Operational Research in Engineering Sciences: Theory and Applications
Vol. 6, Issue 3, 2023, pp. 176-202
ISSN: 2620-1607
eISSN: 2620-1747
DOI: https://doi.org/10.31181/oresta/060309

MULTI-ATTRIBUTE DECISION MAKING BASED ON PROBABILISTIC DUAL HESITANT PYTHAGOREAN FUZZY INFORMATION.

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Received: 18 May 2023
Accepted: 28 October2023
First Online: 20 November2023

Research Paper

Abstract: *In this paper, we propose a new multi-attribute decision-making (MADM) method based on probabilistic dual hesitant Pythagorean fuzzy (PDHPF) sets. The existing PDHPF power weighted Hamy mean operator has the drawback that if one PDHPF element among the PDHPF elements whose non-membership grade equals 0, then the non-membership grade of the aggregated PDHPF element is equal to 0. Thus, the existing MADM method based on the PDHPF power weighted Hamy mean operator has the drawback that it cannot distinguish the ranking orders of alternatives in some situations. To overcome these drawbacks of the existing MADM method, this paper proposes the PDHPF improved power weighted averaging (PDHPFIPWA) operator and proposes a MADM method based on the proposed PDHPFIPWA operator. The proposed MADM method can overcome the drawbacks of the existing MADM method. It offers us a very useful approach for MADM in PDHPF environments.*

Keywords: Probabilistic Dual Hesitant Pythagorean Fuzzy Set, Probabilistic Dual Hesitant Pythagorean Fuzzy Element, Improved Power-Weighted Averaging Operator, Multi-Attribute Decision Making.

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1. Introduction

It is impossible for decision-experts to give evaluation information quantitatively or qualitatively through accurate numerical values in various practical difficulties due to the irrationality of human thought and the growing complexity of day-to-day problems. (Zadeh, 1965) initially developed the idea of fuzzy set (FS) theory, which has been widely used for a variety of applications, to address this issue. In order to quantify the uncertainty in terms of membership degree (MD) and non-membership degree, Atanassov (Atanassov, 1986) suggested an intuitionistic fuzzy set (IFS) as an extension of FS (NMD).

The shortcoming that Zadeh's FSs can only define fuzzy information through MD was remedied by IFS. Numerous academics have done in-depth research since IFSs were first proposed (Chen & Chang, 2016; Chen, Cheng, & Lan, 2016; Chen & Chu, 2020; Chen & Tsai, 2021; Dutta, 2021; Garg, 2017b, 2019; Garg & Kaur, 2020; Garg & Kaur, 2018; Kadian & Kumar, 2021; Kumar & Chen, 2021; Liu & Xiao, 2019; Mishra et al., 2020; Mishra, Singh, & Motwani, 2019; Raj, 2016; Zeng, Chen, & Kuo, 2019; Zou, Chen, & Fan, 2020). Due to the requirement that MD and NMD's total not be more than 1, it can only be used to real-world, dynamic problems. Due to the possibility that certain decision-makers may propose some assessing attribute values that exceed the restriction, many complicated assessment data sets cannot be represented under this criterion. For instance, the IFSs are not appropriate to be used for this kind of issue if the MD and the NMD of an assessment attribute value provided by a decision-maker are 0.8 and 0.6, respectively.

The idea of Pythagorean FSs (PFSs), which Yager (Yager, 2013a, 2013b) established with the development of fuzzy theory, is characterised by the fact that the total of the squares of the MD and the NMD is less than or equal to one. As a result, PFSs are preferred over IFSs for the expression of ambiguous data. Numerous extensive studies have been conducted in light of the PFSs idea. As an illustration, Yager and Abbasov (2013) created a decision-making strategy using PFSs. The TOPSIS approach was expanded by Zhang and Xu (2014) to address decision-making problems with PFSs. To address issues with multi-attribute decision-making (MADM) Ma and Xu (2016), introduced the symmetric Pythagorean fuzzy (PF) weighted aggregation operators (AOs). New generalized Pythagorean fuzzy (PF) information aggregation operators (AOs) were defined by Garg (2016a) and used in decision-making. A decision-making procedure based on PFS correlation coefficients was described by Garg (2016b, 2017a). provided an approach based on confidence levels and PF data. A number of PF information measures were developed by Peng, Yuan, and Yang (2017) and used in decision-making.

Based on PFSs and Einstein operations, Garg created generalised geometric interactive AOs in Garg (2018). Some PF exponential similarity metrics were created by Nguyen et al. (2019) and used in pattern recognition Nie et al. (2019). introduced a decision-making method using PFSs based on Shapley fuzzy measurements and partitioned normalised weighted Bonferroni mean operator. To solve MADM issues Jana, Senapati, and Pal (2019), employed PF Dombi operators Rani et al. (2021). Rani et al. introduced a VIKOR technique in (Rani et al., 2019) hat uses PFS entropy and divergence measurements. A novel decision-making framework was suggested by Rani et al. (2020) to evaluate health-care waste treatment technology. A generalised tri-parametric correlation coefficient for PFSs was proposed by Ejegwa (2021). New distance metrics for PFSs were provided by Ejegwa and Awolola (2021) and used in pattern recognition issues. Rani et al. (2021) evaluated the sustainable bio energy technology for agricultural leftovers using weighted discrimination methods based on PFSs. An MCDM problem was solved by Senapati et al. (2022) under PFSs setting.

Although the PFSs are efficient in handling experts' complex cognitive information, sometimes experts face difficulty in setting MDs and NMDs because they may lie in a set of probable values instead of taking single values. To address this, the notion of dual-hesitant Pythagorean fuzzy sets (DHPFS) was introduced by Wei and Lu (2017). Two sets of some values in the range [0, 1] that represent the possible MDs and NMDs, respectively, are another characteristic of a DHPFS. Thus, compared to PFSSs, DHPFSs depict attribute values with higher flexibility Lu et al. (2019). utilized DHPFSs in the performance evaluation of new rural construction. A MADM approach based on DHPFSs was proposed by Ji, Zhang, and Wang (2021); Tang and Wei (2019b), Tang and Wei presented a decision-making approach for DHPFSs. In addition, a few researchers have worked on the development of aggregation operators on DHPFSs Tang and Wei (2019a). proposed DHPFSs based Bonferroni mean operators Wei et al. (2019). developed weighted Hamy mean operators for DHPFSs.

Although DHPFSs are superior to PFSs, still have a short-coming when dealing with experts' evaluation information. It was assumed that objects of the DHPFSs have an equal weight, which may not be true in reality because different objects may have different significance. To address this problem Ji et al. (2021), proposed the notion of a probabilistic dual-hesitant Pythagorean fuzzy set (PDHPFS). The objects of PDHPFSs are associated with some probabilistic information. Thus, PDHPFS can depict the uncertain information more accurately Ji et al. (2021). proposed probabilistic dual-hesitant Pythagorean fuzzy (PDHPF) power weighted Hamy mean (PDHPFPWHM) operator and used it to solve MADM problems. Ji et al. (2021) method has the drawback that, in some situations, it is unable to discriminate between the priority orders of alternatives or cannot acquire the priority order of alternatives.

To overcome the drawback of the current method, an unique MADM strategy must be created. The followings are the significant contributions of our paper:

1. In order to obtain a logical priority order of options, we first propose some improved operational laws for PDHPF elements and then develop PDHPF improved power-weighted averaging (PDHPFIPWA) operator. The proposed PDHPFIPWA operator can conquer the limitation of the PDHPF power weighted Hamy mean operator.

2. A new MADM strategy based on the PDHPFIPWA operator is suggested. The shortcomings of Ji et al.'s (2021) method can be solved by the suggested methodology Ji et al. (2021). The rest part is arranged as follows:

We concisely discuss some essential concepts related to PDHPFs in section 2. Section 3 investigates the short-coming of Ji et al. (2021) method based on PDHPFPWHM operator. We demonstrate some enhanced operations between PDHPFEs in section 4. Additionally, the PDHPF weighted enhanced AO are shown in this part along with their properties. In Section 5, we offer a mechanism for making decisions based on the created AO. The last section is the conclusions.

2. Preliminaries

We go through a few fundamental ideas that are pertinent to this article in this part.

Definition 2.1 (Ji et al. 2021): A PDHPFS $\Im(\Upsilon)$ on a universe set $U = \{u_1, u_2, ..., u_n\}$ is represented as:

$\Im(\Upsilon) = \{< u_i, \mu_{\Im}(\Upsilon)(u_i), \gamma_{\Im}(\Upsilon)(u_i) >: u_i \in U\}$, where

$\mu_{\Im}(\Upsilon)(u_i) = \bigcup_a \{\Delta_{u_i}^{(a)}(\Upsilon^{(a)})\}$ and $\gamma_{\Im}(\Upsilon)(u_i) = \bigcup_b \{\nabla_{u_i}^{(b)}(\Upsilon^{(b)})\}$

$$(0 \le \Upsilon^{(a)}, \Upsilon^{(b)} \le 1, \sum_a \Upsilon^{(a)} \le 1, \sum_b \Upsilon^{(b)} \le 1)$$

denote the MD and the NMD of the element u_i belonging to the PDHPFS $\Im(\Upsilon)$ with the probabilities $\Upsilon^{(a)}$ and $\Upsilon^{(b)}$, respectively with $0 \le (\Delta_{u_i}^{(a)})^2 + (\nabla_{u_i}^{(b)})^2 \le 1$ for each a and b. $\Im(\Upsilon)$ is transformed into a PDHPF element (PDHPFE) if it is contains only one element.

The PDHPFS $\Im(\Upsilon)$ is expressed as $\Im(\Upsilon) =< \bigcup_a \{\Delta^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla^{(b)}(\Upsilon^{(b)})\} >$.

Definition 2.2: The score value $S_c(\Im(\Upsilon))$ of PDHPFE $\Im(\Upsilon) =< \bigcup_a \{\Delta^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla^{(b)}(\Upsilon^{(b)})\} >$ is calculated by.

$$Sc(\Im(\Upsilon)) = \sum_a (\Delta^{(a)} \times \Upsilon^{(a)}) - \sum_b (\nabla^{(b)} \times \Upsilon^{(b)}) \tag{1}$$

Sometimes the ranking order between PDHPFEs cannot be distinguished if their score values are identical. To address this issue, their accuracy values can be used.

