	24.85
A3	9.17	36.103	32.57
A4	2.036	8.546	20.57
A5	7.16	26.94	39
A6	11.59	43.963	24.85
A7	3.35	13.263	41.14
A8	7.25	26.086	27
A9	11.75	45.376	39.85

The ranking of solutions according to the *PSI* method when combined with eleven different DNMs (N2 to N11) was performed similarly to case 1. The calculation results are presented in Table 10. The ranking results of the solutions when using the *PSI* method in combination with N1 (Prasad et al., 2018) were also summarized in this table.

Table 10. The ranked results of solutions in case 3

No.	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12
A1	6	1	2	3	2	8	9	8	7	1	4	9
A2	7	4	5	5	4	4	5	4	6	8	5	7
A3	5	7	8	7	7	6	6	6	3	5	7	3
A4	2	2	1	2	1	3	1	3	8	9	2	8
A5	3	6	4	4	5	2	3	2	5	4	3	4
A6	9	8	9	9	8	9	8	9	1	7	9	2
A7	1	3	3	1	3	1	2	1	9	3	1	6
A8	8	5	6	6	6	5	7	5	4	6	6	5
A9	4	9	7	8	9	7	4	7	2	2	8	1

The obtained results in Table 10 show that A7 is determined to be the best solution when using the *PSI* method in combination with N1 (Prasad et al., 2018). When four methods N4, N6, N8, and N11 were used in combination with the *PSI* method, it was also determined that A7 was the best solution. In this case, it can be concluded that the five methods N1, N4, N6, N8, and N11 are suitable methods to combine with the *PSI* method.

Case 4

The investigated data on air condition in offices was used in this case (Keshavarz-Ghorabaee et al., 2016). Six criteria were used to evaluate the air condition in the office including the amount of air per head (C1), relative air humidity (C2), air temperature (C3), illumination during work hours (C4), rate of airflow (C5), and dew point (C6). In which, the criteria C1 to C4 are the large the better criteria, whereas C5 and C6 are the smaller the better criteria. The data about the solutions and the criteria in this case are presented in Table 11.

Table 11. The data of case 4 (Keshavarz Ghorabaee et al., 2016)

No.	C1	C2	C3	C4	C5	C6
A1	7.6	46	18	390	0.1	11
A2	5.5	32	21	360	0.05	11
A3	5.3	32	21	290	0.05	11
A4	5.7	37	19	270	0.05	9
A5	4.2	31	19	240	0.1	8
A6	4.4	38	19	260	0.1	8
A7	3.9	42	16	270	0.1	5
A8	7.9	44	20	400	0.05	6
A9	8.1	44	20	380	0.05	6
A10	4.5	46	18	320	0.1	7
A11	5.7	48	20	320	0.05	11
A12	5.2	48	20	310	0.05	11
A13	7.1	49	19	280	0.1	12
A14	6.9	49	16	250	0.05	10

In this case, the ranking of the solutions according to the PSI method in combining with twelve different DNMs (N1 to N12) was performed similarly to case 1. The calculated results are presented in Table 12. The ranking results of the solutions when using the CODAS method (Keshavarz Ghorabaee et al., 2016) were also summarized in this table.

Table 12. The ranked results of solutions in case 4

No.	*CODAS*	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12
A1	3	3	9	7	7	3	3	3	3	1	9	5	6
A2	6	8	6	6	6	8	8	8	8	10	7	6	5
A3	9	12	8	9	9	9	11	11	11	12	8	9	9
A4	10	10	7	8	8	6	10	9	10	14	6	7	8
A5	14	14	14	14	14	13	14	14	14	5	14	14	14
A6	13	13	13	13	13	12	13	13	13	4	13	12	12
A7	12	11	12	10	12	14	12	12	12	6	10	13	13
A8	1	1	1	1	1	2	1	1	1	9	1	1	1
A9	2	2	2	2	2	1	2	2	2	11	2	2	2
A10	11	7	11	11	10	11	6	7	6	3	11	11	11
A11	4	4	3	3	3	7	4	4	4	7	4	3	3
A12	7	5	4	4	4	10	5	5	5	8	5	4	4
A13	8	6	10	12	11	4	7	6	7	2	12	10	10
A14	5	9	5	5	5	5	9	10	9	13	3	8	7

The calculated results in Table 12 show that A8 is determined to be the best solution when using the *CODAS* method (Keshavarz Ghorabaee et al., 2016). A8 was also determined to be the best solution when using other methods N1, N2, N3, N4, N6, N8, N10, and N11 in combination with the *PSI* method. From the analyzed results, it is shown that, in this case, eight methods that include N1, N2, N3, N4, N6, N8, N10, and N11 are suitable methods to combine with the *PSI* method.

