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A ROBUST OPTIMIZATION ASSESSMENT OF INVENTORY ROUTING PROBLEMS WITH ROUTE DISRUPTIONS AND GREEN FACTORS

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Abstract: *This work assesses robust optimization as a solution method for the combined issue of vehicle routing and inventory management, adding different characteristics. Thus, its behavior may resemble the industry's current reality. This work also emphasizes the utilization of time windows, considers possible disruptions to the routes that connect the different nodes, and includes a Green Factor assessment. For these purposes, the minimax regret criteria is applied to a set of pre-established instances from the literature, adding the necessary information requirements to evaluate how the method behaves through different parameter combinations, thus changing the number of nodes, costs, and available alternatives and scenarios. The study compares the solutions achieved against the solutions from the classical model to assess the relation between the results from the exercise and the different parameter modifications implemented, achieving a 40% improvement in the total cost algorithm—proportional to the increase in the alternatives and scenarios assessed. The approach proposed allow us to create disruption scenarios to the distribution process, which are connected to the mathematical optimization problem that allow us to determine the best routing process given the uncertainty associated to the disruption events. Our results also allow us to analyze the trades off between the green factors when they are included in the objective function and the results without them.*

Keywords: *disruption, green factors, inventory, robust optimization, routing, time windows.*

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1. Introduction

The inventory routing problem (IRP) has gained considerable relevance in different industries seeking to become more competitive within their own markets. This is mostly due to its easier application based on the evolution of computational tools and techniques aimed at improving distribution processes.

Applying these mathematical formulations to different real business situations is one of the challenges addressed in the scientific literature, wherein several characteristics have been considered and different solution methods have been proposed to address these issues. Some of these problems include meeting customer demands for a single deposit, fixed setup costs (Lou et al., 2009), meeting demands through partial deliveries (Baita et al., 1998), vehicle fleet heterogeneity, excessive loading and unloading times, and time gaps between visits (Coelho & Laporte, 2015), among others.

However, most of these models neither consider disruptions to the natural process of product distribution or delivery to customers or consumers nor possible route disruptions caused by situations such as heavy traffic, vehicle reliability, and traffic accidents, among others (Morales et al., 2017). Within this context, these models have been mostly aimed at cost minimization through route optimization; however, they have failed to consider the environmental impacts from vehicle emissions (Lou et al., 2009), which are currently one of the main approaches within the industry.

Hence, the route disruptions application to IRPs with time windows, coupled with an assessment of the green factors involved, is an extension of previous research conducted by different authors seeking to provide an environmental approach to everyday issues.

This paper introduces a new mathematical modeling approach combined with minimax regret criteria, wherein the classical modeling applied to this problem has been adapted, thereby extending the decision criterion to include route disruptions as a source of uncertainty. In addition, an environmental factor analysis approach is conducted for the problem under study to assess the behavior of these environmental factors within the decision-making process.

This paper is organized as follows. Section II contains a literature review focusing on the solution characteristics and methods previously applied, thus providing a general approach to the problem. Section III denotes the mathematical formulations along and identifies the additional built-in features. Next, Section IV conducts an analysis and assessment of the model proposed based on instances from the literature, and Section V outlines our conclusions.

1. State of the art and related works

The Inventory Routing Model is an optimization problem in which one or more deposits manage node inventory levels to find the most efficient way to meet their individual demands. This is often achieved by minimizing costs, such as transportation costs and inventory management costs (Eliseo Pérez Kaligari, 2015)

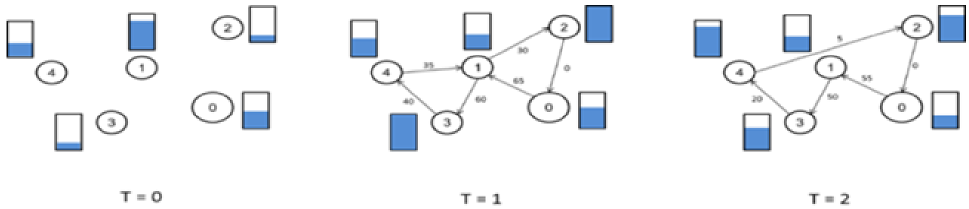

Figure 1. Classic IRP Model.

This model defines vehicle routes at different periods of time to satisfy node demands while maintaining proper inventory levels at each node. As it may be observed in Fig. 1, for each T time interval, inventory levels, therein depicted by the bars, change. These variations are subject to the number of items received by each vehicle minus the demand satisfied at each time period, without exceeding node storage capacities or vehicle transportation capacities. We will present some of the most relevant works related to the Inventory Routing Problem and its extensions, followed by the articles which combines robust optimization as an extension of the IRP (Agyingi et al., 2016).

The IRP is deemed as a variation of the vehicle routing problem (VRP) first introduced by Dantzig and Ramser (Pillac et al., 2013). Likewise the VRP, the IRP has experienced several variations, and different solution methods have been proposed, such as including time windows (Baita et al., 1998), multi-deposit inventory routing problems with n numbers of clients (Lou et al., 2009), cyclic vs reactive planning problems (Raa, 2014), as well as solving bi-objective problems through meta-heuristics (Nekooghadirli et al., 2014).

Throughout different studies, IRPs have focused on two main trends, adding new features to the model and proposing new solution methods, with some works even combining these two trends. Some examples of these works will be discussed below. In 2012, Yugang Yu et al. published a paper on the Stochastic Inventory Routing problem with Split Delivery, wherein demands were met by using two or more vehicles for each node within a certain time interval (Yu et al., 2012). In 2013, Coelho and Laporte proposed a branch-and-cut algorithm to solve several variants of the IRP, especially the multiple vehicle problem with homogeneous and heterogeneous fleets, wherein they conducted an extensive computational analysis (Coelho & Laporte, 2013).

In 2014, Nekooghadirli introduced a bi-objective model that considers a multi-period and multi-vehicle system. As this study seeks to minimize both transportation costs and travel times, a Multi-Objective Imperialist Competitive Algorithm is effectively used to resolve the problem proposed (Nekooghadirli et al., 2014). In 2015, Zhang applied robust optimization to maritime IRPs, providing solutions to high-uncertainty routing logistics problems within the maritime industry (Zhang, 2015).

In 2016, Merakli proposed linear mixed integer programming formulations using min-max criteria and devised two exact solution algorithms based on Bender's decomposition to solve large-scale problems. This work is one of the first ever to model demand uncertainties through a polyhedral set (Meraklı & Yaman, 2016). Li Liao proposed a genetic algorithm model for finite-time problems with a deterministic demand and homogeneous fleet applied to the agricultural product supply chain (Liao et al., 2013). Stefan Treitl also proposed an IRP model algorithm for the petrochemical industry, which involves several environmental aspects (Treitl et al., 2014).

Since Inventory Routing Models are based on distribution models, they are frequently used by service and courier companies. However, their application has been extended to other fields with similar behaviors. Likewise, the different solution models proposed in the literature have also expanded their fields of application. For instance, one model was specifically developed for the distribution of industrial gases, wherein the authors use a three-phase heuristic to solve a finite-time model with deterministic demand and a single deposit serving multiple nodes through a homogeneous fleet (Day et al., 2009).

In 2020, Ortega proposes using meta-heuristic search techniques to solve an IRP with Time windows, adding two variables to the classic model with the intention of matching the problem to the real business world (Ortega et al., 2020). In his work, Yavari includes route disruptions in IRPs through a demand management model through genetic algorithms (Yavari et al., 2020), thus revealing the need to consider aspects that directly affect routing problems.

These meta-heuristics have been widely used in problems, such as the IRP, and several improvement proposals have been developed. For example, Saif-Eddine proposed one using an improved genetic algorithm to optimize supply chains (Saif-Eddine et al., 2019). More recent models, such as the work proposed by Jie Zhu, are starting to consider uncertainty factors. Here, the author applies a Kernel distribution to robust optimization models in order to determine the best scenario within which possible disruptions may occur (Zhu et al., 2020).

These robust optimization models have also been applied to different other fields, such as in the work proposed by Lin, where the model is used to solve manifold inferences (Lin et al., 2020), or the work proposed by Houska, where the model is used in predictive control models (Houska & Villanueva, 2019).

As it may be observed, IRPs have been highly studied in the literature from the standpoint of applicability and computational complexity. They also continue to exhibit considerable relevance in current research work and are commonly implemented at companies because when and how much inventory must be replenished to meet customer demands while still reducing costs to a minimum remains one of the most relevant business questions for companies (Kleywegt et al., 2004).

However, including uncertainty in these models is yet to be widely explored, especially focusing on routes between nodes and representing situations such as heavy traffic, vehicle reliability, and traffic accidents, among others, while still assuming a 100% probability of success for all trips (Morales et al., 2017).