Definition 2.3: The accuracy value of the PDHPFE $\Im(\Upsilon) =< \bigcup_a \{\Delta^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla^{(b)}(\Upsilon^{(b)})\} >$ is calculated by:

$$A_c(\Im(\Upsilon)) = \sum_a (\Delta^{(a)} \times \Upsilon^{(a)}) + \sum_b (\nabla^{(b)} \times \Upsilon^{(b)}) \tag{2}$$

Definition 2.4 (Ji et al, 2021): The ranking rules between the PDHPFEs $\Im^{(1)}(\Upsilon)$ and $\Im^{(2)}(\Upsilon)$ are defined as follows:

(a) If $S_c(\Im^{(1)}(\Upsilon)) > S_c(\Im^{(2)}(\Upsilon))$, then $\Im^{(1)}(\Upsilon) \succ \Im^{(2)}(\Upsilon)$
(b) If $S_c(\Im^{(1)}(\Upsilon)) = S_c(\Im^{(2)}(\Upsilon))$, then
(i) If $A_c(\Im^{(1)}(\Upsilon)) > A_c(\Im^{(2)}(\Upsilon))$, then $\Im^{(1)}(\Upsilon) \succ \Im^{(2)}(\Upsilon)$
(ii) If $A_c(\Im^{(1)}(\Upsilon)) = A_c(\Im^{(2)}(\Upsilon))$, then $\Im^{(1)}(\Upsilon) = \Im^{(2)}(\Upsilon)$

Definition 2.5: Let $\Im^{(j)}(\Upsilon) =< \mu_{\Im^{(j)}}(\Upsilon), \gamma_{\Im^{(j)}}(\Upsilon) >=< \bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_j^{(b)}(\Upsilon^{(b)})\} >$ $(j = 1, 2)$ be two PDHPFEs. The distance between these two PDHPFEs is defined as follows:

$$D(\Im^{(1)}(\Upsilon), \Im^{(1)}(\Upsilon)) = \frac{1}{(r+s)} \left(\begin{array}{l} \sum_{a=1}^{r} \left| (\Delta_1^{(a)})^2 \times (\Upsilon_1^{(a)}) - (\Delta_2^{(a)})^2 \times (\Upsilon_2^{(a)}) \right| \\ + \sum_{b=1}^{s} \left| (\nabla_1^{(b)})^2 \times (\Upsilon_1^{(b)}) - (\nabla_2^{(b)})^2 \times (\Upsilon_2^{(b)}) \right| \end{array} \right) \tag{3}$$

where r and s denote the number of values in $\mu_{\Im^{(j)}}(\Upsilon)$ and $\gamma_{\Im^{(j)}}(\Upsilon)$ respectively.

Definition 2.6 (Yager, 2001): Let be real values between zero and one. The power average (PA) operator of the real values $a_1, a_2, ..., a_n$ is defined by.

$$PA(a_1, a_2, ..., a_n) = \frac{\sum_{i=1}^{n}(1+\psi(a_i))a_i}{\sum_{i=1}^{n}(1+\psi(a_i))} \tag{4}$$

where $\psi(a_i) = \sum_{j=1, j\neq i}^{n} Supp(a_i, a_j)$ and $Supp(a_i, a_j)$ denotes the support for a_i from a_j, which has the following three properties:

(i) $0 \le Supp(a_i, a_j) \le 1$

(ii) $Supp(a_i, a_j) = Supp(a_j, a_i)$

(iii) $Supp(a_i, a_j) \ge Supp(a_k, a_r)$ provided $|a_i - a_j| < |a_k - a_r|$ where $i, j, k, r \in \{1, 2, ..., n\}$

Some applications of the PA operator can be found in (Atanassov, 1986; Ejegwa & Awolola, 2021; Garg, 2016a; Ji et al., 2021; Kumar & Chen, 2021; Liu & Xiao, 2019; Ma & Xu, 2016).

3. Analyzing the drawbacks of Ji et al.'s method (Ji et al., 2021)

To analyze the short-comings of Ji et al.'s method (Ji et al., 2021), we first recall the Ji et al.'s MADM procedure (Ji et al., 2021) described below:

Suppose there are m number of options A_i $(i = 1(1)m)$ and n number of criteria C_j $(j = 1(1)n)$ associated with a MADM problem where each alternative is assessed by an expert under PDHPF setting. Assume that the initial result evaluated by the expert is represented as:

$$M = \left[\Im^{(ij)}(\Upsilon)\right]_{m\times n} = \left[< \bigcup_a \{\Delta_{ij}^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_{ij}^{(b)}(\Upsilon^{(b)})\} > \right]_{m\times n} \quad (i = 1(1)m,\ j = 1(1)n).$$

Then the Ji et al. (2021) methodology encompasses the succeeding steps:

Step 1: Obtain the normalized the PDHPF matrix $\tilde{M} = \left[\tilde{\Im}^{(ij)}(\Upsilon)\right]_{m\times n}$ using the following equation.

$$\tilde{\Im}^{(ij)}(\Upsilon) = \begin{cases} < \bigcup_a \{\Delta_{ij}^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_{ij}^{(b)}(\Upsilon^{(b)})\} >, \text{ if } C_j \text{ is of profit type} \\ < \bigcup_b \{\nabla_{ij}^{(b)}(\Upsilon^{(b)})\}, \bigcup_a \{\Delta_{ij}^{(a)}(\Upsilon^{(a)})\}, >, \text{ if } C_j \text{ is of non-profit type} \end{cases} \tag{5}$$

Step 2: Compute the supports $Supp(\tilde{\Im}^{(ij)}(\Upsilon), \tilde{\Im}^{(ik)}(\Upsilon))$ based on the following formula:

$$Supp(\tilde{\Im}^{(ij)}(\Upsilon), \tilde{\Im}^{(ik)}(\Upsilon)) = 1 - D(\tilde{\Im}^{(ij)}(\Upsilon), \tilde{\Im}^{(ik)}(\Upsilon)) \ \ (j,k = 1(1)n;\ j \neq k) \tag{6}$$

Step 3: Determine the values $\psi(\tilde{\Im}^{(ij)}(\Upsilon))$ utilizing the following formula.

$$\psi(\tilde{\Im}^{(ij)}(\Upsilon)) = \sum_{j=1,\, j\neq k}^{n} Supp(\tilde{\Im}^{(ij)}(\Upsilon), \tilde{\Im}^{(ik)}(\Upsilon)) \quad (i = 1(1)m,\ j = 1(1)n) \tag{7}$$

Step 4: Obtain the power weights $\theta^{(ij)}$ using the following formula.

$$\theta^{(ij)} = \frac{w_j(1+\psi(\tilde{\Im}^{(ij)}(\Upsilon)))}{\sum_{q=1}^{n} w_q(1+\psi(\tilde{\Im}^{(iq)}(\Upsilon)))} \quad (i = 1(1)m,\ j = 1(1)n) \tag{8}$$

where w_j denotes the weight of C_j satisfying $w_j \geq 0\ \forall\, j$ and $\sum_{j=1}^{n} w_j = 1$.

Step 5: Aggregate the PDHPFEs using the PDHPFPWHM operator. Suppose the aggregated PDHPFEs are $\tilde{\Im}^{(i)}(\Upsilon)\ (i = 1(1)m)$ where

$$\tilde{\Im}^{(i)}(\Upsilon) = PDHPFPWHM(\tilde{\Im}^{(i1)}(\Upsilon), \tilde{\Im}^{(i2)}(\Upsilon), \ldots, \tilde{\Im}^{(in)}(\Upsilon)) \quad (i = 1(1)m) \tag{9}$$

Step 6: Calculate the scores (and/or accuracy values) of $\tilde{\Im}^{(i)}(\Upsilon)\ (i = 1(1)m)$.

Step 7: Generate the preference the options $A_i\ (i = 1(1)m)$ using the ranking rule for $\tilde{\Im}^{(i)}(\Upsilon)\ (i = 1(1)m)$ and select the optimal option.

However Ji et al.'s (2021) method has the limitation that it fails to identify the priorities of the alternatives in some situations. To demonstrate this, we consider the following three counterexamples.

Example 3.1: Suppose three alternatives A_1, A_2, and A_3 need to be assessed based on three attributes, namely C_1, C_2, and C_3. Suppose the criteria weights are 0.35, 0.25 and 0.40. The initial evaluation result is in Table -1in terms of PDHFEs.

Then the steps for obtaining ranking order by Ji et al.'s (2021) method are as follows:

Step 1: Due to the absence of non-profit type criteria, normalization is not required. Thus $\tilde{M} = \left[\tilde{\Im}^{(ij)}(\Upsilon)\right]_{3\times 3} = \left[\Im^{(ij)}(\Upsilon)\right]_{3\times 3} = M$.

Step 2: We use $Supp^{(jk)}$ to denote the support between the PDHPFEs $\tilde{\Im}^{(ij)}(\Upsilon)$ and $\tilde{\Im}^{(ik)}(\Upsilon)\ (j,k = 1,2,3;\ j \neq k)$. Then based on Eq. (6), we obtain,

$$Supp^{(12)} = Supp^{(21)} = (0.96375, 0.98085, 0.96148),$$
$$Supp^{(13)} = Supp^{(31)} = (0.97974, 0.96882, 0.95978),$$
$$Supp^{(23)} = Supp^{(32)} = (0.96663, 0.95906, 0.98748).$$

Step 3: The values $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))\ (i, j = 1, 2, 3)$ are calculated based on Eq. (7) and these are presented in the following matrix.

$$\psi = \begin{bmatrix} 1.9435 & 1.9303 & 1.9463 \\ 1.9496 & 1.9399 & 1.9278 \\ 1.9212 & 1.9489 & 1.9472 \end{bmatrix}$$

Table 1: Initial Assessment matrix for Example 3.1.

