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RELIABILITY CHARACTERISTICS OF RAILWAY COMMUNICATION SYSTEM SUBJECT TO SWITCH FAILURE

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Research paper

Abstract. *In the present study, a railway communication system (RCS) reliability model is developed based on system failure. The proposed RCS has control centre and stations which are arranged in such a manner that failure of control centre or a single station stops the working of overall system i.e., all switches must be working for communication to be available. To improve the reliability of the proposed communication system, a ring architecture is employed. In this architecture one additional communication path is connected in parallel configuration. Provision of two path of communication ensures that failure of one path will not cause a communication failure and communication will be available through additional path. All failures of RCS are exponentially distributed. Mathematical modelling of the system is carried out using Markov process by which the differential equations are generated. These differential equations are further used to evaluate the reliability measures like availability, reliability, mean time to failure of the proposed RCS. Likewise, sensitivity analysis is done to determine the impact of failures on RCS's performance measures. The proposed Markov process-based model gives the information about the failure and working of the multi- state railway communication system. Finally, numerical results are provided with graphs to demonstrates the usefulness of the findings.*

Key words*: Railway communication system; Reliability; Mean time to failure; Markov process; Sensitivity*

1. Introduction

In the present day's society demands inexpensive, more secure, and timesaving public transport. Railway transportation systems attracts a lot of passengers because

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of their capacity of transporting the people with high luxury, great comfort and large get-up-and-go efficacy (Ai et al., 2014). A lot of people choose trains for travelling because of the easy understanding, experience and comfort that the rail transport gives. Railway communications systems are needed to develop the communication between train and path equipment for traffic management and dealing with continuous high-data-rate traveler services, hypermedia dispatching video transmissions, railway mobile ticketing, and the internet of things (IOT) for railways (Ai et al., 2015; Guan et al., 2017). The security of railway's employees, passengers and of the general public are the first requirements and is of specific significance in the railway industry. Railway industry looking for many aspects to improve the security / safety and the reliability of the railway systems.

A railway system is a very large and complex stochastic dynamic system. This system is already interesting by itself. Large, stochastic complex systems are generally examined with deterministic methods. Many authors have written about deterministic optimization of railway systems over the last twenty years. A lot of research has been done in the context of the different techniques to increase the reliability of a railway communication system. Aggarwal (1975) obtained reliability expression for communication system. Marquez et al. (2003) discussed about the improvement of a way to deal the use of remote monitoring to the reliability centred maintenance of railway attendances. Tao et al. (2007) used fault tree analysis method for RCS, in which main factors affecting the failure of RCS are determined by minimum cut set analysis.

De Felice and Petrillo (2011) proposed a methodological approach based on human reliability analysis (HRA) and failures modes, effects criticality analysis (FMECA) to calculate the reliability of railway transportation system. HRA gives a logical analysis of factors affecting human performance, prompts suggestions for improvement. Lin (2015) proposed an advanced finite state Markov chain channel model for high-speed railway fading channels and derived the expression of state transition probabilities under different speed modes. Unterhuber et al. (2016) provided a summary of communication systems in trains and discussed about possible direction for future wireless network. Also, authors identified the gaps for station classification in railway environment, total velocity, and relative velocity for radio broadcast measurement. He et al. (2017) studied the propagation characteristics for rapid railway communication system including metropolitan, rural and tunnel with straight and curved route. Zhang et al. (2018) proposed a Markov model for railway communication system and used multi-link transmission communication technique to improve the capacity of RCS. Kumar and Kumar (2019) evaluated the reliability, and mean time to failure of the wireless communication system regarding its component failure. Authors also identified the critical component by sensitivity analysis. Song (2019) described a communication-based train control system and discussed the different constraints that affects the working of communication system. Authors also evaluated system availability and performance by applied stochastic petri nets.

In this research article, a railway communication system with control centre and stations is considered. The considered multi-state repairable system having control centre and stations in such a manner that for communication, all switches must be in working condition. Failure in control centre and any of one station arises a communication failure. To improve the reliability of the proposed communication system, an additional path is connected in parallel configuration. In the proposed model of RCS, probability of each transition states is obtained and reliability measures such as system availability, system reliability, mean time to failure (MTTF), and sensitivity of reliability have been computed.