In terms of include disruptions applied to the Inventory Routing Problem, there are some recent works that have treated the problem. For example, Golsefidi and Jokar (2020) propose the use of robust optimization for a combined problem of production-routing where authors proposed a mixed integer linear programming model contrasting the deterministic version with the stochastic one (Golsefidi & Jokar, 2020), also given the complexity of the problem, some metaheuristics are developed over different instance sizes and their performance are evaluated. In the same context of managing the uncertainty for an extension of the Inventory Routing Problem, Liu, Zhang, and Yuan (2021) proposed a distributionally robust optimization approach where the sailing and waiting times are the parameters with uncertainty (Liu et al., 2021). To track the problem authors proposed the use of a decomposition approach and their results are compared with a real case instance. In the same sense, there are some applications in different areas of robust optimization with inventory routing problems as those presented by Frifita, Afsar, and Hnaien (2022) and Shang et al. (2022) (Frifita et al., 2022; Shang et al., 2022).

The contribution of this paper can be summarized as follows: we propose the inclusion of the routing disruptions over the arcs to modeling real life restrictions when the distribution problems are connected to the availability of roads, also, we have proposed the use of the robust optimization approach to analyze the solutions when disruptions are considered. Finally, we include the environmental factors, that is a crucial characteristic of decision making.

Hence, this study seeks to introduce route disruptions, thereby echoing situations where deliveries are affected by route disruptions and proposing a solution method for this problem as an extension of previously published research studies (Morales et al., 2018; Morales et al., 2017).

2. Mathematical formulation

The model proposed preserves the classic model scheme presented by Archetti et al. (2014) (Archetti et al., 2014), wherein a "Node 1" deposit distributes quantities of the same product to a set of nodes $Vp = \{2,\dots,n\}$, within a planning horizon, P. The deposit releases a certain product quantity rt_t after each $T= \{1,\dots,P\}$ period.

For each t ⧠ T period, the nodes evidence a dt_i demand less than or equal to the warehouse capacity of C_i. Likewise, for each i node, an initial inventory level of $I0_i$ and a safety stock of L_i is defined. The ending inventory I_{It} for each period is a decision variable calculated by the model, where c_{ij} represents the Euclidean distance between two points.

During the t period, each customer is visited by the $k = \{1, \dots, K\}$ vehicle and received a $q_{i,k,t}$ quantity of product. Here, $Y_{i,k,t}= \{0,1\}$ is the binary variable that controls whether the corresponding node was visited through the $X_{i,j,k,t}=\{0,1\}$ route, which represents whether the distance between the two related nodes was covered. Since the fleet is homogeneous, this quantity received must be less than the vehicle capacity of Qk.

In addition to these basic model characteristics, the following are also considered in this work:

1. Here, $e_{i,k,t}$ represents the initial time of service for each i node from each k, where t must exceed the lower limit of the time window ai and not exceed the upper limit of bi. In addition, service times directly impact travel times.
2. As part of the time window restrictions, time windows are linearized by M_{ij}. The value of this variable will be $b_i + s_i + c_{ij} - a_j$ if the $a_i + s_i + c_{ij} > 0$ condition is met. Otherwise, its value will be 0.
3. A new E_{ijktzf} variable is generated to calculate emission levels for each route segment traveled.

The mathematical model is characterized by the following constraints:

$$MIN\ Z = \sum_{i\epsilon V}\sum_{t\epsilon T} h_i I_i^{t,Alt,Esc} + \sum_{i\epsilon V}\sum_{j\epsilon V}\sum_{k\epsilon K}\sum_{t\epsilon T} c_{ij}^{zf}\, X_{ij}^{kt} + \sum_{i\epsilon V}\sum_{j\epsilon V}\sum_{k\epsilon K}\sum_{t\epsilon T} E_{ij}^{kt} \tag{1}$$

Subject to

$$I_1^t = I_1^{t-1} + rt - \textstyle\sum_{i\epsilon Vp}\sum_{j\epsilon Vp}\sum_{k\epsilon K} q_{ij}^{kt} \qquad \forall\, t \in T \tag{2}$$

$$I0^t \geq 0 \quad \forall\, t\ \in\ T \tag{3}$$

$$I_i^{tzf} = I_i^{t-1} + \sum_{k\epsilon K}\sum_{j\epsilon Vp} q_{ji}^{kt} - dt_i^t \qquad \forall\, i \in Vp, t \in T \tag{4}$$

$$I_i^t \geq 0 \qquad \forall\, i \in Vp, t \in T \tag{5}$$

$$I_i^t \geq L_i \qquad \forall\, i \in Vp, t \in T \tag{6}$$

$$I_i^t \geq C_i \qquad \forall\, i\ \in\ Vp, t\ \in\ T \tag{7}$$

$$\sum_{k\epsilon K}\sum_{j\epsilon Vp} q_{ji}^{kt} \leq C_i - I_i^{t-1,zf} \qquad \forall\, i \in Vp, t \in T \tag{8}$$

$$\sum_{k\epsilon K}\sum_{j\epsilon Vp} q_{ji}^{kt} \leq C_i \sum_{j\epsilon V}\sum_{k\epsilon K} X_{ij}^{ktzf} \quad \forall\, i \in Vp, t \in T \tag{9}$$

$$\sum_{k\epsilon K}\sum_{j\epsilon Vp} q_{ji}^{kt} \leq Qk \quad \forall\, t \in T, k \in\ K \tag{10}$$

$$\sum_{j\epsilon Vp} q_{ji}^{kt} \leq Y_i^{kt} C_i \qquad \forall\, i\ \in\ Vp, t \in T, k \in\ K \tag{11}$$

$$\sum_{j\epsilon V} X_{ij}^{kt} = \sum_{j\epsilon V} X_{ji}^{kt} \qquad \forall\, i\ \in\ Vp, t \in T, k \in\ K \tag{12}$$

$$\sum_{j\epsilon V} X_{ij}^{kt} = Y_i^{kt} \qquad \forall\, i\ \in\ Vp, t \in T, k \in\ K \tag{13}$$

$$\sum_{j\epsilon Vp} X_{1j}^{kt} \leq 1 \qquad \forall\, k \in\ K, t \in T \tag{14}$$

$$\sum_{k\epsilon K} Y_i^{kt} \leq 1 \qquad \forall\, i \in Vp, t \in T \tag{15}$$

$$X_{ij}^{kt}, Y_i^{kt} \in \{0,1\} \qquad \forall\, i,j \in Vp, i \neq j, t \in T, k \in K \tag{16}$$

$$e_i^{kt} + s_i + c_{ij} - e_j^{kt}\left(X_{ij}^{kt}\right) M_{ij} \qquad \forall\, i \in V, j \in Vp, k \in K, t \in T, i \neq j \tag{17}$$

$$e_i^{kt} \geq a_i \qquad \forall\, i \in Vp, k \in K, t \in T \tag{18}$$

$$e_i^{kt} \geq b_i \qquad \forall\, i \in Vp, k \in K, t \in T \tag{19}$$

$$e_1^{kt} = 0 \tag{20}$$

$$E_{ij}^{ktzf} = (PO + \frac{PF - P0}{Qk} * q_{ij}^{ktzf} \times c_{ij}^{zf} \times x_{ij}^{ktzf} \qquad \forall\, i \in V, j \in Vp, k \in K, t \in T \mid j \neq i \tag{21}$$

where equations (2)–(6) represent the inventory restrictions which guarantee that the final inventory is within the limits established for each node i, as well as that their values are not negative. Equations (7)–(9) represent the capacity constraints for each node *i*. These equations guarantee that neither the final inventory nor the quantity received by each customer exceeds its storage capacity. Equation (10) guarantees compliance with the vehicle capacities defined as *k*.

Equations (11)–(15) represent routing restrictions, which guarantee the correct functioning of the routes defined and reveal the decisions made by each vehicle in each node at each time interval. Equation (16) represents the type of variables constraints. Equations (17)–(20) guarantee that each node *i* complies with time windows within the *t* period, thus allowing each *i* node to be served within the defined time. For these equations, we are assuming a constant velocity of 1 unit of distance per each unit of time.

These time windows are generated externally and introduced to the algorithm as a parameter. They are randomly created using a uniform distribution based on the work by (Eliseo Pérez Kaligari, 2015). The time window creation process can be expressed as follows:

$$LIV_1 = LI \tag{22}$$

$$LSV_1 = LS \tag{23}$$

$$LIV_i = [A1_i\ (LS - LI)] + LI \forall\, i \in Vp \tag{24}$$

$$DURi = [A2_i\ (RS - RI)] + RI \qquad \forall\, i \in Vp \tag{25}$$

$$Si \begin{cases} LIV_i + DUR_i \geq LS \rightarrow LSV_i = LS \\ LIV_i + DUR_i < LS \rightarrow LSV_i = LIV_i \end{cases} \quad \forall\, i \in Vp \tag{26}$$

Where:

LI = The lower limit for a given time interval
LS = The upper limit for a given time interval
IR = The lower limit of the possible time window range
RS = The upper limit of the possible time window range
$A1_i$= A random number generated for the *i* node
$A2_i$ = A random number generated for the *i* node
LIV_i = The upper limit of the time window for the *i* node
LSV_i= The upper limit of the time window for the *i* node
DUR_i= The duration of the time window for the *i* node

As part of the model's target function, environmental factors will measure the environmental impact generated by the routing proposed. In our case, the environmental impact will be measurement through the vehicle emissions according to the assigned load and the route traveled, as represented by (21). Where:

P0 = *CO2* level x unloaded vehicle
PF = *CO2* level x fully loaded vehicle

These values are input as parameters to the algorithm proposed by Lou et al. (2009). Decision-making problems pose different situations in which decisions must be made based on options and information available (Martínez Ortega, 2017). Due to the nature of these problems, some optimization methods have been developed based on decision-making methods, such as the minimax regret proposed by Savage.