	C1	**C2**	**C3**
A1	<{0.3(0.25), 0.4(0.25), 0.5(0.25), 0.6(0.25)}, {0.1(0.25), 0.2(0.25), 0.25 (0.25), 0.28 (0.25)}>	<{0.7(0.25), 0.6(0.25), 0.4(0.25), 0.2(0.25)}, {0.1(0.25), 0.2(0.25), 0.35 (0.25), 0.41 (0.25)}>	<{0.51(0.25), 0.27(0.25), 0.4(0.25), 0.5 (0.25)}, {0 (1)}>
A2	<{0.1(0.25), 0.3(0.25), 0.4(0.25), 0.5 (0.25)}, {0.1(0.25), 0.2(0.25), 0.3 (0.25), 0.35 (0.25)}>	<{0.3(0.25), 0.4(0.25), 0.5(0.25), 0.6(0.25)}, {0(1)}>	<{0.7(0.25), 0.6(0.25), 0.4(0.25), 0.2(0.25)}, {0.1(0.25), 0.2(0.25), 0.3 (0.25), 0.4 (0.25)}>
A3	<{0.7(0.25), 0.6(0.25), 0.4(0.25), 0.2(0.25)}, {0(1)}>	<{0.1(0.25), 0.3(0.25), 0.4(0.25), 0.5 (0.25)}, {0.1(0.25), 0.2(0.25), 0.25 (0.25), 0.4 (0.25)}>	<{0.3(0.25), 0.4(0.25), 0.5(0.25), 0.6(0.25)}, {0.1(0.25), 0.2(0.25), 0.3 (0.25), 0.37 (0.25)}>

Step 4: The power weights $\theta^{(ij)}\ (i = 1(1)3,\ j = 1(1)3)$ are calculated using Eq. (8) and these are presented in the following matrix.

$$\theta = \begin{bmatrix} 0.3502 & 0.2491 & 0.4006 \\ 0.3513 & 0.2501 & 0.3985 \\ 0.3479 & 0.2508 & 0.4012 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the PDHPFPWHM operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ =<{0.5243 (0.0156), 0.4282 (0.0156), 0.4389 (0.0156), 0.4955 (0.0156)},

{0 (0.0625)}> ,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ =<{0.5057 (0.0156), 0.4741 (0.0156), 0.4283 (0.0156), 0.4524 (0.0156)},

{0 (0.0625)}> ,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ =<{0.4901 (0.0156), 0.4694 (0.0156), 0.4442 (0.0156), 0.4828 (0.0156)},

{0 (0.0625)}> .

Step 6: The scores and accuracy values of the aggregated PDHPFEs are:
$S_c(\tilde{\mathfrak{I}}^{(1)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(1)}(\Upsilon))$ = 0.029, $S_c(\tilde{\mathfrak{I}}^{(2)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(2)}(\Upsilon))$ = 0.029,

$S_c(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = 0.029.$

Step 7: The ranking order obtained by Ji et al. (2021) method is $A_1 = A_2 = A_3$. Hence Ji et al.'s (2021) method fails to generate a proper ranking order.

Example 3.2: Suppose three alternatives A_1, A_2, and A_3 need to be assessed based on three attributes, namely C_1, C_2, and C_3. Suppose the criteria weights are 0.25, 0.35 and 0.40.

Table 2: Initial Assessment Matrix for Example 3.2.

	C1	C2	C3
A1	<{0.4(0.25), 0.5(0.25), 0.3(0.25), 0.2(0.25)}, {0.35(0.25), 0.36(0.25), 0.55(0.25), 0.56(0.25)}>	<{0.5(0.25), 0.7(0.25), 0.1(0.25), 0.4(0.25)}, {0.4(0.25), 0.41(0.25), 0.5(0.25), 0.52(0.25)}>	<{0.33(0.25), 0.3(0.25), 0.48(0.25), 0.75 (0.25)}, {0 (1)}>
A2	<{0.6(0.25), 0.3(0.25), 0.5(0.25), 0.7(0.25)}, {0.5(0.25), 0.52(0.25), 0.6(0.25), 0.61(0.25)}>	<{0.4(0.25), 0.5(0.25), 0.3(0.25), 0.2(0.25)}, {0(1)}>	<{0.5(0.25), 0.7(0.25), 0.1(0.25), 0.4(0.25)}, {0.45(0.25), 0.46(0.25), 0.55(0.25), 0.6(0.25)}>
A3	<{0.5(0.25), 0.7(0.25), 0.1(0.25), 0.4(0.25)}, {0(1)}>	<{0.6(0.25), 0.3(0.25), 0.5(0.25), 0.7(0.25)}, {0.5(0.25), 0.52(0.25), 0.55(0.25), 0.56(0.25)}>	<{0.38(0.25), 0.5(0.25), 0.3(0.25), 0.2(0.25)}, {0.55(0.25), 0.56(0.25), 0.6(0.25), 0.1(0.25)}>

Then the steps for obtaining ranking order Ji et al.'s (2021) method are as follows:

Step 1: Due to the absence of non-profit type criteria, normalization is not required. So $\tilde{M} = \left[\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)\right]_{3\times 3} = \left[\mathfrak{I}^{(ij)}(\Upsilon)\right]_{3\times 3} = M$.

Step 2: For the sake of simplicity, we use the symbol $Supp^{(jk)}$ to denote the support between the PDHPFEs $\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)$ and $\tilde{\mathfrak{I}}^{(ik)}(\Upsilon)$ $(j,k=1,2,3;\ j\neq k)$. Then using Eq. (6) we obtain,

$$Supp^{(12)} = Supp^{(21)} = (0.97807, 0.93055, 0.93073),$$

$$Supp^{(13)} = Supp^{(31)} = (0.94556, 0.96075, 0.95213),$$

$$Supp^{(23)} = Supp^{(32)} = (0.93711, 0.94979, 0.95492).$$

Step 3: We compute $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))\ (i, j=1,2,3)$ using Eq. (7) and present them as:

$$\psi = \begin{bmatrix} 1.9236 & 1.9151 & 1.8826 \\ 1.8913 & 1.8803 & 1.9105 \\ 1.8828 & 1.8856 & 1.9071 \end{bmatrix}$$

Step 4: We determine the power weights $\theta^{(ij)}\ (i, j=1,2,3)$ using Eq. (8) and present them as:

$$\theta = \begin{bmatrix} 0.2516 & 0.3513 & 0.3970 \\ 0.2496 & 0.3482 & 0.4021 \\ 0.2491 & 0.3490 & 0.4018 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the PDHPFPWHM operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ =<{0.4166 (0.0156), 0.5411 (0.0156), 0.3507 (0.0156), 0.5740 (0.0156)},

{0 (0.0625)}> ,
$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ =<{0.5001 (0.0156), 0.5709 (0.0156), 0.3209 (0.0156), 0.4723 (0.0156)},
{0 (0.0625)}> ,
$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ =<{0.5018 (0.0156), 0.5206 (0.0156), 0.3624 (0.0156), 0.5053 (0.0156)},
{0 (0.0625)}> .

Step 6: The scores and accuracy values of the aggregated PDHPFEs are:
$S_c(\tilde{\mathfrak{I}}^{(1)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(1)}(\Upsilon)) = 0.029$, $S_c(\tilde{\mathfrak{I}}^{(2)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(2)}(\Upsilon)) = 0.029$,
$S_c(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = 0.029$.

Step 7: The ranking order obtained by Ji et al. (2021) method is $A_1 = A_2 = A_3$. Hence, Ji et al' (2021) method fails to generate a proper ranking order.

Example 3.3: Suppose three alternatives A_1, A_2, and A_3 need to be assessed based on three attributes, namely C_1, C_2, and C_3. Suppose the criteria weights are 0.25, 0.35 and 0.40. The initial assessment matrix is presented in Table 3.

Table 3: Initial Assessment Matrix for Example 3.3.

	C1	**C2**	**C3**
A1	<{0.2(0.25), 0.3(0.25), 0.5(0.25), 0.4(0.25)}, {0.35(0.25), 0.36(0.25), 0.55(0.25), 0.56(0.25)}>	<{0.1(0.25), 0.4(0.25), 0.5(0.25), 0.7(0.25)}, {0.4(0.25), 0.41(0.25), 0.5(0.25), 0.52(0.25)}>	<{0.25(0.25), 0.5(0.25), 0.6(0.25), 0.2 (0.25)}, {0 (1)}>
A2	<{0.3(0.25), 0.5(0.25), 0.6(0.25), 0.2 (0.25)}, {0.5(0.25), 0.52(0.25), 0.6(0.25), 0.61(0.25)}>	<{0.2(0.25), 0.3(0.25), 0.5(0.25), 0.4(0.25)}, {0(1)}>	<{0.1(0.25), 0.4(0.25), 0.5(0.25), 0.7(0.25)}, {0.45(0.25), 0.46(0.25), 0.55(0.25), 0.6(0.25)}>
A3	<{0.1(0.25), 0.4(0.25), 0.5(0.25), 0.7(0.25)}, {0(1)}>	<{0.3(0.25), 0.5(0.25), 0.6(0.25), 0.35 (0.25)}, {0.5(0.25), 0.52(0.25), 0.55(0.25), 0.56(0.25)}>	<{0.2(0.25), 0.3(0.25), 0.5(0.25), 0.4(0.25)}, {0.55(0.25), 0.56(0.25), 0.6(0.25), 0.1(0.25)}>

Then the steps for obtaining ranking order by Ji et al.'s (2021) method are as follows:

Step 1: Due to the absence of non-profit type criteria, normalization is not required. So $\tilde{M} = \left[\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)\right]_{3\times 3} = \left[\mathfrak{I}^{(ij)}(\Upsilon)\right]_{3\times 3} = M$.

Step 2: Here $Supp^{(jk)}$ is used to denote the support between the PDHPFEs $\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)$ and $\tilde{\mathfrak{I}}^{(ik)}(\Upsilon)$ $(j,k=1,2,3;\ j\neq k)$. Then using Eq. (6) we obtain,

$Supp^{(12)} = Supp^{(21)} = (0.98119, 0.94711, 0.94425)$,
$Supp^{(13)} = Supp^{(31)} = (0.95997, 0.97169, 0.95574)$,
$Supp^{(23)} = Supp^{(32)} = (0.95153, 0.95292, 0.97455)$.

Step 3: We compute $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))\ (i, j = 1, 2, 3)$ using Eq. (7) and present them as:

$$\psi = \begin{bmatrix} 1.9412 & 1.9327 & 1.9115 \\ 1.9188 & 1.9000 & 1.9246 \\ 1.8999 & 1.9188 & 1.9303 \end{bmatrix}$$

Step 4: We determine the power weights $\theta^{(ij)}\ (i, j = 1, 2, 3)$ using Eq. (8) and present them as:

$$\theta = \begin{bmatrix} 0.2512 & 0.3507 & 0.3979 \\ 0.2503 & 0.3482 & 0.4013 \\ 0.2483 & 0.3500 & 0.4015 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the PDHPFPWHM operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ =<{ 0.1967 (0.0156), 0.4251 (0.0156), 0.5439 (0.0156), 0.5063 (0.0156)},

{0 (0.0625)}> ,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ =<{ 0.2024 (0.0156), 0.4005 (0.0156), 0.5283 (0.0156), 0.5376 (0.0156)},

{0 (0.0625)}> ,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ =<{0.2248 (0.0156), 0.4077 (0.0156), 0.5389 (0.0156), 0.4964 (0.0156)},

{0 (0.0625)}> .

Step 6: The scores and accuracy values of the aggregated PDHPFEs are:
$S_c(\tilde{\mathfrak{I}}^{(1)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(1)}(\Upsilon)) = 0.026$, $S_c(\tilde{\mathfrak{I}}^{(2)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(2)}(\Upsilon)) = 0.026$,
$S_c(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = A_c(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = 0.026$.