The rest of the paper is categorised as follows. The description of the proposed RCS with requisite assumptions and notations are presented in section 2. In section 3, the set of differential equations is constructed based on Markov process and also probability of each state is calculated using Laplace transformation. In section 4, the numerical calculations to compute the reliability measures of RCS such as availability, reliability, mean time to failure, and sensitivity analysis is given. In section 5, the behaviour of the reliability measures is discussed with the help of tables and graphs. Finally, section 6 gives the concluding remark to highlight the significant and some future prospects of the present work.

2. System modelling

In this study, a railway communication system with control centre and stations is considered. In this system a communication which is provided in the control centre as well as in each of the stations which are connected in a series configuration. It can be seen in the diagram that all switches must be working for the communication system to be function. If there is a failure in switch at any station, there will be a communication failure at those stations as well as at all stations beyond that point i.e., all switches must be working to continue communication. Further one additional communication path is connected in parallel configuration to improve the reliability of proposed system. Adding an additional path means that the communication will be available after failure of any one of the paths. On failure of both path's switches, the communication system will be failed.

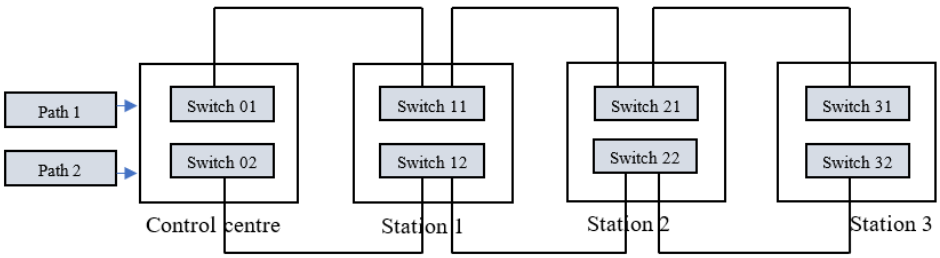

Figure 1. Reliability block diagram of railway communication system

To study and formulation of the system, following assumptions and notations are made (Table 1):

- The communication system consists of two paths namely path 1 and path 2, in which each path has a control centre and 3 stations.
- The proposed communication system has main three states full operation, degradation states and failure state.
- At time t = 0, control centre and stations are in fully operation state and communication system is available.

- This study assumes that the failure rates of path 1 units and path 2 units are statistically independent, constant and are exponentially distributed with failure rates λc_1, λ_1, and λc_2, λ_2.
- Failure of one path will not cause a communication failure.
- Repair service is always available to repair the failed unit, i.e., as soon as an operating unit fail, it is instantaneously detected and sent for repair.
- When a failed unit is repaired it considered to be a new one.

Table 1. Notations

t	Time variable
λc_1	Failure rate of path 1 control centre
λc_2	Failure rate of path 2 control centre
λ_1	Failure rate of all stations of path 1
λ_2	Failure rate of all stations of path 2
$P_i(t)$	The state probability of the system at instant 't' for i = 0 to 7
$P_i(x, t)$	The failed state probability of the system at instant 't' and an elapsed repair time x for i = 8 to 21
x	Elapsed repair time
μ (x)	Repair rate for repaired state