4.2 Sensitivity Analysis

The combined results from the four above cases give an overview of the fit/nonconformity when combining the DNMs with the PSI method and as presented in Table 13. In which, the cells that were marked "" show the suitability of combining the DNM with the PSI method. In contrast, the blank cells represent nonconformities when combining the DNM with the PSI method. However, this suitability only considers the factors that the method of data normalization when combined with the PSI method can determine the best solution in comparing to published studies. In order to confirm that a DNM is appropriate in combination with the PSI method, it is necessary to analyze the sensitivity in ranking the solutions. Of course, the sensitivity analysis only needs to be performed for the data normalized methods that was jointly identified the best solution. With above four cases, these methods were N1, N6, N8, and N11.

Table 13. Suitable normalization methods for combining with the PSI method

Examples	Normalization method											
	N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	N11	N12
Example 1	✓	✓	✓			✓		✓		✓	✓	✓
Example 2	✓			✓		✓		✓			✓	
Example 3	✓			✓		✓		✓			✓	
Example 4	✓	✓	✓	✓		✓		✓		✓	✓	✓

The sensitivity analysis is the determination of the degree of variation in the ranking results of the solutions under the different scenarios. The scenarios that were commonly used for sensitivity analysis include changing the weight of the criteria, removing one/several solutions from the list of solutions, and changing the criterion type (Božanić et al., 2021; Zopounidis & Doumpos, 2017). In this case, the generation of different scenarios is done by eliminating a certain solution. In each case, the eliminated solution will also be selected differently. For case 1, solution A5 was removed from the list of solutions. According to the ranking results of the solutions in case 1 (section 4.1), A5 ranked 4, A1 ranked 7, and A4 ranked 1 (when using N1, N4, N8, and N11). Therefore, if removing A5 from the list of solutions does not affect on the ranking of the solutions, then A4 is still the best solution and A1 is still the worst solution. After removing A5 from the list of solutions, the ranking results of solutions are shown in Figure 1.

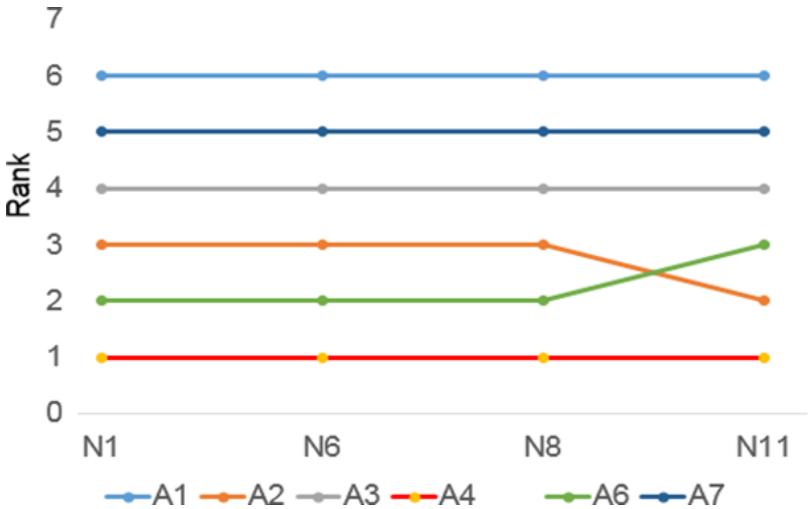

Figure 1. Ranked results of the solutions without A5 solution in case 1

It is seen that although the rank inversion occurred in some solutions, however, A4 is still the best solution, and A1 is still the worst solution for all four different DNMs. It shows that the removal of A5 from the list of solutions does not change the best solution and the worst solution. In this case, it can be concluded that N1, N6, N8, and N11 methods are suitable methods to combine with the *PSI* method.

For case 2, solution A6 was removed from the list of solutions. According to the ranking of the solutions in case 2 (section 4.2), A6 is the worst solution and A2 is the best solution (when using N1, N4, N8, and N11). Therefore, if removing A6 from the list of solutions does not influence on the ranking of solutions, then A2 is still the best solution. On the other hand, currently, A7 ranks 6, so if A6 is removed from the list of solutions, A7 will rank last. After removing A6 from the list of solutions, the ranking results of the solutions are shown in Figure 2.