Step 7: The ranking order obtained by Ji et al. (2021) method is $A_1 = A_2 = A_3$. Hence, Ji et al.'s (2021) method fails to generate a proper ranking order.

The outcomes of examples 3.1, 3.2, and 3.3 show that in some situations, Ji et al.'s (2021) method fails to identify the proper priority order of alternatives due to the drawback of the PDHPWHM operator. So, there is a need for an improved MADM procedure to tackle such scenarios.

4 Improved operations between PDHPFEs and associated AO

To understand the drawbacks of the existing operations between PDHPFEs, let us recall the basic operations that are given below.

Definition 4.1: Let $\mathfrak{I}^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_j^{(b)}(\Upsilon^{(b)})\} > (j = 1, 2)$ be two

PDHPFEs. Then

$$\text{(i) } \Im^{(1)}(\Upsilon)\oplus\Im^{(2)}(\Upsilon)=\left\langle \begin{array}{c} \bigcup_a\left\{\sqrt{(\Delta_1^{(a)})^2+(\Delta_2^{(a)})^2-(\Delta_1^{(a)})^2+(\Delta_2^{(a)})^2}\,(\Upsilon_1^{(a)}\Upsilon_2^{(a)})\right\}, \\ \bigcup_b\left\{\nabla_1^{(b)}\nabla_2^{(b)}\right\}(\Upsilon_1^{(b)}\Upsilon_2^{(b)}) \end{array}\right\rangle$$

$$\text{(ii) } \lambda\Im^{(1)}(\Upsilon)=\left\langle \bigcup_a\left\{\sqrt{1-\left(1-(\Delta_1^{(a)})^2\right)^{\lambda}}\,(\Upsilon_1^{(a)})\right\},\bigcup_b\left\{\nabla_1^{(b)}(\Upsilon_1^{(b)})\right\}\right\rangle$$

The following example shows that the operations defined by Ji et al. (2021) are not reasonable.

Example 4.1: Take $\Im^{(1)}(\Upsilon)=<\{0.4(0.5),0.7(0.5)\},\{0(1)\}>$ and $\Im^{(2)}(\Upsilon)=<\{0.5(0.3)\},0.6(0.7)\},\{0.2(0.6),\quad 0.5(0.4)\}>$. Then we have, $\Im^{(1)}(\Upsilon)\oplus\Im^{(2)}(\Upsilon)=<\{0.6082(0.15),0.68(0.35),0.7858(0.15),0.8207(0.35)\},\ \{0(1)\}>$ and $\lambda\Im^{(1)}(\Upsilon)=<\{0.1851(0.5),0.3549(0.5)\},\{0(1)\}>$. So, non-zero NMDs have no influence on the final results. Thus the operations defined by Ji et al. (2021) are not reasonable.

Next, To show that the $PDHPFWHM$ operator proposed by Ji et al. (2021) is not reasonable, the following example is given.

Example 4.2: Let us consider the PDHPFEs $\Im^{(1)}(\Upsilon)$ =<{0.2 (0.25), 0.3 (0.25), 0.5 (0.25), 0.4 (0.25)}, {0.35 (0.25), 0.36 (0.25), 0.55 (0.25), 0.56 (0.25)}>, $\Im^{(2)}(\Upsilon)$ =<{0.1 (0.25), 0.4 (0.25), 0.5 (0.25), 0.7 (0.25)}, {0.4 (0.25), 0.41 (0.25), 0.5 (0.25), 0.52 (0.25)}>, and $\Im^{(3)}(\Upsilon)$ =<{0.3 (0.25), 0.5 (0.25), 0.6 (0.25), 0.2 (0.25)}, {0 (1)}>. If their weights are respective 0.25, 0.35 and 0.40, then, we have $PDHPFWHM(\Im^{(1)}(\Upsilon),\Im^{(2)}(\Upsilon),\Im^{(3)}(\Upsilon))$ = <{0.2173 (0.01), 0.4116 (0.01), 0.5376 (0.01), 0.5075 (0.01)}, {0 (0.06)}> .Thus, So, non-zero NMDs have no influence on the final results. Thus the PDFPFWHM operator defined by Ji et al. (2021) is not reasonable.

Hence, our first target is to define some improved operations to overcome the above-mentioned issues. To do so, we first introduce the concept of adjustment of PDHPFEs. Let us take two PDHPFEs $\Im^{(1)}(\Upsilon)$ and $\Im^{(2)}(\Upsilon)$ Then their probabilities can be adjusted in the following manner.

Example 4.3: Suppose $\Im^{(1)}(\Upsilon)=<\{0.4(0.8),0.3(0.2)\},\{0.7(0.5),0.8(0.5)\}>$ and $\Im^{(2)}(\Upsilon)=<\{0.5(1)\},\{0.6(0.4),\quad 0.8(0.6)\}>$. Then their corresponding adjusted PDHPFEs are: $\Im^{(1)}(\Upsilon)=<\{0.4(0.4),0.4(0.1),0.4(0.3),0.3(0.2)\},$ $\{0.7(0.4),0.7(0.1),0.8(0.3),0.8(0.2)\}>$ and $\Im^{(2)}(\Upsilon)=<\{0.5(0.4),0.5(0.1),0.5(0.3),0.5(0.2)\},\{0.6(0.4),0.8(0.1),$ $0.8(0.3),0.8(0.2)\}>$.

We use the symbol $\aleph$ to represent the set of all adjusted PDHPFEs. Now, based on the adjusted PDHPFEs, we propose improved operations and develop the corresponding

AOs.

Definition 4.2: Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_j^{(b)}(\Upsilon^{(b)})\}> (j=1,2)$ be two adjusted PDHPFEs. Then, for any $\lambda, \lambda_1, \lambda_2 > 0$, the improved operations between the PDHPFEs are defined as:

$$1.\ \Im^{(1)}(\Upsilon) \oplus \Im^{(2)}(\Upsilon) = \left\langle \bigcup_a \left\{ \sqrt{\frac{\prod_{j=1}^{2}(1+(\Delta_j^{(a)})^2) - \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)}{\prod_{j=1}^{2}(1+(\Delta_j^{(a)})^2) + \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)}} (\Upsilon^{(a)}) \right\}, \right.$$
$$\left. \bigcup_b \left\{ \sqrt{\frac{2\left(\prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2) - \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2 - (\nabla_j^{(b)})^2)\right)}{\prod_{j=1}^{2}(1+(\Delta_j^{(a)})^2) + \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)}} (\Upsilon^{(b)}) \right\} \right\rangle$$

$$2.\ \lambda\Im^{(1)}(\Upsilon) = \left\langle \bigcup_a \left\{ \sqrt{\frac{(1+(\Delta_1^{(a)})^2)^\lambda - (1-(\Delta_1^{(a)})^2)^\lambda}{(1+(\Delta_1^{(a)})^2)^\lambda + (1-(\Delta_1^{(a)})^2)^\lambda}} (\Upsilon^{(a)}) \right\}, \right.$$
$$\left. \bigcup_b \left\{ \sqrt{\frac{2((1-(\Delta_1^{(a)})^2)^\lambda - (1-(\Delta_1^{(a)})^2 - (\nabla_1^{(b)})^2)^\lambda)}{(1+(\Delta_1^{(a)})^2)^\lambda + (1-(\Delta_1^{(a)})^2)^\lambda}} (\Upsilon^{(b)}) \right\} \right\rangle$$

Theorem 4.1: Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\Delta_j^{(b)}(\Upsilon^{(b)})\}> (j=1,2)$ be two adjusted PDHPFEs. Then for $\lambda, \lambda_1, \lambda_2 > 0$, we have,

(i) $\Im^{(1)}(\Upsilon) \oplus \Im^{(2)}(\Upsilon) = \Im^{(2)}(\Upsilon) \oplus \Im^{(1)}(\Upsilon)$

(ii) $\lambda(\Im^{(1)}(\Upsilon) \oplus \Im^{(2)}(\Upsilon)) = \lambda\Im^{(1)}(\Upsilon) \oplus \lambda\Im^{(2)}(\Upsilon)$

(iii) $(\lambda_1 + \lambda_2)\Im^{(1)}(\Upsilon) = \lambda_1\Im^{(1)}(\Upsilon) \oplus \lambda_2\Im^{(1)}(\Upsilon)$

Proof: Added in the Supplementary material.

Next, based on the improved operational laws for PDHPFEs, we propose probabilistic dual hesitant Pythagorean fuzzy improved power weighted averaging (PDHPFIPWA) operator and study it's properties.

Definition 4.3: Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\Delta_j^{(b)}(\Upsilon^{(b)})\}> \in \aleph\ (j = 1(1)n)$. Then the improved operations based weighted AO on PDHPFEs is denoted by PDHPFIPWA and is defined by:

$$PDHPFIPWA(\Im^{(1)}(\Upsilon), \Im^{(2)}(\Upsilon), \ldots, \Im^{(n)}(\Upsilon)) = \bigoplus_{j=1}^{n} \left(\theta^{(j)}\Im^{(j)}(\Upsilon)\right)$$

where $\theta^{(j)} = \dfrac{w_j(1+\psi(\tilde{\Im}^{(j)}(\Upsilon)))}{\sum_{j=1}^{n} w_j(1+\psi(\tilde{\Im}^{(j)}(\Upsilon)))}$ and w_j denotes the weight of $\Im^{(j)}(\Upsilon)$ with $w_j \geq 0$ and $\sum_j w_j = 1$.

Theorem 4.2: Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\Delta_j^{(b)}(\Upsilon^{(b)})\}> \in \aleph$ $(j = 1(1)n)$. Then $PDHPFIPWA(\Im^{(1)}(\Upsilon), \Im^{(2)}(\Upsilon), \ldots, \Im^{(n)}(\Upsilon))$ is again a PDHPFE and

$$PDHPFIPWA(\Im^{(1)}(\Upsilon), \Im^{(2)}(\Upsilon), \ldots, \Im^{(n)}(\Upsilon))$$
$$= \left\langle \bigcup_a \left\{ \sqrt{\frac{\prod_{j=1}^{2}(1+(\Delta_j^{(a)})^2)^{\theta^{(j)}} - \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)^{\theta^{(j)}}}{\prod_{j=1}^{2}(1+(\Delta_j^{(a)})^2)^{\theta^{(j)}} + \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)^{\theta^{(j)}}}}(\Upsilon^{(a)}) \right\}, \right.$$
$$\left. \bigcup_b \left\{ \sqrt{\frac{2\left(\prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)^{\theta^{(j)}} - \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2-(\nabla_j^{(b)})^2)^{\theta^{(j)}}\right)}{\prod_{j=1}^{2}(1+(\Delta_j^{(a)})^2)^{\theta^{(j)}} + \prod_{j=1}^{2}(1-(\Delta_j^{(a)})^2)^{\theta^{(j)}}}}(\Upsilon^{(b)}) \right\} \right\rangle$$

Proof: Follows from Theorem 4.1.