This method considers a set of options and uses the one that generates the least regret based on the results from the decision that would have been made if all the information had been known a priori. To adapt this method to our model, two additional sets are established as “alternatives” and “scenarios” in the base model above. This way, route disruptions can be represented, and the required parameters can be created for the minimax regret robust optimization model. A description of these adaptations is discussed below.

Alternatives: In the proposed approximation, route disruptions are used to represent the routing delays as shown in Fig. 2 below. Here, we can observe disruptions in routes 1–2, 3–6, and 4–5, which means that the routes specified have experienced a type of disruption.

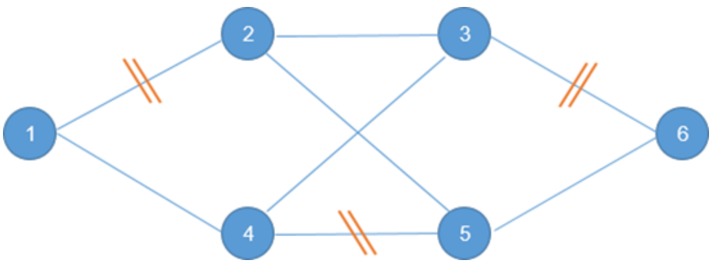

Figure 2. Routing with Disruptions.

Scenarios: These are the possible scenarios created by the mathematical model to support decision making, e.g., working hours and distribution shifts. Each scenario may consider different alternatives as denoted in Fig. 3.

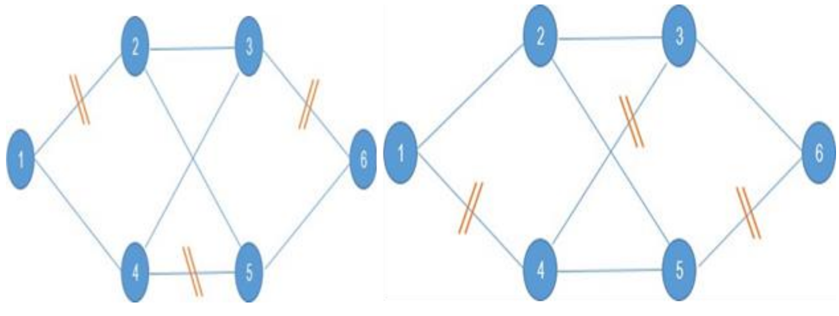

Figure 3. Scenarios with Different Alternatives (left alternative 1, right alternative 2).

In this way, the model assesses the impact from disruptions on transfer times, from node a to node b, and the impact generated in the target function. This way, we can compare routing costs and their corresponding increase or decrease to select the best possible option.

In our model, alternatives and scenarios are externally generated and introduced into the algorithm as parameters for the following equation:

$$c_{ij}^{zf} = \left[\left|py_j - py_i\right| + \left|px_j - px_i\right|\right] \times ALT_{zf} \times \ ESC_{zf} \tag{27}$$

Where:

Alt_{zf} = Random number generated for the different alternatives
Esc_{zf} = Random number generated for the different scenarios
px_i / pv_i = Node *i* coordinates on the x and y axes, respectively
px_j / pv_j = Node *j* coordinates on the x and y axes, respectively

With these values, regret will be calculated through the minimax regret algorithm. For the purposes hereof, “regret” shall be understood as the loss of benefits compared to the decision that should have been made if we had known the scenario that was going to occur according to the model proposed by Savage (Martínez Ortega, 2017). These new elements of disruption are formulated through the following equations, wherein scenario and alternative selections are proposed as follows:

$$Sol^{z,f} = MIN\ Z \tag{28}$$

$$Aux^{z,f} = \max_{z\ =\ 1,\ldots NumAlt}\{Sol^{z,f}\} - Sol^{z,f} \qquad \forall\ z \in Alt, f \in Esc \tag{29}$$

$$Vec2^{z} = \max_{f\ =\ 1,\ldots NumEsc}\{Aus^{z,f}\} \quad 0Aux^{z,f} \qquad \forall\ z \in Alt, f \in Esc \tag{30}$$

$$MaxValue = \min_{z\ =\ 1,\ldots NumAlt}\{Vec2^{z}\} = \min_{z\ =\ 1,\ldots NumAlt}\begin{Bmatrix}\max\{Aux^{z,f}\}\\ f\ =\ 1,\ldots NumEsc\end{Bmatrix} \tag{31}$$

Where:

$Sol_{(Alt,Esc)}$ = Model solutions for each z and f

$Vec_{(Esc)}$ = Vector that stores the maximum value for each scenario

$Aux_{(Alt,Esc)}$ = Auxiliary Matrix that stores the difference between the Maximum Vector for each scenario and the model solution for each z and f

$Vec2_{(Alt)}$ = Vector that stores the maximum value for each Alternative from the Auxiliary Matrix

MaxValue = Variable that stores the minimax regret

$TotEm_{(Alt,Esc)}$ = Matrix that stores the total emissions for each z, f

MinValue = Variable that stores the minimum value for the selected scenario Hence, we can describe the model as follows:

$$Min\ \left\{\max_{f=1,\dots NumEsc}\left\{Aux^{z,f}\right\}\right\} \tag{32}$$

Subject to:

where X is a restriction set [(1)–(21)]

$(x, y) \in X$

3. Results and analysis

This section assesses the results obtained from the algorithm according to the processes above. Here, we test different alternatives and scenarios to determine whether a relationship exists between the number of alternatives and scenarios and the result obtained. This experiment was programmed using the FICO XPRESS 8.8 optimization software where we use the disruption information as an input for the mathematical optimization model.

In addition, this study seeks to analyze emission behavior in relation to the number of alternatives and scenarios, as well as how they impact the target function. For this, the instances from Cohelo and Laporte available at http://www.leandro-coelho.com/instances/inventory-routing/ were considered, these instances provides the demand of nodes, the coordinates between each echelon, capacity of transportation, inventory and transportation costs as well inventory limits. These instances are split into 3 and 6 time periods and high and low costs. As model input parameters, these instances were modified by adding the following:

- Time window limits
- Service times at each node
- Route disruptions

Based on the design of the test instances previously detailed, four types of instances will be assessed. with the following characteristics, as shown in Table 1, Table shows the characteristics of test instances with the different elements considered per type of cost and time periods.

Table 1. Test instances

	HighCost H3	LowCost H3	HighCost H6	LowCost H6
Planning Horizon	3	3	6	6
Warehouse Inventory Cost	0.3	0.03	0.3	0.03
Node Inventory Cost	0,1<= x <= 0,5	0,01<= x <= 0,05	0,1<= x <= 0,5	0,01<= x <= 0,05
Number of Nodes	n = 5, 10, 15, 20	n = 5, 10, 15, 20	n = 5, 10, 15, 20	n = 5, 10, 15, 20
Alternatives	5	5	5	5
Scenarios	5	5	5	5

Below is an example of the application of the High Cost H3 instances for variations with 1 alternative and 1, 2 and 3 scenarios, respectively, this instance is composed by three time periods.

In Figs. 4–6, the resulting routes can be differentiated for each vehicle k in each period t for each scenario/alternative combination. These figures denote the value of the target function obtained as well as the total emissions according to the conventions specified. In Figs. 4-6 we analyze the contrast between different scenarios (disruptions) and alternatives, therefore, after running the optimization model with the information related, we can calculate the total objective function and the emissions estimated. In this sense, we can choose which is the best alternative for each scenario. The three figures shows for each alternative and the same scenario, the results of which period of time and which vehicle is used to distribute product to a specific customer.