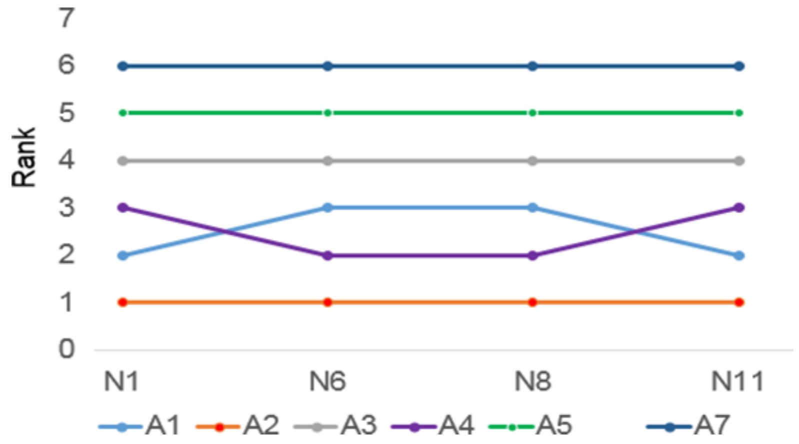

Figure 2. Ranked results of the solutions without A6 solution in case 2

It is seen that although the rank inversion was occurred in some solutions, however, A2 is still the best solution, and A7 is still the worst solution when using four different DNMs. That shows that the removal of A6 from the list of solutions was not changed the best and worst solution. In this case, it is again certainty established that methods N1, N6, N8, and N11 are suitable methods to combine with the *PSI* method.

For case 3, once again, the worst solution is removed from the list of solutions (solution A6). According to the ranking of solutions in case 3 (section 4.3), A7 is the best solution. If removing A6 from the list of solutions does not affect on the ranking of solutions, then A7 is still the best solution. After removing A6 from the list of solutions, the results of ranking solutions are shown in Figure 3. It is seen that although the rank inversion also occurred in some solutions, however, A7 is still the best solution and ranks 2, 3, and 4 are the same those when using DNMs. In this case, we can again confirm that N1, N6, N8, and N11 are suitable methods to combine with the *PSI* method

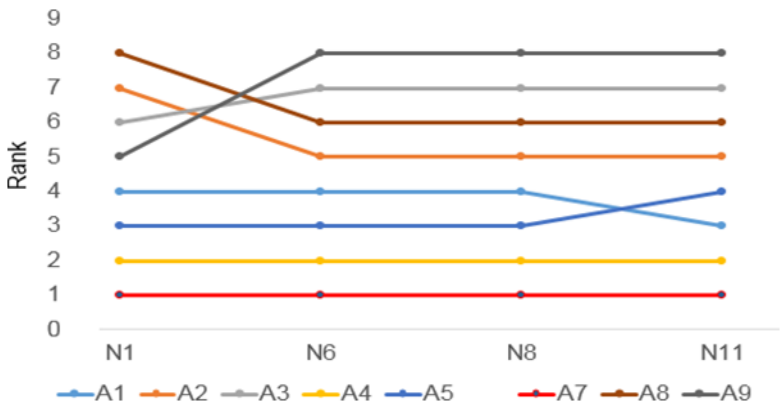

Figure 3. Ranked results of the solutions without A6 solution in case 3

For case 4, the best solution was removed from the list of solutions, (solution A8). According to the ranking results of the solutions in case 4 (item 3.4), A9 ranked 2nd, and A5 ranked last.

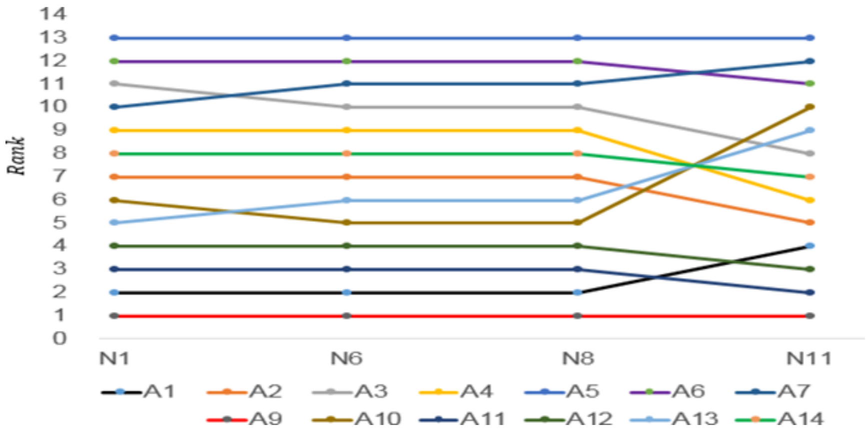

Figure 4. Ranked results of the solutions without A8 solution in case 4

Therefore, if removing A8 from the list of solutions does not affect on the ranking of solutions, then A9 will rank 1, and A5 will still rank last. After removing A8 from the list of solutions, the ranking results of solutions are shown in Figure 4. It is seen that rank inversion also occurred in some solutions. However, A9 is always the best solution, and A5 is always the worst solution. So, the removal of A8 from the list of solutions does not change the best solution and the worst solution. Once again, we can confirm that methods N1, N6, N8, and N11 are suitable methods to combine with the *PSI* method.