In the following, some vital properties of the $PDHPFIPWA$ operator are presented.

Theorem 4.3: (Idempotency) Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_j^{(b)}(\Upsilon^{(b)})\}> \in \aleph$ $(j = 1(1)n)$. If $\Im^{(j)}(\Upsilon) = \Im^{(L)}(\Upsilon)$ $\forall j$ (L being a fixed natural number), then $PDHPFIPWA(\Im^{(1)}(\Upsilon), \Im^{(2)}(\Upsilon), \ldots, \Im^{(n)}(\Upsilon)) = \Im^{(L)}(\Upsilon)$.

Theorem 4.4: (Monotonicity) Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_j^{(b)}(\Upsilon^{(b)})\}> \in \aleph$ $(j = 1(1)n)$ and $\Im'^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j'^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\Delta_j'^{(b)}(\Upsilon^{(b)})\}> \in \aleph$ $(j = 1(1)n)$ such that $\forall j,\ \Delta_j^{(a)} \leq \Delta_j'^{(a)}$ and $\nabla_j^{(b)} \geq \nabla_j'^{(b)}$. Then, $PDHPFIPWA(\Im^{(1)}(\Upsilon), \Im^{(2)}(\Upsilon), \ldots, \Im^{(n)}(\Upsilon)) \prec PDHPFIPWA$ $(\Im'^{(1)}(\Upsilon), \Im'^{(2)}(\Upsilon), \ldots, \Im'^{(n)}(\Upsilon))$.

Theorem 4.5: (Boundedness**)** Let $\Im^{(j)}(\Upsilon) = <\bigcup_a \{\Delta_j^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_j^{(b)}(\Upsilon^{(b)})\}> \in \aleph$ $(j = 1(1)n)$. If

$\Im^{(j-)}(\Upsilon) =< \{\min_a \Delta_j^{(a)}(\Upsilon^{(a)})\}, \{\max_b \nabla_j^{(b)}(\Upsilon^{(b)})\} >$ and $\Im^{(j+)}(\Upsilon) =< \{\max_a \Delta_j^{(a)}(\Upsilon^{(a)})\}, \{\min_b \nabla_j^{(b)}(\Upsilon^{(b)})\} >$, then $\Im^{(j-)}(\Upsilon) \prec PDHPFIPWA(\Im^{(1)}(\Upsilon), \Im^{(2)}(\Upsilon), ..., \Im^{(n)}(\Upsilon)) \prec \Im_{\wp}^{(j+)}(\Upsilon)$.

5 Improved decision-making methodology with PDHPFEs

5.1 Proposed MADM approach

Suppose there are m number of options A_i $(i = 1(1)m)$ and n number of criteria C_j $(j = 1(1)n)$ associated with a MADM problem where each alternative is assessed by an expert under probabilistic dual hesitant Pythagorean fuzzy (PDHPF) setting. Assume that the initial result evaluated by the expert is represented by:

$$M = \left[\Im^{(ij)}(\Upsilon) \right]_{m\times n} = \left[< \bigcup_a \{\Delta_{ij}^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_{ij}^{(b)}(\Upsilon^{(b)})\} > \right]_{m\times n}$$

$(j = 1, 2, ..., n;\ i = 1, 2, ..., m)$.

Then the proposed methodology encompasses the succeeding steps:

Step 1: Obtain the normalized the PDHPF matrix $\tilde{M} = \left[\tilde{\Im}^{(ij)}(\Upsilon) \right]_{m\times n}$ using the following equation.

$$\tilde{\Im}^{(ij)}(\Upsilon) = \begin{cases} < \bigcup_a \{\Delta_{ij}^{(a)}(\Upsilon^{(a)})\}, \bigcup_b \{\nabla_{ij}^{(b)}(\Upsilon^{(b)})\} >, \text{ if } C_j \text{ is of profit type} \\ < \bigcup_b \{\nabla_{ij}^{(b)}(\Upsilon^{(b)})\}, \bigcup_a \{\Delta_{ij}^{(a)}(\Upsilon^{(a)})\}, >, \text{ if } C_j \text{ is of non-profit type} \end{cases} \tag{10}$$

Then construct the normalized adjusted PDHPF matrix $\hat{M} = \left[\tilde{\Im}^{(ij)}(\hat{\Upsilon}) \right]_{m\times n}$.

Step 2: Compute the supports $Supp(\tilde{\Im}^{(ij)}(\hat{\Upsilon}), \tilde{\Im}^{(ik)}(\hat{\Upsilon}))$ using:

$$Supp(\tilde{\Im}^{(ij)}(\hat{\Upsilon}), \tilde{\Im}^{(ik)}(\hat{\Upsilon})) = 1 - D(\tilde{\Im}^{(ij)}(\hat{\Upsilon}), \tilde{\Im}^{(ik)}(\hat{\Upsilon}))\ \ (j, k = 1(1)n;\ j \neq k) \tag{11}$$

Step 3: Determine the values $\psi(\tilde{\Im}^{(ij)}(\hat{\Upsilon}))$ utilizing the following formula.

$$\psi(\tilde{\Im}^{(ij)}(\hat{\Upsilon})) = \sum_{j=1, j\neq k}^{n} Supp(\tilde{\Im}^{(ij)}(\hat{\Upsilon}), \tilde{\Im}^{(ik)}(\hat{\Upsilon}))\quad (j = 1, 2, ..., n;\ i = 1, 2, ..., m) \tag{12}$$

Step 4: Determine $\theta^{(ij)}$ using the following formula.

$$\theta^{(ij)} = \frac{w_j(1 + \psi(\tilde{\Im}^{(ij)}(\hat{\Upsilon})))}{\sum_{q=1}^{n} w_q(1 + \psi(\tilde{\Im}^{(iq)}(\hat{\Upsilon})))}\quad (j = 1, 2, ..., n;\ i = 1, 2, ..., m) \tag{13}$$

where w_j denotes the weight of C_j satisfying $w_j \geq 0 \ \forall j$ and $\sum_{j=1}^{n} w_j = 1$.

Step 5: Aggregate the PDHPFEs using the $PDHPFIPWA$ operator. Suppose the aggregated PDHPFEs are $\tilde{\Im}^{(i)}(\Upsilon)\ (i=1(1)m)$ where.

$$\tilde{\Im}^{(i)}(\Upsilon)$$
$$= PDHPFIPWA(\tilde{\Im}^{(i1)}(\hat{\Upsilon}), \tilde{\Im}^{(i2)}(\hat{\Upsilon}), \ldots, \tilde{\Im}^{(in)}(\hat{\Upsilon}))$$
$$= \left\langle \bigcup_{a} \left\{ \sqrt{\frac{\prod_{j=1}^{n}(1+(\Delta_j^{(a)})^2)^{\theta^{(ij)}} - \prod_{j=1}^{n}(1-(\Delta_j^{(a)})^2)^{\theta^{(ij)}}}{\prod_{j=1}^{n}(1+(\Delta_j^{(a)})^2)^{\theta^{(ij)}} + \prod_{j=1}^{n}(1-(\Delta_j^{(a)})^2)^{\theta^{(ij)}}}} (\hat{\Upsilon}^{(a)}) \right\}, \right. \tag{14}$$
$$\left. \bigcup_{b} \left\{ \sqrt{\frac{2\left(\prod_{j=1}^{n}(1-(\Delta_j^{(a)})^2)^{\theta^{(ij)}} - \prod_{j=1}^{n}(1-(\Delta_j^{(a)})^2-(\nabla_j^{(b)})^2)^{\theta^{(ij)}}\right)}{\prod_{j=1}^{n}(1+(\Delta_j^{(a)})^2)^{\theta^{(ij)}} + \prod_{j=1}^{n}(1-(\Delta_j^{(a)})^2)^{\theta^{(ij)}}}} (\hat{\Upsilon}^{(b)}) \right\} \right\rangle \ (i=1(1)m)$$

Step 6: Calculate the scores (and/or accuracy values) of $\tilde{\Im}^{(i)}(\hat{\Upsilon})\ (i=1(1)m)$.

Step 7: Generate the preference the options $A_i\ (i=1(1)m)$ using the ranking rule for $\tilde{\Im}^{(i)}(\hat{\Upsilon})\ (i=1(1)m)$ and select the optimal option.

5.2 Applications of the proposed MADM approach:

We use it to address the instances provided in section 3 to show that the suggested technique may overcome the restrictions of Ji et al.'s (2021) method.

Example 3.1: The steps for obtaining ranking order by the proposed method are as follows:

Step 1: Due to absence of non-profit type criteria, normalization is not required. So $\tilde{M} = \left[\tilde{\Im}^{(ij)}(\Upsilon)\right]_{3\times3} = \left[\Im^{(ij)}(\Upsilon)\right]_{3\times3} = M$. Since the elements of $\tilde{M}(=M)$ are in adjusted form, so $\tilde{M} = M = \hat{M}$.

Step 2: The supports between the PDHPFEs $\tilde{\Im}^{(ij)}(\Upsilon)$ and $\tilde{\Im}^{(ik)}(\Upsilon)$ $(j,k=1,2,3;\ j\neq k)$ are calculated using Eq. (11) and these are given by:

$$Supp^{(12)} = Supp^{(21)} = (0.96375, 0.98085, 0.96148),$$
$$Supp^{(13)} = Supp^{(31)} = (0.97974, 0.96882, 0.95978),$$
$$Supp^{(23)} = Supp^{(32)} = (0.96663, 0.95906, 0.98748).$$

Step 3: We present the values $\psi(\tilde{\Im}^{(ij)}(\Upsilon))$ $(i, j = 1,2,3)$ by

$$\psi = \begin{bmatrix} 1.9435 & 1.9303 & 1.9463 \\ 1.9496 & 1.9399 & 1.9278 \\ 1.9212 & 1.9489 & 1.9472 \end{bmatrix}$$

Step 4: The values $\theta^{(ij)}$ $(i, j = 1,2,3)$ are presented by

$$\theta = \begin{bmatrix} 0.3502 & 0.2491 & 0.4006 \\ 0.3513 & 0.2501 & 0.3985 \\ 0.3479 & 0.2508 & 0.4012 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the $PDHPFIPWA$ operator are:

$\tilde{\Im}^{(1)}(\Upsilon)$ =<{0.5151 (0.25), 0.4216 (0.25), 0.4381 (0.25), 0.4894 (0.25)},

{ 0.0802 (0.25), 0.1637 (0.25), 0.2333 (0.25), 0.2608 (0.25)}> ,

$\tilde{\Im}^{(2)}(\Upsilon)$ =<{ 0.4850 (0.25), 0.4676 (0.25), 0.4276 (0.25), 0.4442 (0.25)},

{ 0.0933 (0.25), 0.1782 (0.25), 0.2580 (0.25), 0.3187 (0.25)}> ,

$\tilde{\Im}^{(3)}(\Upsilon)$ =<{ 0.4706 (0.25), 0.4637 (0.25), 0.4433 (0.25), 0.4746 (0.25)},

{0.0736 (0.25), 0.1544 (0.25), 0.2341 (0.25), 0.3352 (0.25)}> .