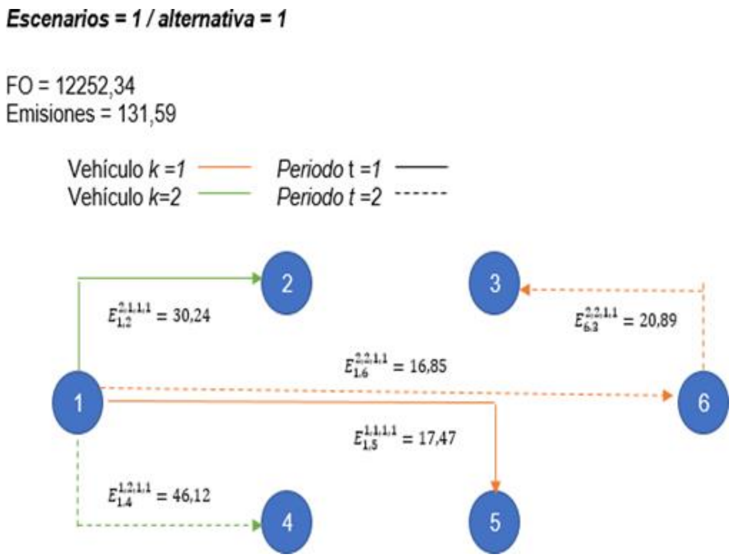

Figure 4. High Cost H3 E1A1

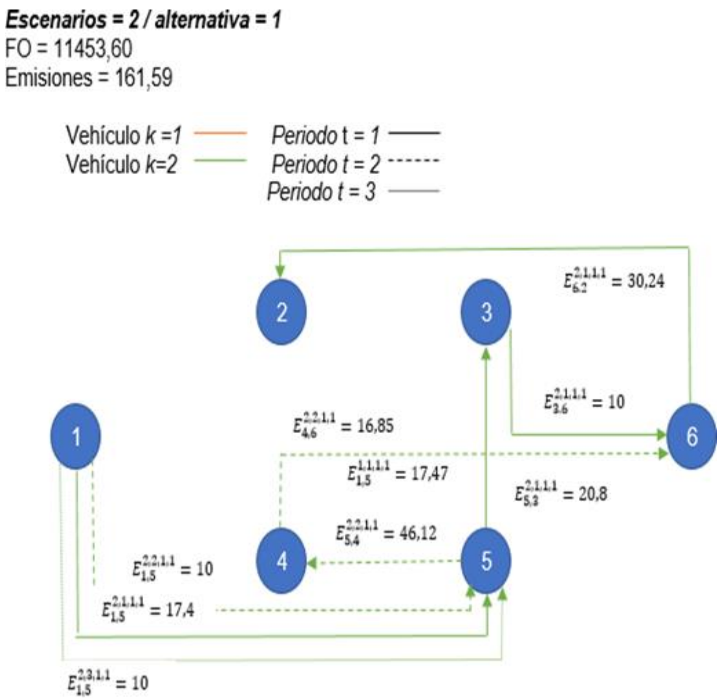

Figure 5. High Cost H3 E2A1

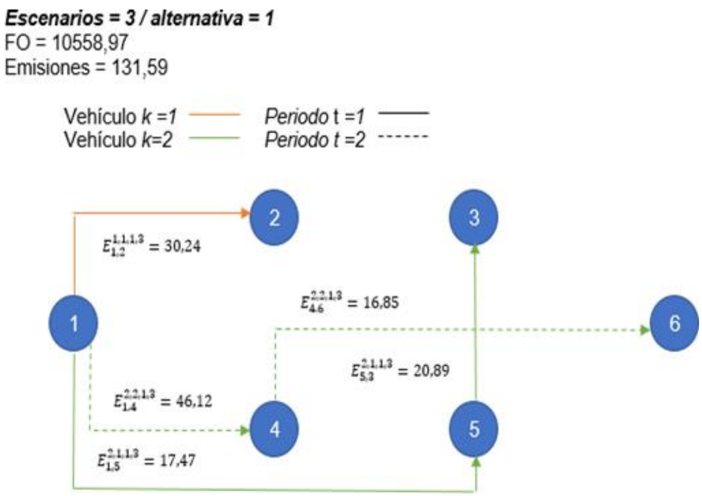

Figure 6. High Cost H3 E3A1

Here, it can be observed that Scenario 3 with Alternative 1 achieves a lower value in the target function while maintaining lower emission costs than Scenario 1, thus being considered as the best decision based on the different proposed conditions. The minimax regret algorithm presented in Section III selects the alternative and scenario combination that provides the best results, as has just been observed. In the same way, as the number of scenarios increases, the target function decreases. For this reason, whether this same behavior is generated for all the instances will be verified.

For this evaluation, Table 2 below denotes the results from each scenario and alternative iteration where columns represent the scenario and rows the alternatives and the results obtained in costs for each combination. Based on these results, the percentage impact when increasing these parameters can be assessed, as it may be observed in Table 3.

Table 2. Target function summary

Alt / Esc	1	2	3	4	5	Total
1	474.094	426.781	403.302	389.555	384.307	2.078.040
2	441.710	384.529	372.297	374.411	363.222	1.936.170
3	389.3592	349.262	354.410	323.558	313.773	1.730.364
4	434.693	350.333	314.869	330.498	326.435	1.756.830
5	396.028	332.475	356.620	283.644	282.352	1.651.122
Total	2.135.886	1.843.381	1.801.499	1.701.669	1.670.091	9.152.528

Table 3. Increases vs. No. of Alternative/Scenarios

Alt / Esc	1	2	3	4	5
1	0%	-10%	-15%	-18%	-19%
2	-7%	-19%	-21%	-21%	-23%
3	-18%	-26%	-25%	-32%	-34%
4	-8%	-26%	-34%	-30%	-31%
5	-16%	-30%	-25%	-40%	-40%

The 1 Alternative–1 Scenario combination represents the original model against which we can observe a cost decrease proportional to the number of scenarios and alternatives entered as parameters in the proposed methodology. Here, the

combination of five scenarios and five alternatives decreases costs by 40%, which represents the best results for the minimization objective, and it is consistent with the results from in Figs. 2–4.

The total costs from the target function decrease proportionally as the number of alternatives and/or scenarios entered as parameters increase, thus guaranteeing better results. This is consistent with the results from Figs. 4–6. On the other hand, Tables 4 and 5 present the results of the emissions and the increasing values of these emissions in each combination of alternative/scenario.

Table 4. Emissions Summary Table

Alt / Esc	1	2	3	4	5	General Total
1	$6.537	$7.632	$8.303	$7.995	$7.943	$38.412
2	$7.743	$7.647	$8.998	$8.705	$8.493	$41.588
3	$8.166	$8.460	$8.291	$7.916	$8.522	$41.358
4	$7.769	$8.694	$9.040	$8.543	$8.765	$42.814
5	$8.527	$8.754	$8.491	$7.441	$7.745	$40.960
General Total	$38.744	$41.190	$43.125	$40.602	$41.471	$205.134

Table 5. Emissions Summary Table Scenarios

Alt / Esc	1	2	3	4	5
1	0%	17%	27%	22%	22%
2	18%	17%	38%	33%	30%
3	25%	29%	27%	21%	30%
4	19%	33%	38%	31%	34%
5	30%	34%	30%	14%	18%

The emissions assessment is conducted in the same way as the target function value (see Tables 4 and 5). However, the opposite behavior can be observed as emissions increase as the number of alternatives and/or scenarios proposed also increases. Still, its percentage increase is lower than the improvement from the total results, which means that the model is still beneficial.

Owing to these results, the contribution from both aspects in the development of the problem can be assessed. Here, robust optimization is used as a solution method for assessing problem behavior under different circumstances and as input for decision-making considering the effects of uncertainty. In addition, the inclusion of characteristics such as environmental factors within the target function allowed providing this contribution to the existing literature. Detailed experimentation is listed as an appendix.

4. Theoretical and managerial insights

From a theoretical perspective, we highlight the following aspects: first, we present a new combination of the Inventory Routing Problem when disruptions occur, this modeling contributes to the understanding of the routing processes in cases of perturbations of the roads. Secondly, we analyze the trades off of the green factors in the distribution processes. Finally, we adapt a present an approach of robust optimization to analyze the different alternatives included in the mathematical modeling.

From the managerial perspective, our model support decision making when disruptions are present, in the same way the inclusion of the green factors when managing inventory and distribution problems allows to analyze the trades off between costs and emissions.

5. Conclusions

This work demonstrates the advantages of applying robust optimization to IRPTW (Inventory Routing Problem with Time Windows) problems, while still meeting customer demands within problem restrictions, but improving costs by considering the possible scenarios and alternatives in each context.

The foregoing allows us to simulate a more realistic environment of the needs and restrictions of current logistics problems, thus providing a more accurate solution at the time of its implementation. However, one of its limitations is precisely the definition of the scenarios and alternatives assessed at the time of algorithm application, which were simulated under pre-defined parameters in this work.

This implies a need for previous knowledge on the system implemented to clearly identify the scenarios and alternatives input into the model, as well as their intrinsic parameters and how they must be entered to prevent losing the advantages of the scheme proposed.

As a method for evaluating route uncertainties, the minimax regret criteria foster making decisions based on occurrence probabilities, denoting adequate performance by improving results as the number of alternatives and/or scenarios assessed increases and reducing costs, thus supporting decision-making processes in situations of uncertainty.