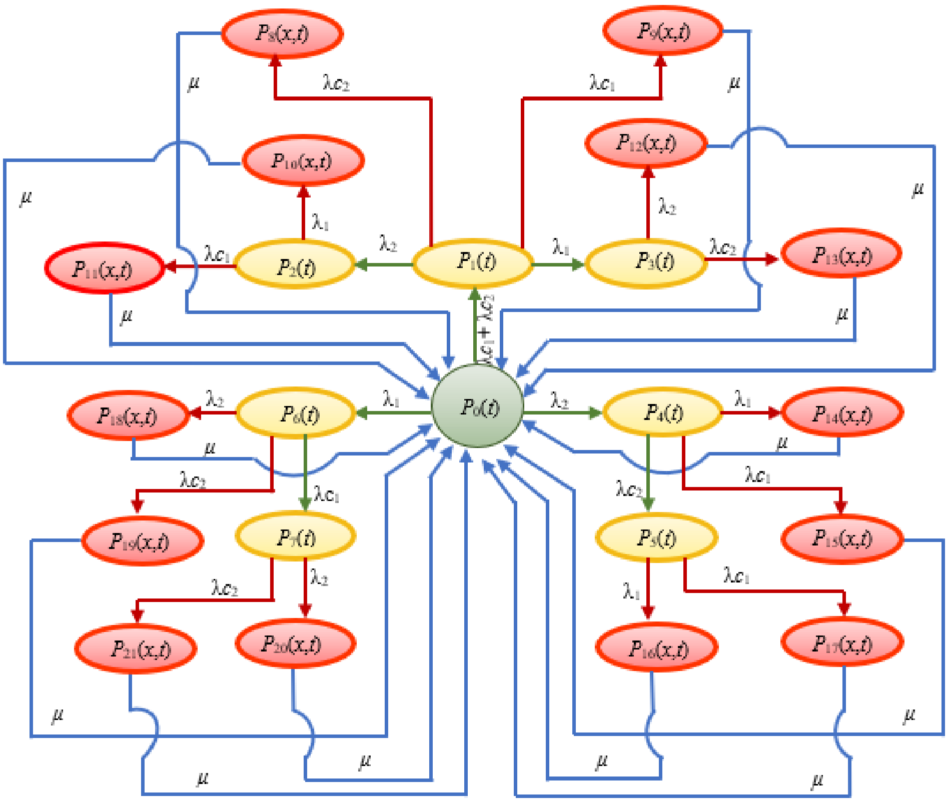

Figure 2. Transition diagram

3. Governing Equations

The following set of equations have been derived for the proposed communication system by using Markov process.

Original State

$$\left[\frac{\partial}{\partial t}+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2\right]P_0(t)=\int_0^\infty\sum_{i=8}^{21}\mu P_i(x,t)dx \tag{1}$$

Degraded States

$$\left[\frac{\partial}{\partial t}+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2\right]P_1(t)=\left(\lambda c_1+\lambda c_2\right)P_0(t) \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\lambda c_1+\lambda_1\right]P_2(t)=\lambda_2 P_1(t) \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\lambda c_2+\lambda_2\right]P_3(t)=\lambda_1 P_1(t) \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\lambda c_1+\lambda c_2+\lambda_1\right]P_4(t)=\lambda_2 P_0(t) \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\lambda c_1+\lambda_1\right]P_5(t)=\lambda c_2 P_4(t) \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\lambda c_1+\lambda c_2+\lambda_2\right]P_6(t)=\lambda_1 P_0(t) \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\lambda c_2+\lambda_2\right]P_7(t)=\lambda c_1 P_6(t) \tag{8}$$

Failed States

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu\right]P_i(x,t)=0,\quad i=8,9,\ldots 20,21 \tag{9}$$

Boundary Conditions

$$P_8(0,t)=\lambda c_2 P_1(t) \tag{10}$$

$$P_9(0,t)=\lambda c_1 P_1(t) \tag{11}$$

$$P_{10}(0,t)=\lambda_1 P_2(t) \tag{12}$$

$$P_{11}(0,t)=\lambda c_1 P_2(t) \tag{13}$$

$$P_{12}(0,t)=\lambda_2 P_3(t) \tag{14}$$

$$P_{13}(0,t)=\lambda c_2 P_3(t) \tag{15}$$

$$P_{14}(0,t)=\lambda_1 P_4(t) \tag{16}$$

$$P_{15}(0,t)=\lambda c_1 P_4(t) \tag{17}$$

$$P_{16}(0,t)=\lambda_1 P_5(t) \tag{18}$$

$$P_{17}(0,t)=\lambda c_1 P_5(t) \tag{19}$$

$$P_{18}(0,t)=\lambda_2 P_6(t) \tag{20}$$

$$P_{19}(0,t)=\lambda c_2 P_6(t) \tag{21}$$

$$P_{20}(0,t)=\lambda_2 P_7(t) \tag{22}$$

$$P_{21}(0,t)=\lambda c_2 P_7(t) \tag{23}$$

Initial Condition

$$\begin{aligned} &P_0(0)=1 \\ &P_i(0)=0, \qquad i=1,2,\ldots,21 \end{aligned} \tag{24}$$