Step 6: The scores of the aggregated PDHPFEs are:

$Sc(\tilde{\Im}^{(1)}(\Upsilon))$ =0.2815, $Sc(\tilde{\Im}^{(2)}(\Upsilon))$ =0.2440, $Sc(\tilde{\Im}^{(3)}(\Upsilon))$ =0.2637.

Step 7: The ranking order is $A_1 \succ A_3 \succ A_2$. Hence, the best alternative is A_1 .

Therefore, the disadvantage of Ji et al.'s (2021) technique may be overcome by the proposed way.

Example 3.2: Then the steps for obtaining ranking order by the proposed method are as follows:

Step 1: Due to absence of non-profit type criteria, normalization is not required. So $\tilde{M} = \left[\tilde{\Im}^{(ij)}(\Upsilon)\right]_{3\times3} = \left[\Im^{(ij)}(\Upsilon)\right]_{3\times3} = M$. Since the elements of $\tilde{M}(=M)$ .are in adjusted form, so $\tilde{M} = M = \hat{M}$.

Step 2: The supports between the PDHPFEs $\tilde{\Im}^{(ij)}(\Upsilon)$ and $\tilde{\Im}^{(ik)}(\Upsilon)$ $(j,k = 1,2,3;\ j \neq k)$ are calculated using Eq. (11) and these are given by:

$Supp^{(12)} = Supp^{(21)} = (0.97807, 0.93055, 0.93073)$,

$Supp^{(13)} = Supp^{(31)} = (0.94556, 0.96075, 0.95213)$,

$Supp^{(23)} = Supp^{(32)} = (0.93711, 0.94979, 0.95492)$.

Step 3: We present the values $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))$ $(i, j = 1,2,3)$ by.

$$\psi = \begin{bmatrix} 1.9236 & 1.9151 & 1.8826 \\ 1.8913 & 1.8803 & 1.9105 \\ 1.8828 & 1.8856 & 1.9071 \end{bmatrix}$$

Step 4: The values $\theta^{(ij)}$ $(i, j = 1,2,3)$ are presented by

$$\theta = \begin{bmatrix} 0.2516 & 0.3513 & 0.3970 \\ 0.2496 & 0.3482 & 0.4021 \\ 0.2491 & 0.3490 & 0.4018 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the *PDHPFIPWA* operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ =<{ 0.4147 (0.25), 0.5298 (0.25), 0.3446 (0.25), 0.5559 (0.25)},

{0.3107 (0.25), 0.3513 (0.25), 0.4032 (0.25), 0.3811 (0.25)}> ,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ =<{0.4977 (0.25), 0.5620 (0.25), 0.3144 (0.25), 0.4577 (0.25)},

{0.4081 (0.25), 0.4232 (0.25), 0.4900 (0.25), 0.5789 (0.25)}> ,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ =<{0.4983 (0.25), 0.5113 (0.25), 0.3566 (0.25), 0.4885 (0.25)},

{0.4701 (0.25), 0.4618 (0.25), 0.5267 (0.25), 0.4668 (0.25)}> .

Step 6: The scores of the aggregated PDHPFEs are:

$Sc(\tilde{\mathfrak{I}}^{(1)}(\Upsilon))$ =0.0997, $Sc(\tilde{\mathfrak{I}}^{(2)}(\Upsilon))$ =-0.0171, $Sc(\tilde{\mathfrak{I}}^{(3)}(\Upsilon))$ =-0.0177.

Step 7: The ranking order is $A_1 \succ A_2 \succ A_3$. Hence, the best alternative is A_1 .

Therefore, the disadvantage of Ji et al.'s (2021) technique may be overcome by the proposed way.

Example 3.3: The steps for obtaining ranking order by the proposed method are as follows:

Step 1: Due to absence of non-profit type criteria, normalization is not required. So $\tilde{M} = \left[\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)\right]_{3\times 3} = \left[\mathfrak{I}^{(ij)}(\Upsilon)\right]_{3\times 3} = M$. Since the elements of $\tilde{M}(=M)$ are in adjusted form, so $\tilde{M} = M = \hat{M}$.

Step 2: The supports between the PDHPFEs $\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)$ and $\tilde{\mathfrak{I}}^{(ik)}(\Upsilon)$ $(j,k = 1,2,3;\ j \neq k)$ are calculated using Eq. (11) and these are given by:

$$Supp^{(12)} = Supp^{(21)} = (0.98119, 0.94711, 0.94425),$$
$$Supp^{(13)} = Supp^{(31)} = (0.95997, 0.97169, 0.95574),$$
$$Supp^{(23)} = Supp^{(32)} = (0.95153, 0.95292, 0.97455).$$

Step 3: We present the values $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))$ $(i, j = 1, 2, 3)$ by $\psi =$

$$\begin{bmatrix} 1.9412 & 1.9327 & 1.9115 \\ 1.9188 & 1.9000 & 1.9246 \\ 1.8999 & 1.9188 & 1.9303 \end{bmatrix}$$

Step 4: The values $\theta^{(ij)}$ $(i, j = 1, 2, 3)$ are presented by.

$$\theta = \begin{bmatrix} 0.2512 & 0.3507 & 0.3979 \\ 0.2503 & 0.3482 & 0.4013 \\ 0.2483 & 0.3500 & 0.4015 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the $PDHPFIPWA$ operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ =<{0.1961 (0.25), 0.4231 (0.25), 0.5429 (0.25), 0.4896 (0.25)},

{0.2968 (0.25), 0.3009 (0.25), 0.4098 (0.25), 0.4915 (0.25)}> ,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ =<{ 0.2013 (0.25), 0.3985 (0.25), 0.5276 (0.25), 0.5236 (0.25)},

{0.3896 (0.25), 0.4145 (0.25), 0.4947 (0.25), 0.5771 (0.25)}> ,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ =<{0.2238 (0.25), 0.4052 (0.25), 0.5379 (0.25), 0.4863 (0.25)},

{0.4697 (0.25), 0.4823 (0.25), 0.5232 (0.25), 0.3356 (0.25)}> .

Step 6: The scores of the aggregated PDHPFEs are:

$Sc(\tilde{\mathfrak{I}}^{(1)}(\Upsilon))$ =0.381, $Sc(\tilde{\mathfrak{I}}^{(2)}(\Upsilon))$ =-0.0562, $Sc(\tilde{\mathfrak{I}}^{(3)}(\Upsilon))$ =-0.0393.

Step 7: The ranking order is $A_1 \succ A_3 \succ A_2$. Hence, the best alternative is A_1 .

Therefore, the disadvantage of Ji et al.'s (2021) technique may be overcome by the proposed way.

To demonstrate the similarity of outcomes derived through Ji et al.'s (2021) method and the proposed method, we consider two more examples furnished below. In these two examples we have assumed that none of the PDHPFEs has a non-membership value equals to "0".

Example 5.1: Suppose an Educational Institute wants to appoint a placement officer against a vacant post. Assume that three candidates $A_i (i = 1, 2, 3)$ get nominated for further evaluation after pre-elimination. In this regard, three criteria, namely 'communication skill', 'experience' and 'overall knowledge' are considered for final assessment. The criteria weights are respectively 0.25, 0.35, and 0.40. The initial assessment matrix is presented in Table 4.

Table 4: Initial Assessment Matrix for Example 5.1.

	C_1	C_2	C_3
A_1	<{0.1(0.3), 0.2(0.4), 0.4(0.1), 0.5(0.2)}, {0.4(0.3), 0.5(0.4), 0.55(0.1), 0. 6(0.2)}>	<{0.3(0.2), 0.5(0.4), 0.52(0.3), 0.53(0.1)}, {0.4(0.2), 0.45(0.4), 0.55(0.3), 0.6(0.1)}>	<{0.2(0.4), 0.3(0.1), 0.4(0.1), 0.45 (0.4)}, {0.6(0.4), 0.63(0.1), 0.65(0.1), 0.7(0.4)}>

A2	<{0.68(0.2), 0.7(0.3), 0.71(0.4), 0.73 (0.1)}, {0.1(0.2), 0.2(0.3), 0.4(0.4), 0.5(0.1)}>	<{0.82(0.3), 0.8(0.4), 0.73(0.1), 0.7(0.2)}, {0.4(0.3), 0.5(0.4), 0.55(0.1), 0.6(0.2)}>	<{0.2(0.3), 0.3(0.4), 0.5(0.1), 0.6(0.2)}, {0.3(0.3), 0.35(0.4), 0.45(0.1), 0.5(0.2)}>
A3	<{0.25(0.4), 0.3(0.1), 0.33(0.1), 0.4(0.4)}, {0.4(0.4), 0.6(0.1), 0.65(0.1), 0.7(0.4)}>	<{0.3(0.3), 0.4(0.4), 0.55(0.1), 0.6 (0.2)}, {0.2(0.3), 0.3(0.4), 0.5(0.1), 0.6(0.2)}>	<{0.6(0.3), 0.65(0.4), 0.7(0.1), 0.75(0.2)}, {0.3(0.3), 0.4(0.4), 0.42(0.1), 0.5(0.2)}>

Then the steps for obtaining ranking order by Ji et al. (2021) method are as follows:

Step 1: Due to absence of non-profit type criteria, the normalization process is not required. Therefore, $\tilde{M} = \left[\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)\right]_{3\times 3} = \left[\mathfrak{I}^{(ij)}(\Upsilon)\right]_{3\times 3} = M$.