The contribution of this work is related to the combination of robust optimization applied to the Inventory Routing Problem combined with the evaluation of green factors, this becomes a powerful tool to analyze distribution problems when disruptions to roads are presented, the combination of these elements allows decision makers to contrast which is the best routing process when disruptions and green factors are included. Limitations of the proposal are related to the knowledge necessary to identify scenarios to the mathematical optimization model as they are considered as an input. Future work, implies to model disruption with different types of approximations and to develop algorithms to accelerate the solution of different size of instances.

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Appendix

This Appendix contains the experimentation conducted with the following characteristics:

- Type: Identifies the type of instance used according to the descriptions presented in section 3
- Abs: Identifies the number of Nodes assessed in the experiment
- Alternatives No.: Identifies the number of possible alternatives
- Scenarios No.: Identifies the number of possible scenarios
- Selected Alternative: Identifies the alternative selected from all available alternatives
- Selected Scenario: Identifies the scenario selected from all available scenarios
- Emissions: Denotes the emissions for the selected alternative and scenario
- Results: Denotes the experiment results for the selected alternative and scenario
- Pure "No Emissions" Results: Denotes experiment results for the selected alternative and scenario minus emission costs

Type	*Abs*	*Alternatives No.*	*Scenarios No.*	*Selected Alternative*	*Selected Scenario*	*Emissions*	*Results*	*Pure "No Emissions" Results*
HC H3	Abs1n5	1	1	1	1	131.6	12252.3	12120.8
HC H3	Abs1n5	1	2	1	1	161.6	11453.6	11292.0
HC H3	Abs1n5	1	3	1	3	131.6	10560.0	10428.4
HC H3	Abs1n5	1	4	1	2	181.6	9073.0	8891.4
HC H3	Abs1n5	1	5	1	1	151.6	9183.6	9032.0
HC H3	Abs1n5	2	1	1	1	191.6	12527.5	12335.9
HC H3	Abs1n5	2	2	1	2	161.6	8828.6	8667.0
HC H3	Abs1n5	2	3	2	1	171.6	4928.2	4756.6
HC H3	Abs1n5	2	4	1	1	161.6	6636.2	6474.7
HC H3	Abs1n5	2	5	2	4	161.6	9022.4	8860.8
HC H3	Abs1n5	3	1	3	1	131.6	8636.4	8504.8
HC H3	Abs1n5	3	2	3	1	161.6	8440.7	8279.1
HC H3	Abs1n5	3	3	3	2	161.6	5462.4	5300.8
HC H3	Abs1n5	3	4	3	2	161.6	4389.9	4228.3
HC H3	Abs1n5	3	5	3	2	161.6	4389.9	4228.3
HC H3	Abs1n5	4	1	3	1	161.6	9829.7	9668.1
HC H3	Abs1n5	4	2	1	2	161.6	7206.6	7045.0
HC H3	Abs1n5	4	3	3	2	151.6	6386.9	6235.3
HC H3	Abs1n5	4	4	3	4	161.6	4052.7	3891.1
HC H3	Abs1n5	4	5	1	3	151.6	3817.2	3665.6
HC H3	Abs1n5	5	1	4	1	151.6	6714.3	6562.7
HC H3	Abs1n5	5	2	2	1	161.6	7979.7	7818.1
HC H3	Abs1n5	5	3	1	3	151.6	4019.3	3867.7
HC H3	Abs1n5	5	4	4	4	171.6	7595.0	7423.4

HC H3	Abs1n5	5	5	2	4	161.6	6887.2	6725.6
HC H3	Abs1n10	1	1	1	1	200.5	14338.9	14138.5
HC H3	Abs1n10	1	2	1	1	260.5	18636.0	18375.5
HC H3	Abs1n10	1	3	1	2	220.5	15021.4	14800.9
HC H3	Abs1n10	1	4	1	3	250.5	16579.2	16328.7
HC H3	Abs1n10	1	5	1	5	200.5	15095.5	14895.0
HC H3	Abs1n10	2	1	1	1	270.5	13802.8	13532.3
HC H3	Abs1n10	2	2	1	2	240.5	14625.0	14384.5
HC H3	Abs1n10	2	3	2	2	240.5	15326.7	15086.2
HC H3	Abs1n10	2	4	2	4	190.5	17252.6	17062.1
HC H3	Abs1n10	2	5	2	3	246.4	12752.1	12505.7
HC H3	Abs1n10	3	1	3	1	190.5	11776.4	11585.9
HC H3	Abs1n10	3	2	2	2	254.4	14847.0	14592.5
HC H3	Abs1n10	3	3	1	1	230.5	13895.8	13665.3
HC H3	Abs1n10	3	4	2	1	220.5	11024.6	10804.1
HC H3	Abs1n10	3	5	3	5	190.5	13790.1	13599.6
HC H3	Abs1n10	4	1	1	1	260.5	18764.5	18504.0
HC H3	Abs1n10	4	2	4	1	240.5	20373.3	20132.8
HC H3	Abs1n10	4	3	1	2	230.5	15100.0	14869.5
HC H3	Abs1n10	4	4	4	2	270.5	10683.3	10412.8
HC H3	Abs1n10	4	5	1	4	260.5	11563.9	11303.5
HC H3	Abs1n10	5	1	2	1	250.5	14828.6	14578.1
HC H3	Abs1n10	5	2	2	1	250.5	11345.9	11095.5
HC H3	Abs1n10	5	3	4	3	250.5	17212.8	16962.4
HC/H3	Abs1n10	5	4	3	1	230.5	12224.1	11993.6

HC/H3	Abs1n10	5	5	2	1	290.5	14368.4	14078.0
HC/H3	Abs1n15	1	1	1	1	251.7	25777.2	25525.6
HC/H3	Abs1n15	1	2	1	1	331.7	16036.2	15704.6
HC/H3	Abs1n15	1	3	1	3	285.7	22479.4	22193.7
HC/H3	Abs1n15	1	4	1	4	261.7	15885.5	15623.9
HC/H3	Abs1n15	1	5	1	5	261.7	17727.6	17465.9
HC/H3	Abs1n15	2	1	1	1	311.7	16094.6	15783.0
HC/H3	Abs1n15	2	2	2	2	321.7	15376.7	15055.0
HC/H3	Abs1n15	2	3	1	1	391.7	17686.6	17295.0
HC/H3	Abs1n15	2	4	1	4	421.7	13219.6	12798.0
HC/H3	Abs1n15	2	5	1	2	371.7	13762.8	13391.1
HC/H3	Abs1n15	3	1	1	1	311.7	18087.9	17776.2
HC/H3	Abs1n15	3	2	2	2	331.7	17738.5	17406.8
HC/H3	Abs1n15	3	3	1	1	391.7	15405.2	15013.5
HC/H3	Abs1n15	3	4	1	4	431.7	13353.1	12921.4
HC/H3	Abs1n15	3	5	3	5	271.7	13253.8	12982.1
HC/H3	Abs1n15	4	1	3	1	321.7	15634.3	15312.6
HC/H3	Abs1n15	4	2	1	1	371.7	18705.2	18333.6
HC/H3	Abs1n15	4	3	1	3	381.7	14716.0	14334.3
HC/H3	Abs1n15	4	4	2	3	338.3	13597.5	13259.2
HC/H3	Abs1n15	4	5	4	2	347.2	14789.4	14442.2
HC/H3	Abs1n15	5	1	4	1	341.7	15974.4	15632.7
HC/H3	Abs1n15	5	2	1	1	381.7	13841.6	13460.0
HC/H3	Abs1n15	5	3	5	3	291.7	13830.8	13539.1

HC/H3	Abs1n15	5	4	3	4	385.7	14192.4	13806.7
HC/H3	Abs1n15	5	5	5	1	418.3	13932.5	13514.2
HC/H3	Abs1n20	1	1	1	1	299.4	19570.7	19271.2
HC/H3	Abs1n20	1	2	1	2	299.4	20498.2	20198.7
HC/H3	Abs1n20	1	3	1	1	419.4	22790.9	22371.4
HC/H3	Abs1n20	1	4	1	1	459.4	18046.1	17586.6
HC/H3	Abs1n20	1	5	1	2	459.4	22370.0	21910.5
HC/H3	Abs1n20	2	1	1	1	479.4	23109.9	22630.5
HC/H3	Abs1n20	2	2	2	1	499.4	26819.3	26319.8
HC/H3	Abs1n20	2	3	1	3	535.0	18883.7	18348.7
HC/H3	Abs1n20	2	4	1	3	535.0	18883.7	18348.7
HC H3	Abs1n20	2	5	2	5	319.4	17216.8	16897.4
HC/H3	Abs1n20	3	1	2	1	499.4	20839.7	20340.3
HC/H3	Abs1n20	3	2	1	1	518.8	17854.6	17335.8
HC/H3	Abs1n20	3	3	3	3	309.4	23771.3	23461.9
HC/H3	Abs1n20	3	4	2	1	464.6	21563.2	21098.6
HC/H3	Abs1n20	3	5	3	4	469.4	19488.9	19019.4
HC/H3	Abs1n20	4	1	1	1	459.4	22221.7	21762.3
HC/H3	Abs1n20	4	2	1	1	559.4	21455.9	20896.4
HC/H3	Abs1n20	4	3	1	3	504.6	22766.0	22261.4
HC/H3	Abs1n20	4	4	4	1	494.6	17375.8	16881.2
HC/H3	Abs1n20	4	5	3	2	494.6	19733.3	19238.7
HC/H3	Abs1n20	5	1	2	1	429.4	21299.9	20870.5
HC/H3	Abs1n20	5	2	2	1	519.4	15610.7	15091.2