After taking the Laplace transform from Equations (1) to (23), one can get the following set of equations:

$$\left[s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2\right]\bar{P}_0(s)=\int_0^\infty \sum_{i=8}^{21} \mu \bar{P}_i(x,s)dx \tag{25}$$

$$\left[s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2\right]\bar{P}_1(s)=\left(\lambda c_1+\lambda c_2\right)\bar{P}_0(s) \tag{26}$$

$$\left[s+\lambda c_1+\lambda_1\right]\bar{P}_2(s)=\lambda_2 \bar{P}_1(s) \tag{27}$$

$$\left[s+\lambda c_2+\lambda_2\right]\bar{P}_3(s)=\lambda_1 \bar{P}_1(s) \tag{28}$$

$$\left[s+\lambda c_1+\lambda c_2+\lambda_1\right]\bar{P}_4(s)=\lambda_2 \bar{P}_0(s) \tag{29}$$

$$\left[s+\lambda c_1+\lambda_1\right]\bar{P}_5(s)=\lambda c_2 \bar{P}_4(s) \tag{30}$$

$$\left[s+\lambda c_1+\lambda c_2+\lambda_2\right]\bar{P}_6(s)=\lambda_1 \bar{P}_0(s) \tag{31}$$

$$\left[s+\lambda c_2+\lambda_2\right]\bar{P}_7(s)=\lambda c_1 \bar{P}_6(s) \tag{32}$$

$$\left[s+\frac{\partial}{\partial x}+\mu\right]\bar{P}_i(x,s)=0, \quad i=8,9,\ldots 20,21 \tag{33}$$

$$\bar{P}_8(0,s)=\lambda c_2 \bar{P}_1(s) \tag{34}$$

$$\bar{P}_9(0,s)=\lambda c_1 \bar{P}_1(s) \tag{35}$$

$$\bar{P}_{10}(0,s)=\lambda_1 \bar{P}_2(s) \tag{36}$$

$$\bar{P}_{11}(0,s)=\lambda c_1 \bar{P}_2(s) \tag{37}$$

$$\bar{P}_{12}(0,s)=\lambda_2 \bar{P}_3(s) \tag{38}$$

$$\bar{P}_{13}(0,s)=\lambda c_2\bar{P}_3(s) \tag{39}$$

$$\bar{P}_{14}(0,s)=\lambda_1\bar{P}_4(s) \tag{40}$$

$$\bar{P}_{15}(0,s)=\lambda c_1\bar{P}_4(s) \tag{41}$$

$$\bar{P}_{16}(0,s)=\lambda_1\bar{P}_5(s) \tag{42}$$

$$\bar{P}_{17}(0,s)=\lambda c_1\bar{P}_5(s) \tag{43}$$

$$\bar{P}_{18}(0,s)=\lambda_2\bar{P}_6(s) \tag{44}$$

$$\bar{P}_{19}(0,s)=\lambda c_2\bar{P}_6(s) \tag{45}$$

$$\bar{P}_{20}(0,s)=\lambda_2\bar{P}_7(s) \tag{46}$$

$$\bar{P}_{21}(0,s)=\lambda c_2\bar{P}_7(s) \tag{47}$$

Now, solving Equation (25) - (33) with the help of (34) - (47), the following state transition probabilities are obtained:

$$\bar{P}_0(s)=\frac{1}{(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)-\bar{S}(s)\left[\bar{U}(s)+\bar{V}(s)+\bar{W}(s)\right]} \tag{48}$$

$$\bar{P}_1(s)=\frac{(\lambda c_1+\lambda c_2)}{s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2}\bar{P}_0(s) \tag{49}$$

$$\bar{P}_2(s)=\frac{\lambda_2(\lambda c_1+\lambda c_2)}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\bar{P}_0(s) \tag{50}$$

$$\bar{P}_3(s)=\frac{\lambda_1(\lambda c_1+\lambda c_2)}{(s+\lambda c_2+\lambda_2)(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\bar{P}_0(s) \tag{51}$$

$$\bar{P}_4(s)=\frac{\lambda_2}{s+\lambda c_1+\lambda c_2+\lambda_1}\bar{P}_0(s) \tag{52}$$

$$\bar{P}_5(s)=\frac{\lambda_2\,\lambda c_2}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1)}\bar{P}_0(s) \tag{53}$$

$$\bar{P}_6(s)=\frac{\lambda_1}{s+\lambda c_1+\lambda c_2+\lambda_2}\bar{P}_0(s) \tag{54}$$