Step 2: The symbol $Supp^{(jk)}$ is used to denote the support between the PDHPFEs $\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)$ and $\tilde{\mathfrak{I}}^{(ik)}(\Upsilon)$ $(j,k=1,2,3;\, j\neq k)$. Then, based on Eq. (6), we obtain,

$$Supp^{(12)} = Supp^{(21)} = (0.96031, 0.92174, 0.96529),$$
$$Supp^{(13)} = Supp^{(31)} = (0.95708, 0.93530, 0.92934),$$
$$Supp^{(23)} = Supp^{(32)} = (0.92839, 0.92899, 0.96029).$$

Step 3: We present the values $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))$ $(i,j=1,2,3)$ by $\psi =$
$$\begin{bmatrix} 1.9173 & 1.8887 & 1.8855 \\ 1.8570 & 1.8507 & 1.8643 \\ 1.8946 & 1.9256 & 1.8896 \end{bmatrix}$$

Step 4: The values $\theta^{(ij)}$ $(i,j=1,2,3)$ are presented by

$$\theta = \begin{bmatrix} 0.2519 & 0.3493 & 0.3987 \\ 0.2499 & 0.3491 & 0.4009 \\ 0.2492 & 0.3526 & 0.3981 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the PDHPFPWHM operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ = <{0.2245 (0.024), 0.3713 (0.016), 0.4475 (0.003), 0.4925 (0.008)},

{0.4702 (0.024), 0.5284 (0.016), 0.5878 (0.003), 0.6380 (0.008)}>,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ = <{0.6553 (0.018), 0.6560 (0.048), 0.6532 (0.004), 0.6734 (0.004)},

{0.2520 (0.018), 0.3446 (0.048), 0.4686 (0.004), 0.5328 (0.004)}>,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ = <{0.4506 (0.036), 0.5116 (0.016), 0.5877 (0.001), 0.6414 (0.016)},

{0.2793 (0.036), 0.3998 (0.016), 0.4979 (0.001), 0.5798 (0.016)}>.

Step 6: The scores of the aggregated PDHPFEs are: $Sc(\tilde{\mathfrak{I}}^{(1)}(\Upsilon))$ = 0.0166, $Sc(\tilde{\mathfrak{I}}^{(2)}(\Upsilon))$ = 0.0485, $Sc(\tilde{\mathfrak{I}}^{(3)}(\Upsilon))$ = 0.0352.

Step 7: The ranking order is $A_2 \succ A_3 \succ A_1$. Hence, by Ji et al.'s (2021) method, A_2 comes out as the best alternative.

Next, we apply our proposed method to Example 5.1 to investigate whether the best alternative remains the same or not. Steps are given below.

Step 1: Due to absence of non-profit type criteria, normalization is not required. So $\tilde{M} = \left[\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)\right]_{3\times 3} = \left[\mathfrak{I}^{(ij)}(\Upsilon)\right]_{3\times 3} = M$. Since the elements of $\tilde{M}(=M)$ are in not adjusted form, so we apply the techniques of adjustment of probabilities. The adjusted matrix is given below.

Table 5: The Adjusted Matrix $\hat{M}$ for Example 5.1.

	C1	C2	C3
A1	<{0.1(0.2), 0.1(0.1), 0.2(0.1), 0.2(0.1), 0.2(0.1), 0.2(0.1), 0.4(0.1), 0.5(0.1), 0.5(0.1)}, <{0.4(0.2), 0.4(0.1), 0.5(0.1), 0.5(0.1), 0.5(0.1), 0.5(0.1), 0.55(0.1), 0.6(0.1), 0.6(0.1)}>	<{0.3(0.2), 0.5(0.1), 0.5(0.1), 0.5(0.1), 0.5(0.1), 0.52(0.1), 0.52(0.1), 0.52(0.1), 0.53(0.1)}, <{0.4(0.2), 0.45(0.1), 0.45(0.1), 0.45(0.1), 0.45(0.1), 0.55(0.1), 0.55(0.1), 0.55(0.1), 0.6(0.1)}>	<{0.2(0.2), 0.2(0.1), 0.2(0.1), 0.3(0.1), 0.4(0.1), 0.45(0.1), 0.45(0.1), 0.45(0.1), 0.45(0.1)}, <{0.6(0.2), 0.6(0.1), 0.6(0.1), 0.63(0.1), 0.65(0.1), 0.7(0.1), 0.7(0.1), 0.7(0.1), 0.7(0.1)}>
A2	<{0.68(0.2), 0.7(0.1), 0.7 (0.2), 0.71 (0.2), 0.71 (0.1), 0.71(0.1), 0.73 (0.1)}, {<{0.1(0.2), 0.2(0.1), 0.2 (0.2), 0.4 (0.2), 0.4 (0.1), 0.4(0.1), 0.5 (0.1)}>	<{0.82(0.2), 0.82(0.1), 0.8 (0.2), 0.8 (0.2), 0.73 (0.1), 0.7(0.1), 0.7 (0.1)}, {<{0.4(0.2), 0.4(0.1), 0.5 (0.2), 0.5 (0.2), 0.55 (0.1), 0.6(0.1), 0.6 (0.1)}>	<{0.2(0.2), 0.2(0.1), 0.3 (0.2), 0.3 (0.2), 0.5 (0.1), 0.6(0.1), 0.6 (0.1)}, {<{0.3(0.2), 0.3(0.1), 0.35 (0.2), 0.35 (0.2), 0.45 (0.1), 0.5(0.1), 0.5 (0.1)}>
A3	<{0.25(0.3), 0.25(0.1), 0.3 (0.1), 0.33 (0.1), 0.4(0.1), 0.4(0.1), 0.4 (0.2)}, {0.4(0.3), 0.4(0.1), 0.6 (0.1), 0.65 (0.1), 0.7(0.1), 0.7(0.1), 0.7 (0.2)}>	<{0.3(0.3), 0.4(0.1), 0.4 (0.1), 0.4 (0.1), 0.4(0.1), 0.55(0.1), 0.6 (0.2)}, {0.2(0.3), 0.3(0.1), 0.3 (0.1), 0.3 (0.1), 0.3(0.1), 0.5(0.1), 0.6 (0.2)}>	<{0.6(0.3), 0.65(0.1), 0.65 (0.1), 0.65 (0.1), 0.65(0.1), 0.7(0.1), 0.75 (0.2)}, {0.3(0.3), 0.4(0.1), 0.4 (0.1), 0.4 (0.1), 0.4(0.1), 0.42(0.1), 0.5(0.2)}>

Step 2: The supports between the PDHPFEs $\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)$ and $\tilde{\mathfrak{I}}^{(ik)}(\Upsilon)$ $(j,k=1,2,3;\ j\neq k)$ are calculated based on Eq. (11) and these are given by:

$$Supp^{(12)} = Supp^{(21)} = (0.99046, 0.98048, 0.98017),$$
$$Supp^{(13)} = Supp^{(31)} = (0.98735, 0.97088, 0.96362),$$
$$Supp^{(23)} = Supp^{(32)} = (0.98403, 0.95942, 0.97731).$$

Step 3: We present the values $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))$ $(i,j=1,2,3)$ by $\psi =$
$$\begin{bmatrix} 1.9778 & 1.9745 & 1.9714 \\ 1.9513 & 1.9399 & 1.9303 \\ 1.9437 & 1.9574 & 1.9409 \end{bmatrix}$$

Step 4: The values $\theta^{(ij)}$ $(i,j=1,2,3)$ are presented by.
$$\theta = \begin{bmatrix} 0.2503 & 0.3500 & 0.3996 \\ 0.2510 & 0.3501 & 0.3988 \\ 0.2496 & 0.3512 & 0.3991 \end{bmatrix}$$

Step 5: The aggregated PDHPFEs obtained through utilizing the $PDHPFIPWA$ operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ = <{0.2237 (0.2), 0.3278 (0.1), 0.3389 (0.1), 0.3669 (0.1), 0.4031 (0.1), 0.4323 (0.1), 0.4648 (0.1), 0.4882 (0.1), 0.4921 (0.1)},{0.4959 (0.2), 0.5085 (0.1), 0.5252 (0.1), 0.5419 (0.1), 0.5581 (0.1), 0.6213 (0.1), 0.6219 (0.1), 0.6287 (0.1), 0.6427 (0.1)}> ,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ = <{0.6346 (0.2), 0.6405 (0.1), 0.6405 (0.2), 0.6436 (0.2), 0.6490 (0.1), 0.6664 (0.1), 0.6723 (0.1)},{<{0.3811 (0.2), 0.3878 (0.1), 0.4791 (0.2), 0.5046 (0.2), 0.5074 (0.1), 0.5351 (0.1), 0.5534 (0.1)}>,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ = <{0.4418 (0.3), 0.4969 (0.1), 0.5034 (0.1), 0.5078 (0.1), 0.5196 (0.1), 0.5904(0.1), 0.6344 (0.2)},{0.3052 (0.3), 0.3824 (0.1), 0.4367 (0.1), 0.4565 (0.1), 0.4844 (0.1), 0.5227 (0.1), 0.5842 (0.2)}> .

Step 6: The scores of the aggregated PDHPFEs are:

$Sc(\tilde{\mathfrak{I}}^{(1)}(\Upsilon))$ =-0.1878, $Sc(\tilde{\mathfrak{I}}^{(2)}(\Upsilon))$ =0.1752, $Sc(\tilde{\mathfrak{I}}^{(3)}(\Upsilon))$ =0.0845.

Step 7: The ranking order is $A_2 \succ A_3 \succ A_1$. Hence, the best alternative is A_2 .

Hence, by our proposed method, A_2 comes out as the best alternative which is exactly the same obtained by using Ji et al. (2021) method. Thus, our developed method is effective.

Example 5.2 (Adapted from Ji et al. (2021)): "Suppose four patients $A_i (i = 1,2,3)$ with acute respiratory distress syndrome (ARDS) are admitted in the same ICU and all of them show four conditions of different degree: cardio-palmonary function (C_1), hepatorenal function (C_2), complication risk (C_3) and total vital signs (C_4). The weighted vector of criteria is W=(0.32, 0.26, 0.18, 0.24) satisfying $\sum_{j=1}^{4} w_j = 1$ and $0 \le w_j \le 1\, (j = 1,2,3,4)$.

In order to evaluate the most suitable patient, the doctor performing extra-vascular membrane oxygenation (ECMO) is invited to evaluate the situation of four patients from the four conditions respectively." The initial assessment matrix is presented in Table 6.

Table 6: Initial Assessment Matrix for Example 5.2 (adapted from Ji et al. (2021).