HC/H3	Abs1n20	5	3	4	3	525.5	19679.6	19154.1
HC/H3	Abs1n20	5	4	1	3	519.4	17327.8	16808.4
HC/H3	Abs1n20	5	5	1	2	474.4	18160.5	17686.0
HC/H6	Abs1n5	1	1	1	1	424.7	30862.3	30437.6
HC/H6	Abs1n5	1	2	1	2	404.7	26839.6	26434.9
HC/H6	Abs1n5	1	3	1	1	444.7	28278.9	27834.2
HC/H6	Abs1n5	1	4	1	4	444.7	20676.8	20232.1
HC/H6	Abs1n5	1	5	1	2	416.2	31333.3	30917.0
HC/H6	Abs1n5	2	1	1	1	444.7	53720.6	53275.9
HC/H6	Abs1n5	2	2	2	1	464.7	33683.8	33219.1
HC/H6	Abs1n5	2	3	1	1	444.7	17562.0	17117.3
HC/H6	Abs1n5	2	4	1	4	444.7	25168.2	24723.5
HC/H6	Abs1n5	2	5	2	3	432.3	23759.0	23326.6
HC/H6	Abs1n5	3	1	3	1	424.7	35071.2	34646.5
HC/H6	Abs1n5	3	2	1	2	535.5	14218.2	13682.7
HC/H6	Abs1n5	3	3	3	2	464.7	16707.8	16243.1
HC/H6	Abs1n5	3	4	3	1	474.7	20269.8	19795.2
HC/H6	Abs1n5	3	5	3	1	446.2	14615.8	14169.6
HC/H6	Abs1n5	4	1	1	1	455.7	46058.1	45602.5
HC/H6	Abs1n5	4	2	4	1	442.3	18101.3	17659.0
HC/H6	Abs1n5	4	3	4	1	464.8	12699.6	12234.8
HC/H6	Abs1n5	4	4	1	1	532.4	11923.5	11391.1
HC/H6	Abs1n5	4	5	1	1	482.4	16526.3	16044.0
HC/H6	Abs1n5	5	1	2	1	454.7	32506.8	32052.1

HC H6	Abs1n5	5	2	3	2	474.7	20467.8	19993.1
HC/H6	Abs1n5	5	3	1	1	454.7	21751.6	21296.9
HC/H6	Abs1n5	5	4	3	1	474.7	11241.5	10766.8
HC/H6	Abs1n5	5	5	4	3	492.4	19276.2	18783.8
HC/H6	Abs1n10	1	1	1	1	592.7	34624.3	34031.6
HC/H6	Abs1n10	1	2	1	2	560.0	28849.9	28289.9
HC/H6	Abs1n10	1	3	1	1	653.5	34332.8	33679.3
HC/H6	Abs1n10	1	4	1	2	652.7	36037.9	35385.2
HC/H6	Abs1n10	1	5	1	5	552.7	27529.3	26976.6
HC/H6	Abs1n10	2	1	2	1	542.7	33716.8	33174.1
HC/H6	Abs1n10	2	2	1	1	663.7	29631.2	28967.5
HC/H6	Abs1n10	2	3	2	2	735.3	36692.3	35957.0
HC/H6	Abs1n10	2	4	2	3	722.7	39115.0	38392.3
HC/H6	Abs1n10	2	5	1	5	673.6	36792.5	36118.9
HC/H6	Abs1n10	3	1	3	1	552.7	38539.7	37987.0
HC/H6	Abs1n10	3	2	2	2	644.6	31766.4	31121.9
HC/H6	Abs1n10	3	3	3	1	642.7	34309.7	33667.0
HC/H6	Abs1n10	3	4	1	1	781.9	38166.9	37385.0
HC/H6	Abs1n10	3	5	1	4	672.7	23176.2	22503.5
HC/H6	Abs1n10	4	1	1	1	629.2	41564.0	40934.8
HC/H6	Abs1n10	4	2	3	2	652.7	30105.8	29453.1
HC/H6	Abs1n10	4	3	3	1	678.2	25510.9	24832.7
HC/H6	Abs1n10	4	4	4	3	749.0	36824.2	36075.2
HC/H6	Abs1n10	4	5	2	1	662.7	40450.9	39788.2

HC/H6	Abs1n10	5	1	2	1	651.1	37378.5	36727.5
HC/H6	Abs1n10	5	2	1	2	752.7	25830.1	25077.4
HC/H6	Abs1n10	5	3	2	3	642.7	33608.9	32966.2
HC/H6	Abs1n10	5	4	3	2	690.0	25708.2	25018.2
HC/H6	Abs1n10	5	5	4	5	692.7	32429.9	31737.3
HC/H6	Abs1n15	1	1	1	1	711.3	49837.7	49126.4
HC/H6	Abs1n15	1	2	1	1	931.3	46103.3	45172.0
HC/H6	Abs1n15	1	3	1	1	1081.3	37584.8	36503.4
HC/H6	Abs1n15	1	4	1	3	958.2	44088.1	43129.9
HC/H6	Abs1n15	1	5	1	1	878.5	41920.0	41041.5
HC/H6	Abs1n15	2	1	1	1	888.0	39483.7	38595.7
HC/H6	Abs1n15	2	2	2	2	731.3	27879.5	27148.2
HC/H6	Abs1n15	2	3	1	2	1015.8	33299.7	32283.9
HC H6	Abs1n15	2	4	2	1	841.3	36590.2	35748.9
HC/H6	Abs1n15	2	5	1	4	931.3	40585.1	39653.8
HC/H6	Abs1n15	3	1	2	1	951.3	46139.1	45187.7
HC/H6	Abs1n15	3	2	3	2	691.3	36703.3	36012.0
HC/H6	Abs1n15	3	3	1	1	901.3	32048.1	31146.8
HC/H6	Abs1n15	3	4					0.0
HC/H6	Abs1n15	3	5	2	3	941.3	31093.3	30152.0
HC/H6	Abs1n15	4	1	4	1	731.3	47019.0	46287.7
HC/H6	Abs1n15	4	2	2	2	891.3	46052.4	45161.1
HC/H6	Abs1n15	4	3	3	1	916.2	26687.2	25771.0
HC/H6	Abs1n15	4	4	3	1	847.8	33849.5	33001.7

HC/H6	Abs1n15	4	5	4	1	1021.3	39407.8	38386.5
HC/H6	Abs1n15	5	1	4	1	1021.3	39407.8	38386.5
HC/H6	Abs1n15	5	2	2	1	831.3	36115.0	35283.7
HC/H6	Abs1n15	5	3	1	3	871.3	33351.9	32480.6
HC/H6	Abs1n15	5	4	2	2	871.3	41766.9	40895.6
HC/H6	Abs1n15	5	5	5	1	923.7	29935.5	29011.8
HC/H6	Abs1n20	1	1	1	1	816.2	53427.1	52611.0
HC/H6	Abs1n20	1	2	1	1	1140.6	65484.5	64343.9
HC/H6	Abs1n20	1	3	1	2	1182.9	62913.9	61731.0
HC/H6	Abs1n20	1	4	1	4	800.6	47526.0	46725.4
HC/H6	Abs1n20	1	5	1	1	1253.4	50799.8	49546.4
HC/H6	Abs1n20	2	1	2	1	840.6	53475.2	52634.6
HC/H6	Abs1n20	2	2	2	1	1242.7	45556.0	44313.3
HC/H6	Abs1n20	2	3	2	2	1260.6	41172.7	39912.1
HC/H6	Abs1n20	2	4	2	3	1227.2	46476.5	45249.3
HC/H6	Abs1n20	2	5	1	2	1252.7	42376.9	41124.2
HC/H6	Abs1n20	3	1	2	1	1255.4	37256.5	36001.2
HC/H6	Abs1n20	3	2	1	1	1228.2	47709.7	46481.5
HC/H6	Abs1n20	3	3	2	1	1090.6	46143.3	45052.7
HC/H6	Abs1n20	3	4	1	2	1190.6	53723.2	52532.6
HC/H6	Abs1n20	3	5	1	3	1195.8	49584.4	48388.7
HC/H6	Abs1n20	4	1	3	1	1270.6	41444.9	40174.3
HC/H6	Abs1n20	4	2	4	1	1239.5	43861.0	42621.5
HC/H6	Abs1n20	4	3	2	3	1270.6	50727.0	49456.4