$$\bar{P}_7(s)=\frac{\lambda_1\,\lambda c_1}{(s+\lambda c_2+\lambda_2)(s+\lambda c_1+\lambda c_2+\lambda_2)}\bar{P}_0(s) \tag{55}$$

$$\bar{P}_8(s)=\frac{\lambda c_2(\lambda c_1+\lambda c_2)}{(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{56}$$

$$\bar{P}_9(s)=\frac{\lambda c_1(\lambda c_1+\lambda c_2)}{(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{57}$$

$$\bar{P}_{10}(s)=\frac{\lambda_1\lambda_2(\lambda c_1+\lambda c_2)}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{58}$$

$$\bar{P}_{11}(s)=\frac{\lambda c_1\lambda_2(\lambda c_1+\lambda c_2)}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{59}$$

$$\bar{P}_{12}(s)=\frac{\lambda_1\lambda_2(\lambda c_1+\lambda c_2)}{(s+\lambda c_2+\lambda_2)(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{60}$$

$$\bar{P}_{13}(s)=\frac{\lambda c_2\lambda_1(\lambda c_1+\lambda c_2)}{(s+\lambda c_2+\lambda_2)(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{61}$$

$$\bar{P}_{14}(s)=\frac{\lambda_1\lambda_2}{(s+\lambda c_1+\lambda c_2+\lambda_1)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{62}$$

$$\bar{P}_{15}(s)=\frac{\lambda_1\lambda_2}{(s+\lambda c_1+\lambda c_2+\lambda_1)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{63}$$

$$\bar{P}_{16}(s)=\frac{\lambda c_2\,\lambda_1\,\lambda_2}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{64}$$

$$\bar{P}_{17}(s)=\frac{\lambda c_2\,\lambda c_1\,\lambda_2}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{65}$$

$$\bar{P}_{18}(s)=\frac{\lambda_1\,\lambda_2}{(s+\lambda c_1+\lambda_1)(s+\lambda c_1+\lambda c_2+\lambda_1)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{66}$$

$$\bar{P}_{19}(s)=\frac{\lambda_1\,\lambda c_2}{(s+\lambda c_1+\lambda c_2+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{67}$$

$$\bar{P}_{20}(s)=\frac{\lambda c_1\lambda_1\,\lambda_2}{(s+\lambda c_2+\lambda_2)(s+\lambda c_1+\lambda c_2+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{68}$$

$$\bar{P}_{21}(s)=\frac{\lambda c_2\lambda_1\,\lambda c_1}{(s+\lambda c_2+\lambda_2)(s+\lambda c_1+\lambda c_2+\lambda_2)}\left(\frac{1-\bar{S}(s)}{s}\right)\bar{P}_0(s) \tag{69}$$

The probabilities of up and down states are as follows:

$$\bar{P}_{up}(s)=\bar{P}_0(s)\left[\begin{array}{l}1+\dfrac{\lambda c_1+\lambda c_2}{(s+\lambda_1+\lambda c_1+\lambda c_2+\lambda_2)}\left\{1+\dfrac{\lambda_2}{(s+\lambda_1+\lambda c_1)}+\dfrac{\lambda_1}{(s+\lambda_2+\lambda c_2)}\right\}+\dfrac{\lambda_2}{(s+\lambda_1+\lambda c_1+\lambda c_2)}\\ \left\{1+\dfrac{\lambda c_2}{(s+\lambda_1+\lambda c_1)}\right\}+\dfrac{\lambda_1}{(s+\lambda_2+\lambda c_1+\lambda c_2)}\left\{1+\dfrac{\lambda c_1}{(s+\lambda_2+\lambda c_2)}\right\}\end{array}\right] \tag{70}$$