	C1	**C2**	**C3**	**C4**
A1	<{0.7(0.3), 0.6(0.3), 0.5(0.4)},{0.2(1)}>	<{0.7(1)},{0.25(1)>	<{0.2(1)},{0.2(1)}>	{0.7(0.5), 0.6(0.5)}, {0.3(1)}>
A2	<{0.1(1)},{0.4(1)}>	<{0.3(1)},{0.7(1)}>	<{0.3(0.5), 0.2(0.5)}, {0.7(1)}>	<{0.3(1)},{0.3(1)}>
A3	<{0.6(1)},{0.35(1)}>	<{0.56(1)},{0.2(1)}>	<{0.7(1)},{0.1(1)}>	{0.2(0.6), 0.4(0.4)}, {0.4(1)}>
A4	<{0.05(0.7), 0.2(0.3)},{0.5(1)}>	<{0.3(0.5),0.2(0.5)}, {0.6(0.5),0.5(0.5)}>	<{0.15(1)},{0.8(1)}>	<{0.2(1)},{0.6(1)}>

Then the steps for obtaining ranking order by our proposed method are as follows:

Step 1: Since C_3 is a non-profit type criterion, so normalization is required. The normalized matrix is given in Table 7.

Table 7: Normalized Assessment Matrix.

	C_1	C_2	C_3	C_4
A_1	<{0.7(0.3), 0.6(0.3), 0.5(0.4)},{0.2(1)}>	<{0.7(1)},{0.25(1)}>	<{0.2(1)},{0.2(1)}>	{0.7(0.5), 0.6(0.5)}, {0.3(1)}>
A_2	<{0.1(1)},{0.4(1)}>	<{0.3(1)},{0.7(1)}>	<{0.7(1)},{0.3(0.5), 0.2(0.5)}>	<{0.3(1)},{0.3(1)}>
A_3	<{0.6(1)},{0.35(1)}>	<{0.56(1)},{0.2(1)}>	<{0.1(1)},{0.7(1)}>	{0.2(0.6), 0.4(0.4)}, {0.4(1)}>
A_4	<{0.05(0.7), 0.2(0.3)},{0.5(1)}>	<{0.3(0.5), 0.2(0.5)}, {0.6(0.5),0.5(0.5)}>	<{0.8(1)},{0.15(1)}>	<{0.2(1)},{0.6(1)}>

Since the elements of the normalized matrix are in not adjusted form, so we apply the techniques of adjustment of probabilities (Example 4.3). The adjusted matrix is given below.

Step 2: The supports between the PDHPFEs $\tilde{\mathfrak{I}}^{(ij)}(\Upsilon)$ and $\tilde{\mathfrak{I}}^{(ik)}(\Upsilon)$ $(j,k=1,2,3;\ j\neq k)$ are calculated based on Eq. (11) and these are given by:

$$\begin{aligned}
Supp^{(12)} &= Supp^{(21)} = (0.98031,0.89750,0.96777,0.97344),\\
Supp^{(13)} &= Supp^{(31)} = (0.96062,0.85625,0.82062,0.78656),\\
Supp^{(14)} &= Supp^{(41)} = (0.98500,0.96250,0.92262,0.96593),\\
Supp^{(23)} &= Supp^{(32)} = (0.94094,0.79375,0.81160,0.78562),\\
Supp^{(24)} &= Supp^{(42)} = (0.98843,0.90000,0.91360,0.98000),\\
Supp^{(34)} &= Supp^{(43)} = (0.94562,0.89375,0.89800,0.76562).
\end{aligned}$$

Step 3: We present the values $\psi(\tilde{\mathfrak{I}}^{(ij)}(\Upsilon))$ $(i,j=1,2,3)$ by $\psi =$

$$\begin{bmatrix} 2.9259 & 2.9096 & 2.8472 & 2.9191 \\ 2.7162 & 2.5912 & 2.5437 & 2.7562 \\ 2.7110 & 2.6929 & 2.5302 & 2.7342 \\ 2.7259 & 2.7391 & 2.3378 & 2.7115 \end{bmatrix}$$

Step 4: The values $\theta^{(ij)}$ $(i,j=1,2,3)$ are presented by

$$\theta = \begin{bmatrix} 0.3216 & 0.2602 & 0.1773 & 0.2408 \\ 0.3247 & 0.2549 & 0.1742 & 0.2461 \\ 0.3227 & 0.2609 & 0.1727 & 0.2436 \\ 0.3261 & 0.2659 & 0.1643 & 0.2436 \end{bmatrix}$$

Table 8: The Adjusted Normalized Matrix.

	C_1	C_2	C_3	C_4
A_1	<{0.7(0.3), 0.6(0.2), 0.6(0.1), 0.5(0.4)}, {0.2(0.3), 0.2(0.2), 0.2(0.1), 0.2(0.4)}>	<{0.7(0.3), 0.7(0.2), 0.7(0.1), 0.7(0.4)}, {0.25(0.3), 0.25(0.2), 0.25(0.1), 0.25(0.4)}>	<{0.2(0.3), 0.2(0.2), 0.2(0.1), 0.2(0.4)}, {0.2(0.3), 0.2(0.2), 0.2(0.1), 0.2(0.4)}>	<{0.7(0.3), 0.7(0.2), 0.6(0.1), 0.6(0.4)}, {0.3(0.3), 0.3(0.2), 0.3(0.1), 0.3(0.4)}>
A_2	<{0.1(0.5), 0.1(0.5) }, {0.4(0.5), 0.4(0.5)}>	<{0.3(0.5), 0.3(0.5)}, {0.7(0.5), 0.7(0.5)}>	<{0.7(0.5), 0.7(0.5)}, {0.3(0.5), 0.2(0.5)}>	<{0.3(0.5), 0.3(0.5) }, {0.3(0.5), 0.3(0.5)}>
A_3	<{0.6(0.6), 0.6(0.4)}, {0.35(0.6), 0.35(0.4)}>	<{0.56(0.6), 0.56(0.4)}, {0.2(0.6), 0.2(0.4)}>	<{0.1(0.6), 0.1(0.4) }, {0.7(0.6), 0.7(0.4)}>	<{0.2(0.6), 0.4(0.4) }, {0.4(0.6), 0.4(0.4)}>
A_4	<{0.05(0.5), 0.05(0.2), 0.2(0.3)}, {0.5(0.5), 0.5(0.2), 0.5(0.3)}>	<{0.3(0.5), 0.2(0.2), 0.2(0.3)}, {0.6(0.5), 0.5(0.2), 0.5(0.3)}>	<{0.8(0.5), 0.8(0.2), 0.8(0.3)}, {0.15(0.5), 0.15(0.2), 0.15(0.3)}>	<{0.2(0.5), 0.2(0.2), 0.2(0.3)}, {0.6(0.5), 0.6(0.2), 0.6(0.3)}>

Step 5: The aggregated PDHPFEs obtained through utilizing the $PDHPFIPWA$ operator are:

$\tilde{\mathfrak{I}}^{(1)}(\Upsilon)$ = <{0.6483 (0.3), 0.6142 (0.2), 0.5864 (0.1), 0.5559 (0.4)},{ 0.2505 (0.3), 0.2536 (0.2), 0.2484 (0.1), 0.2511 (0.4)}> ,

$\tilde{\mathfrak{I}}^{(2)}(\Upsilon)$ = <{0.3753 (0.5), 0.3753 (0.5)},{ 0.4810 (0.5), 0.4679 (0.5) }> ,

$\tilde{\mathfrak{I}}^{(3)}(\Upsilon)$ =<{0.4631 (0.6), 0.4924 (0.4)},{ 0.4144 (0.6), 0.4119 (0.4) }> ,

$\tilde{\mathfrak{I}}^{(4)}(\Upsilon)$ = <{0.3973 (0.5), 0.3805 (0.2), 0.3958 (0.3)},{ 0.5005 (0.5), 0.4704 (0.2), 0.4699 (0.3)}> .

Step 6: The scores of the aggregated PDHPFEs are:

$$Sc(\tilde{\mathfrak{I}}^{(1)}(\Upsilon)) = 0.3472,\ Sc(\tilde{\mathfrak{I}}^{(2)}(\Upsilon)) = -0.0991,\ Sc(\tilde{\mathfrak{I}}^{(3)}(\Upsilon)) = 0.0613,\ Sc(\tilde{\mathfrak{I}}^{(4)}(\Upsilon)) = -0.0918.$$

Step 7: The ranking order is $A_1 \succ A_3 \succ A_4 \succ A_2$. Hence, the best alternative is A_1.

On the other hand, by Ji et al.'s (2021) method, we obtain the ranking order $A_1 \succ A_3 \succ A_2 \succ A_4$ (for k=1) which is slightly different from what we obtained, but the best alternative remains the same for both methods. This means that our method is effective and credible.

6 Conclusion

PDHPFSs can effectively portray the dubiousness and uncertainty due to the inclusion of the MDs and NMDs with their corresponding probabilities. The joint occurrence of the stochastic and the non-stochastic ambiguity makes the PDHPFSs more realistic and superior compared to Pythagorean fuzzy information, and hesitant Pythagorean fuzzy information (we refer Table 9 for characteristic comparison). The basic operations for PDHFPFEs proposed by Ji et al. (2021) are not reasonable in the case when among the PDHPFEs considered, one PDHPFE has a non-belongingness grade equal to '0'. Also the PDHPF power weighted Hamy mean operator proposed by Ji et al. (2021) gives unreasonable output in the case when among the PDHPFEs considered, one PDHPFE has a non-belongingness grade equal to '0'. So, firstly, to get a reasonable output, we have proposed the adjustments of probabilities of the PDHPFEs based on which we have defined improved operational laws for PDHPFEs, and improved power weighted averaging operator. Then we present it's pivotal qualities like idempotency, boundedness, and monotonicity under PDHPF setting. Since Ji et al.'s (2021) method based on PDHPF power weighted Hamy mean operator for dealing with MADM issues sometimes generates unreasonable ranking order, so we have developed an improved MADM approach to generate the reasonable ranking order and to track down the best option in PDHPF setting.

The limitations of our work are: (i) the proposed $PDHPFIPWA$ operator doesn't consider the dependency among various criteria, and (ii) the proposed model is not suitable for multi-expert MCDM problems. In the future, the proposed operator can be merged with Bonferroni mean operators (Saha, Garg, & Dutta, 2021; Saha, Senapati, & Yager, 2021), and Maclaurin symmetric mean operators (Saha et al., 2022) to generate hybrid operators to consider the dependency among different criteria. Moreover, the developed model can be extended to multi-expert MCDM model upon consideration of experts' weights and their consensus reaching. Besides, one can extend the proposed approach to probabilistic dual hesitant q-rung orthopair fuzzy sets, and its linguistic version namely probabilistic linguistic dual hesitant q-rung orthopair fuzzy sets.

Table 9: The Characteristic Comparison.

Information type	***Whether deals with hesitancy?***	***Whether deals with probabilistic information?***	***Generality and flexibility***
Pythagorean fuzzy information	No	No	Medium
Hesitant Pythagorean fuzzy information	Yes	No	High
Probabilistic dual hesitant Pythagorean fuzzy information	Yes	Yes	Very high

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