HC/H6	Abs1n20	4	4	2	1	1163.4	57469.3	56305.9
HC/H6	Abs1n20	4	5	2	3	1173.8	48314.5	47140.8
HC H6	Abs1n20	5	1	4	1	1192.9	49253.2	48060.3
HC/H6	Abs1n20	5	2	4	1	1160.6	49720.0	48559.4
HC/H6	Abs1n20	5	3	4	1	1160.6	49720.0	48559.4
HC/H6	Abs1n20	5	4					0.0
HC/H6	Abs1n20	5	5					0.0
LC/H3	Abs1n5	1	1	1	1	131.6	14476.7	14345.1
LC/H3	Abs1n5	1	2	1	1	161.6	10461.0	10299.4
LC/H3	Abs1n5	1	3	1	1	161.6	4974.5	4812.9
LC/H3	Abs1n5	1	4	1	4	151.6	5242.7	5091.1
LC/H3	Abs1n5	1	5	1	2	151.6	5253.9	5102.3
LC/H3	Abs1n5	2	1	1	1	161.6	9434.5	9272.9
LC/H3	Abs1n5	2	2	2	2	141.6	10072.3	9930.7
LC/H3	Abs1n5	2	3	1	2	172.5	6864.9	6692.5
LC/H3	Abs1n5	2	4	1	4	161.6	7455.4	7293.8
LC/H3	Abs1n5	2	5	2	1	131.6	5929.9	5798.3
LC/H3	Abs1n5	3	1	1	1	161.6	4312.8	4151.2
LC/H3	Abs1n5	3	2	2	1	151.6	7635.6	7484.0
LC/H3	Abs1n5	3	3	1	2	161.6	8540.9	8379.3
LC/H3	Abs1n5	3	4	3	1	159.1	6321.8	6162.7
LC/H3	Abs1n5	3	5	3	2	171.6	7205.6	7034.0
LC/H3	Abs1n5	4	1	2	1	151.6	10136.4	9984.8
LC/H3	Abs1n5	4	2	2	1	161.6	6748.2	6586.6

LC/H3	Abs1n5	4	3	1	2	141.6	9790.5	9648.9
LC/H3	Abs1n5	4	4	2	4	151.6	7323.5	7171.9
LC/H3	Abs1n5	4	5	3	2	151.6	8124.4	7972.8
LC/H3	Abs1n5	5	1	5	1	131.6	8162.2	8030.6
LC/H3	Abs1n5	5	2	2	1	151.6	6181.0	6029.4
LC/H3	Abs1n5	5	3	3	3	151.6	6231.8	6080.2
LC/H3	Abs1n5	5	4	5	4	131.6	6119.0	5987.4
LC/H3	Abs1n5	5	5	3	3	151.6	7818.1	7666.5
LC/H3	Abs1n10	1	1	1	1	200.5	16246.8	16046.3
LC/H3	Abs1n10	1	2	1	2	240.5	15552.0	15311.6
LC/H3	Abs1n10	1	3	1	1	230.5	12318.0	12087.6
LC/H3	Abs1n10	1	4	1	3	220.5	12554.9	12334.4
LC/H3	Abs1n10	1	5	1	2	230.5	14669.7	14439.3
LC/H3	Abs1n10	2	1	1	1	280.5	13703.7	13423.2
LC/H3	Abs1n10	2	2	2	2	230.5	10162.2	9931.8
LC H3	Abs1n10	2	3	2	3	200.5	16161.5	15961.0
LC/H3	Abs1n10	2	4	2	4	210.5	14148.0	13937.6
LC/H3	Abs1n10	2	5	1	4	260.5	11047.2	10786.7
LC/H3	Abs1n10	3	1	2	1	250.5	15157.6	14907.1
LC/H3	Abs1n10	3	2	1	2	240.5	11367.8	11127.4
LC/H3	Abs1n10	3	3	3	3	200.5	11543.9	11343.4
LC/H3	Abs1n10	3	4	2	4	260.5	15547.0	15286.5
LC/H3	Abs1n10	3	5	2	5	280.5	10625.4	10344.9
LC/H3	Abs1n10	4	1	2	1	280.5	12027.0	11746.6

LC/H3	Abs1n10	4	2	4	1	240.5	11492.5	11252.0
LC/H3	Abs1n10	4	3	1	3	240.5	12102.9	11862.4
LC/H3	Abs1n10	4	4	2	3	268.6	11059.6	10791.0
LC/H3	Abs1n10	4	5	1	4	278.6	12370.0	12091.4
LC/H3	Abs1n10	5	1	5	1	220.5	16330.3	16109.8
LC/H3	Abs1n10	5	2	5	2	220.5	10899.1	10678.7
LC/H3	Abs1n10	5	3	3	2	270.5	11639.1	11368.6
LC/H3	Abs1n10	5	4	4	2	240.5	7273.1	7032.6
LC/H3	Abs1n10	5	5	3	5	270.5	8923.9	8653.5
LC/H3	Abs1n15	1	1	1	1	251.7	17720.8	17469.1
LC/H3	Abs1n15	1	2	1	2	281.7	17309.4	17027.7
LC/H3	Abs1n15	1	3	1	1	321.7	13079.0	12757.3
LC/H3	Abs1n15	1	4	1	3	291.7	14738.7	14447.1
LC/H3	Abs1n15	1	5	1	2	351.7	11192.5	10840.8
LC/H3	Abs1n15	2	1	1	1	381.7	14626.0	14244.3
LC/H3	Abs1n15	2	2	2	2	281.7	16487.0	16205.3
LC/H3	Abs1n15	2	3	2	3	271.7	11025.9	10754.2
LC/H3	Abs1n15	2	4	2	2	341.7	16217.8	15876.1
LC/H3	Abs1n15	2	5	1	3	341.7	14360.3	14018.7
LC/H3	Abs1n15	3	1	3	1	281.7	11261.9	10980.3
LC/H3	Abs1n15	3	2	1	2	416.9	13369.8	12953.0
LC/H3	Abs1n15	3	3	1	3	361.7	13508.1	13146.4
LC/H3	Abs1n15	3	4	1	4	415.8	13564.3	13148.6
LC/H3	Abs1n15	3	5	2	4	398.3	12972.9	12574.6

LC/H3	Abs1n15	4	1	4	1	311.7	12161.3	11849.6
LC/H3	Abs1n15	4	2	2	2	348.3	10196.4	9848.1
LC/H3	Abs1n15	4	3	2	2	437.0	15324.6	14887.6
LC/H3	Abs1n15	4	4	2	3	358.3	12024.1	11665.8
LC H3	Abs1n15	4	5	3	5	321.7	15166.9	14845.2
LC/H3	Abs1n15	5	1	4	1	421.7	17153.2	16731.5
LC/H3	Abs1n15	5	2	1	2	441.7	10101.7	9660.1
LC/H3	Abs1n15	5	3	3	1	388.3	9326.1	8937.9
LC/H3	Abs1n15	5	4	1	3	358.3	11548.0	11189.7
LC/H3	Abs1n15	5	5	1	3	377.2	12276.6	11899.4
LC/H3	Abs1n20	1	1	1	1	319.4	25973.2	25653.7
LC/H3	Abs1n20	1	2	1	2	289.4	17410.4	17121.0
LC/H3	Abs1n20	1	3	1	2	479.4	23132.5	22653.1
LC/H3	Abs1n20	1	4	1	1	540.0	15432.5	14892.5
LC/H3	Abs1n20	1	5	1	5	319.4	21472.5	21153.0
LC/H3	Abs1n20	2	1	2	1	309.4	25843.2	25533.7
LC/H3	Abs1n20	2	2	2	2	322.3	24863.8	24541.5
LC/H3	Abs1n20	2	3	2	2	459.4	15318.2	14858.8
LC/H3	Abs1n20	2	4	2	2	499.4	15349.2	14849.7
LC/H3	Abs1n20	2	5	2	3	494.6	12568.4	12073.7
LC/H3	Abs1n20	3	1	1	1	459.4	21525.4	21066.0
LC/H3	Abs1n20	3	2	1	2	519.4	15014.5	14495.1
LC/H3	Abs1n20	3	3	2	3	529.4	15185.6	14656.2
LC/H3	Abs1n20	3	4	1	4	519.4	14188.8	13669.4