$$\bar{P}_{down}(s)=\bar{P}_0(s)\left(\frac{1-\bar{S}(s)}{s}\right)\left[\begin{array}{l}\frac{\lambda c_1+\lambda c_2}{(s+\lambda_1+\lambda c_1+\lambda c_2+\lambda_2)}\left\{\begin{array}{l}\lambda c_1+\lambda c_2+\frac{\lambda_1\lambda_2}{(s+\lambda_1+\lambda c_1)}+\frac{\lambda c_1\lambda_2}{(s+\lambda_1+\lambda c_1)}+\\ \frac{\lambda_1\lambda_2}{(s+\lambda_2+\lambda c_2)}+\frac{\lambda c_2\lambda_1}{(s+\lambda_2+\lambda c_2)}\end{array}\right\}+\\ \frac{\lambda_2}{(s+\lambda_1+\lambda c_1+\lambda c_2)}\left\{\lambda_1+\lambda c_1+\frac{\lambda c_2\lambda_1}{(s+\lambda_1+\lambda c_1)}+\frac{\lambda c_1\lambda c_2}{(s+\lambda_1+\lambda c_1)}\right\}+\\ \frac{\lambda_1}{(s+\lambda_2+\lambda c_1+\lambda c_2)}\left\{\lambda_2+\lambda c_2+\frac{\lambda c_1\lambda_1}{(s+\lambda_2+\lambda c_2)}+\frac{\lambda c_1\lambda c_2}{(s+\lambda_2+\lambda c_2)}\right\}\end{array}\right] \quad (71)$$

where,

$$\bar{U}(s)=\frac{(\lambda c_1+\lambda c_2)}{(s+\lambda c_1+\lambda c_2+\lambda_1+\lambda_2)}\left\{\begin{array}{l}\lambda c_1+\lambda c_2+\frac{\lambda_1\lambda_2}{(s+\lambda c_1+\lambda_1)}+\frac{\lambda c_1\lambda_2}{(s+\lambda c_1+\lambda_1)}\\ +\frac{\lambda_1\lambda_2}{(s+\lambda c_2+\lambda_2)}+\frac{\lambda c_2\lambda_1}{(s+\lambda c_2+\lambda_2)}\end{array}\right\}$$

$$\bar{V}(s)=\frac{\lambda_2}{(s+\lambda c_1+\lambda c_2+\lambda_1)}\left\{\lambda_1+\lambda c_1+\frac{\lambda_1\lambda c_2}{(s+\lambda c_1+\lambda_1)}+\frac{\lambda c_1\lambda c_2}{(s+\lambda c_1+\lambda_1)}\right\}$$

$$\bar{W}(s)=\frac{\lambda_1}{(s+\lambda c_1+\lambda c_2+\lambda_2)}\left\{\lambda_2+\lambda c_2+\frac{\lambda_2\lambda c_1}{(s+\lambda c_2+\lambda_2)}+\frac{\lambda c_1\lambda c_2}{(s+\lambda c_2+\lambda_2)}\right\}$$

4. Numerical Calculations

For computing the reliability measures of the proposed railway communication system, following failure and repair rates will be assumed as given by Table 2.

Table 2. Assumed failure and repair rate of proposed railway communication system.

Failure and repair rate/per hour
$\lambda_1 = 0.009$
$\lambda c_1 = 0.06$
$\lambda_2 = 0.007$
$\lambda c_2 = 0.03$
$\mu = 1$

4.1. Availability

The availability of the proposed system is computed by substituted the values of failure and repair rates as given in Table 2, in Equation (70), after putting these values, availability of the proposed RCS in terms of time t is given as follows:

$$A(t) = -0.01158e^{(-0.9887t)}+0.05608e^{(-0.2075t)}-0.00085e^{(-0.0779t)}-0.00277e^{(-0.0439t)}+0.95911 \quad (72)$$

Now after varying time from 0 to 50 with the interval of 5 units of time, one can get the numerical values of availability of the proposed system which are demonstrates by table 3.

Table 3. Availability of the proposed system

Time (t)	Availability
0	1.00000
5	0.97611
10	0.96398
15	0.95991
20	0.95867
25	0.95838
30	0.95829
35	0.95815
40	0.95811
45	0.95808
50	0.95801