4.3. The appropriate DNM for combination with PSI Method

From the above-performed analyzed results, it is seen that in the above-mentioned twelve DNMs, there are only four DNMs including N1, N6, N8, and N11 are suitable methods to combine with the PSI method in all studied cases. These combinations not only consistently identified the same best solution, but also gave equivalent results in comparing to other methods (CODAS, R, and CURLI) as analyzed in each case. The sensitivity analysis of the ranking results of the solutions was also performed with different scenarios. The results all confirmed that N1, N6, N8, and N11 are suitable methods to combine with the PSI method.

These obtained results could open a wide application range for the PSI method. It can be said that because in the cases, there does not exist any value of yij equal to 0, all four methods of data normalization can be applied. However, when there exists a certain value yij = 0, then the method N1 cannot be applied, the remaining three methods (N6, N8, and N11) can still be applied. Even when there exists a value max(yij) = 0, then all three methods N1, N6, and N8 cannot be applied, there is still an alternative method (N11). This can be considered a great discovery to be able to apply the PSI method in all cases. The case that was applied immediately below will make this statement clearer.

In this case, there are 3 different solutions A1, A2, and A3. Each solution is evaluated through 5 criteria C1, C2, C3, C4, and C5. In which, C1, C2, and C3 are criteria as the larger the better, whereas C4 and C5 are criteria as the smaller the better. The values of the criteria at the solutions are selected at random, in which, there are both positive values, zero values, and negative values (Table 14). It is clear that in this case, methods N1, N6, and N8 cannot be applied, but only method N11 can be applied to rank the solutions. Using the PSI method with the DNM (N11) to rank solutions, the ranking results were summarized in table 14. In addition, to verify the ranking results, *R* and *CURLI* methods were also applied with the ranked results as summarized in Table 14.

Table 14. Ranked results when using PSI+N11, CURLI, and R methods

No.	Criteria					Rank		
	C1	C2	C3	C4	C5	*PSI* + N11	*CURLI*	*R*
A1	5	-3	10	1	0	1	1	1
A2	6	-2	8	0	2	2	2	2
A3	3	0	6	3	1	3	3	3

The calculated results in Table 14 show that when ranking the solutions by *PSI* method in combining with N11, the ranking results are completely consistent with those ones when using *CURLI* and *R* methods. Once again, we see that the N11 method is perfectly suited to combine with the *PSI* method. This combination will create more effective when other DNMs (N1, N6, and N8) cannot be applied. The identification of the appropriate DNMs when combined with a specific MCDM method is a suitable research direction in studying on the MCDM. Therefore, in this case, the first time the *PSI* method was selected as the research object both showing the correctness of the approach as well as the novelty of this work. This study identified four DNMs suitable to combine with the *PSI* method. This discovery has expanded the *PSI* method application scope that has not been considered in previous studies.

5. Conclusion

With the simplicity of application and no need to determine the weights for the criteria, the PSI method has been widely applied for MCDM in many different fields. However, the proponent of the PSI method as well as all the studies that applied this method all normalized the data according to the N1 method. It is clear that in all mentioned cases, the author has not considered cases when a certain criterion has a value of 0 in a certain solution. In these cases, the N1 method cannot be applied, and then the PSI method also cannot be applied. To overcome this limitation, this study investigated the suitability of combining twelve different DNMs with the PSI method. All those combinations were tested in four cases in four different fields. The number of solutions, the number of criteria, and the type of criteria (the larger the better, the smaller the better) are not the same in all cases. In this study, it was determined that in all four cases, four methods including N1, N6, N8, and N11 were identified as suitable methods to combine with the PSI method. These results from this study open a wide application range for the PSI method. Specifically, when there exists yij = 0 and/or max(yij) = 0, then the N1, N6, and N8 methods cannot be applied, the N11 method can still be applied for multi-criteria decision making.

However, all twelve DNMs that were mentioned in this study cannot be applied if the criteria are in the qualitative form (color, preferences, etc.). In these cases, the assignment of these qualitative criteria to the numbers is necessary to be done before performing the data normalization. In these cases, the studies that apply the PSI method for MCDM when having the qualitative criteria are the next research direction of this study.

When the value of the criteria at each solution is a fuzzy set, the evaluation of the suitable degree when combining the DNMs (N1, N6, N8, and N11) with the *PSI* method, which is also a new research direction should be performed as soon as possible.

All twelve used DNMs in this study should also be tested to determine the methods that are suitable when combined with other MCMD methods.

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