LC/H3	Abs1n20	3	5	2	4	474.6	13616.9	13142.3
LC/H3	Abs1n20	4	1	4	1	359.4	14662.0	14302.6
LC/H3	Abs1n20	4	2	3	1	529.4	12423.4	11894.0
LC/H3	Abs1n20	4	3	2	2	484.8	15191.6	14706.8
LC/H3	Abs1n20	4	4	4	2	439.4	16714.0	16274.6
LC/H3	Abs1n20	4	5	1	1	489.4	11438.0	10948.5
LC/H3	Abs1n20	5	1	2	1	349.4	24305.3	23955.9
LC/H3	Abs1n20	5	2	1	1	499.4	17129.7	16630.3
LC/H3	Abs1n20	5	3	2	3	456.6	15396.6	14940.0
LC/H3	Abs1n20	5	4	1	1	459.4	16412.8	15953.4
LC/H3	Abs1n20	5	5	2	2	474.6	17790.9	17316.3
LC/H6	Abs1n5	1	1	1	1	141.6	10252.5	10110.9
LC/H6	Abs1n5	1	2	1	2	131.6	7953.8	7822.2
LC/H6	Abs1n5	1	3	1	3	131.6	8557.1	8425.5
LC/H6	Abs1n5	1	4	1	3	151.6	11299.7	11148.1
LC/H6	Abs1n5	1	5	1	1	161.6	6384.1	6222.5
LC/H6	Abs1n5	2	1	2	1	131.6	13880.9	13749.3
LC H6	Abs1n5	2	2	1	2	171.6	8511.3	8339.7
LC/H6	Abs1n5	2	3	2	2	161.6	7185.3	7023.7
LC/H6	Abs1n5	2	4	2	3	151.6	7078.0	6926.5
LC/H6	Abs1n5	2	5	1	5	151.6	9698.4	9546.8
LC/H6	Abs1n5	3	1	1	1	151.6	10756.8	10605.2
LC/H6	Abs1n5	3	2	2	1	151.6	7989.7	7838.1
LC/H6	Abs1n5	3	3	2	3	151.6	6287.1	6135.5
LC/H6	Abs1n5	3	4	2	3	151.6	6287.1	6135.5

LC/H6	Abs1n5	3	5	2	5	151.6	9495.1	9343.5
LC/H6	Abs1n5	4	1	1	1	151.6	8069.9	7918.3
LC/H6	Abs1n5	4	2	3	2	171.6	9137.9	8966.3
LC/H6	Abs1n5	4	3	4	2	151.6	5977.3	5825.7
LC/H6	Abs1n5	4	4	2	2	161.6	6280.7	6119.1
LC/H6	Abs1n5	4	5	2	5	151.6	7357.5	7205.9
LC/H6	Abs1n5	5	1	1	1	161.6	5084.9	4923.3
LC/H6	Abs1n5	5	2	1	1	171.6	10702.2	10530.6
LC/H6	Abs1n5	5	3	1	2	151.6	10323.5	10171.9
LC/H6	Abs1n5	5	4	4	4	161.6	5909.9	5748.3
LC/H6	Abs1n5	5	5	1	4	151.6	6674.8	6523.2
LC/H6	Abs1n10	1	1	1	1	522.7	58606.9	58084.2
LC/H6	Abs1n10	1	2	1	1	713.8	40693.1	39979.3
LC/H6	Abs1n10	1	3	1	1	707.1	44579.2	43872.1
LC/H6	Abs1n10	1	4	1	3	668.5	39515.4	38847.0
LC/H6	Abs1n10	1	5	1	4	692.7	30294.7	29602.0
LC/H6	Abs1n10	2	1	2	1	582.7	37553.8	36971.1
LC/H6	Abs1n10	2	2	1	1	622.7	32990.4	32367.7
LC/H6	Abs1n10	2	3	1	2	772.7	45256.6	44483.9
LC/H6	Abs1n10	2	4	1	4	709.0	31485.6	30776.5
LC/H6	Abs1n10	2	5	1	2	652.7	39084.0	38431.3
LC/H6	Abs1n10	3	1	1	1	652.7	43667.5	43014.8
LC/H6	Abs1n10	3	2	2	1	732.7	34454.7	33722.0
LC/H6	Abs1n10	3	3	1	3	672.7	34135.0	33462.3

LC/H6	Abs1n10	3	4	3	2	692.7	34529.4	33836.7
LC/H6	Abs1n10	3	5	2	5	682.7	21159.7	20477.1
LC/H6	Abs1n10	4	1	4	1	602.7	58216.7	57614.0
LC/H6	Abs1n10	4	2	4	1	802.7	28969.9	28167.2
LC/H6	Abs1n10	4	3	2	2	782.7	17134.6	16351.9
LC H6	Abs1n10	4	4	2	1	702.7	29211.6	28508.9
LC/H6	Abs1n10	4	5	4	1	637.1	18887.2	18250.1
LC/H6	Abs1n10	5	1	4	1	662.7	26141.4	25478.7
LC/H6	Abs1n10	5	2	5	2	562.7	32296.6	31734.0
LC/H6	Abs1n10	5	3	1	2	702.7	39830.5	39127.8
LC/H6	Abs1n10	5	4	1	2	682.7	25463.2	24780.5
LC/H6	Abs1n10	5	5	1	1	702.7	23390.8	22688.1
LC/H6	Abs1n15	1	1	1	1	731.3	44834.9	44103.6
LC/H6	Abs1n15	1	2	1	1	893.7	31917.6	31023.9
LC/H6	Abs1n15	1	3	1	3	691.3	24594.6	23903.3
LC/H6	Abs1n15	1	4	1	4	721.3	34875.7	34154.4
LC/H6	Abs1n15	1	5	1	3	991.3	33713.2	32721.9
LC/H6	Abs1n15	2	1	2	1	736.2	34248.9	33512.8
LC/H6	Abs1n15	2	2	2	2	721.3	39656.5	38935.2
LC/H6	Abs1n15	2	3	1	3	960.7	41314.5	40353.8
LC/H6	Abs1n15	2	4	1	5	896.2	35023.7	34127.5
LC/H6	Abs1n15	2	5	2	1	908.5	31017.7	30109.2
LC/H6	Abs1n15	3	1	3	1	701.3	30806.6	30105.3
LC/H6	Abs1n15	3	2	3	2	691.3	34627.7	33936.3

LC/H6	Abs1n15	3	3	3	2	871.3	29754.6	28883.3
LC/H6	Abs1n15	3	4	1	4	861.3	28351.7	27490.4
LC/H6	Abs1n15	3	5	2	4	883.7	31113.4	30229.7
LC/H6	Abs1n15	4	1	4	1	711.3	33594.2	32882.9
LC/H6	Abs1n15	4	2	4	2	711.3	33463.2	32751.9
LC/H6	Abs1n15	4	3	1	1	943.7	30000.9	29057.1
LC/H6	Abs1n15	4	4	4	4	701.3	28119.3	27418.0
LC/H6	Abs1n15	4	5	3	2	951.3	27018.8	26067.5
LC/H6	Abs1n15	5	1	2	1	891.3	29226.7	28335.4
LC/H6	Abs1n15	5	2	1	2	901.3	27985.5	27084.2
LC/H6	Abs1n15	5	3	4	3	861.3	31768.1	30906.8
LC/H6	Abs1n15	5	4	1	4	851.3	30556.5	29705.2
LC/H6	Abs1n15	5	5	1	5	951.3	28889.8	27938.5
LC/H6	Abs1n20	1	1	1	1	810.6	45292.1	44481.5
LC/H6	Abs1n20	1	2	1	2	830.6	51582.5	50751.9
LC/H6	Abs1n20	1	3	1	2	1160.7	38105.5	36944.7
LC/H6	Abs1n20	1	4	1	3	1241.3	47983.1	46741.9
LC/H6	Abs1n20	1	5	1	5	870.6	45367.5	44496.9
LC H6	Abs1n20	2	1	1	1	1190.6	46488.1	45297.5
LC/H6	Abs1n20	2	2	2	2	830.6	39386.3	38555.7
LC/H6	Abs1n20	2	3	2	1	1204.4	43618.3	42413.9
LC/H6	Abs1n20	2	4	1	2	1190.6	44311.5	43120.9
LC/H6	Abs1n20	2	5	1	4	1163.4	43249.2	42085.8
LC/H6	Abs1n20	3	1	1	1	1190.6	35523.9	34333.3

LC/H6	Abs1n20	3	2	1	1	1190.6	35523.9	34333.3
LC/H6	Abs1n20	3	3	2	2	1150.6	47711.7	46561.1
LC/H6	Abs1n20	3	4	2	3	1130.6	42277.8	41147.2
LC/H6	Abs1n20	3	5	3	3	1130.6	38192.4	37061.8
LC/H6	Abs1n20	4	1	4	1	910.6	43290.1	42379.5
LC/H6	Abs1n20	4	2	3	2	1170.6	32040.3	30869.7
LC/H6	Abs1n20	4	3	3	2	1260.6	34753.3	33492.7
LC/H6	Abs1n20	4	4	3	2	1202.9	33990.4	32787.5
LC/H6	Abs1n20	4	5	2	4	1190.6	31469.0	30278.4
LC/H6	Abs1n20	5	1	3	1	1195.6	52261.4	51065.8
LC/H6	Abs1n20	5	2	1	2	1272.9	36268.7	34995.7
LC/H6	Abs1n20	5	3	4	3	1160.2	38929.7	37769.5
LC/H6	Abs1n20	5	4	1	2	1213.0	50306.5	49093.5
LC/H6	Abs1n20	5	5	1	2	1212.7	41597.5	40384.7