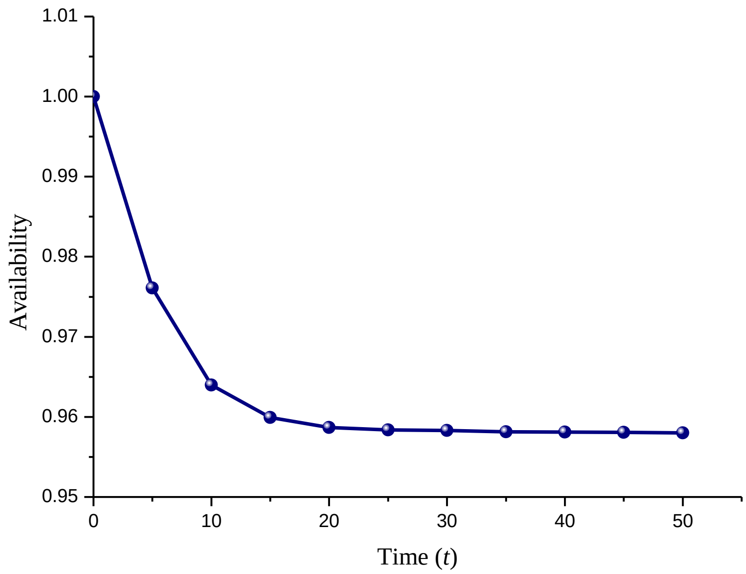

Figure 3. Availability of the proposed system as time vary from 0 to 50 units

4.2. Reliability

For the proposed system, reliability is calculated by substitute the values of failures as given in Table 2 and repair rate equal to zero in Equation (70), after substitute these failures and repairs values, the reliability function in terms of t is given by

$$R(t) = 0.30057e^{(-0.0370t)} + 0.64938e^{(-0.6900t)} + (0.06123t + 0.05005)e^{(-0.1060t)} \quad (73)$$

Now varying time t from 0 to 10 unit with an interval of 1 unit, one can analyzed the reliability behavior of proposed system as tabulated in Table 4 and shown in Figure 4.

Table 4. Reliability of the system

Time (t)	Reliability
0	1.00000
1	0.91938
2	0.76281
3	0.60074
4	0.45978
5	0.34658
6	0.25947
7	0.19404
8	0.14553
9	0.10976
10	0.08341

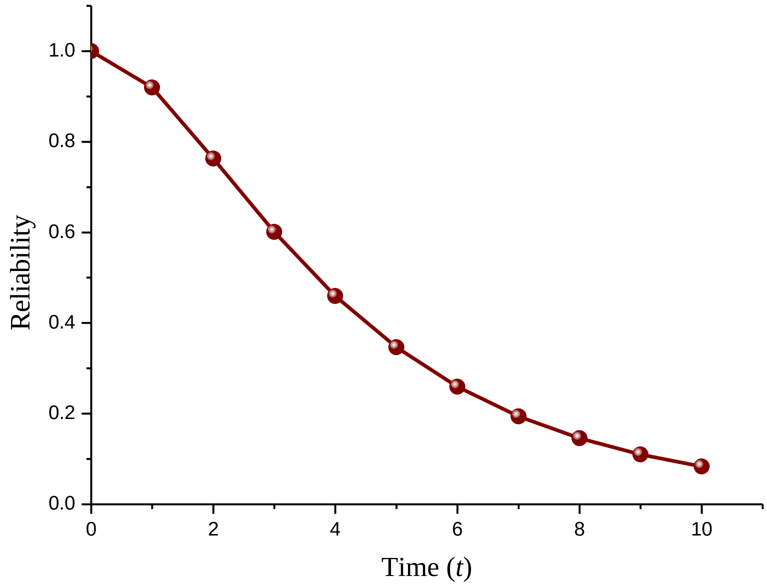

Figure 4. Reliability of the system w.r.t. time

4.3. Mean time to system Failure

The mean time to failure is calculated by taking μ = 0 and limit $s \to 0$ (Tyagi et al., 2021) in Equation (70). So, the MTTF of the proposed system in terms of failure rate is given by Equation (74).

$$MTTF = \frac{1}{(\lambda c_1 + \lambda c_2 + \lambda_1 + \lambda_2)} \left[\begin{array}{l} 1 + \frac{\lambda c_1 + \lambda c_2}{(\lambda_1 + \lambda c_1 + \lambda c_2 + \lambda_2)} \left\{ 1 + \frac{\lambda_2}{(\lambda_1 + \lambda c_1)} + \frac{\lambda_1}{(\lambda_2 + \lambda c_2)} \right\} + \frac{\lambda_2}{(\lambda_1 + \lambda c_1 + \lambda c_2)} \\ \left\{ 1 + \frac{\lambda c_2}{(\lambda_1 + \lambda c_1)} \right\} + \frac{\lambda_1}{(\lambda_2 + \lambda c_1 + \lambda c_2)} \left\{ 1 + \frac{\lambda c_1}{(\lambda_2 + \lambda c_2)} \right\} \end{array} \right] \quad (74)$$

Now setting all failure rates values as given by Table 2 and vary each failure rate one by one from 0.001 to 0.04 in Equation (74) to get the MTTF of the proposed system with respect to variation in the failure rates. The variation in MTTF can be seen from Table 5 and corresponding Figure 5 shows the behaviour of MTTF regarding variation in failure rates.

Table 5. MTTF of the system

Variation in Failure rates	MTTF			
	λ_{C1}	λ_{C2}	λ_1	λ_2
0.001	68.6161	51.82156	22.35057	24.79149
0.003	62.38204	45.53850	21.65165	24.28984
0.005	57.63895	41.19976	21.29690	23.84834
0.007	53.85080	37.98025	21.12371	23.45672
0.009	50.71749	35.46688	20.45672	23.10686
0.02	39.53540	27.40036	19.61863	21.69336
0.04	29.23630	20.75466	19.01606	20.24722
0.06	23.45672	17.10419	17.34855	19.39883
0.08	19.65024	14.65523	17.21551	18.81630
0.1	16.92906	12.85991	16.70574	18.37954

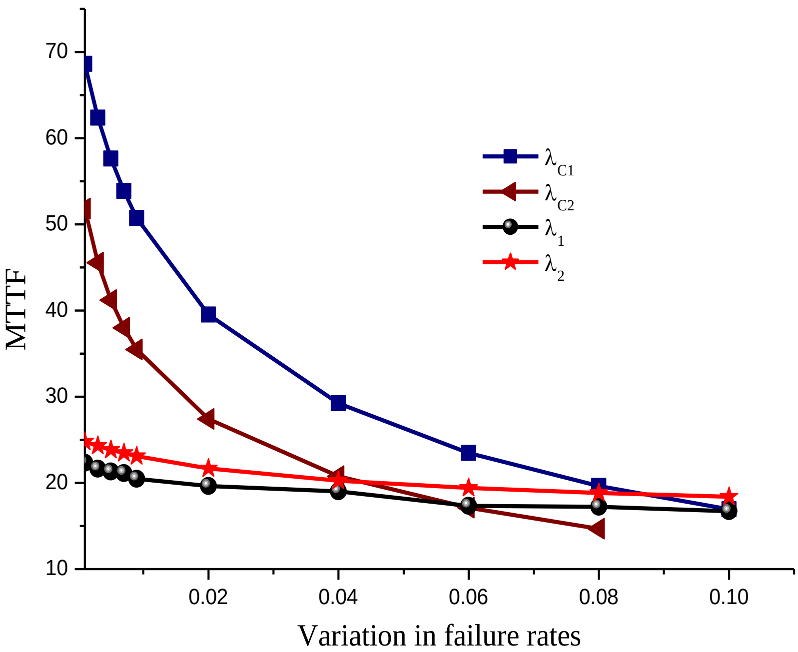

Figure 5. MTTF with respect to variation in failure rates

4.4. Sensitivity of reliability

In reliability sensitivity study, the effect of failure rates on reliability has been studied. The reliability function is the dependent variable. It's dependent because it depends on failures rates. Those failure rates are the independent variable. Sensitivity of reliability is obtained by partial differentiation of reliability function with respect to all the failure rates. Now putting all failure rates as given by Table 2 and repair rate equal to zero in these derivatives, one can get the Table 6 and corresponding Figure 6.

Table 6. Sensitivity of reliability of the proposed communication system

Time (t)	Sensitivity of reliability			
	$\frac{\partial R(t)}{\partial \lambda c_1}$	$\frac{\partial R(t)}{\partial \lambda c_2}$	$\frac{\partial R(t)}{\partial \lambda_1}$	$\frac{\partial R(t)}{\partial \lambda_2}$
0	0	0	0	0
1	-0.08468	-0.08575	-0.01886	-0.03526
2	-0.30679	-0.31133	-0.06439	-0.12756
3	-0.62535	-0.636	-0.123	-0.25976
4	-1.00731	-1.02689	-0.18454	-0.41824
5	-1.42634	-1.45771	-0.24159	-0.59228
6	-1.86164	-1.90766	-0.28897	-0.77354
7	-2.29709	-2.36052	-0.32325	-0.95566
8	-2.72038	-2.80387	-0.34243	-1.13384
9	-3.12234	-3.22838	-0.34555	-1.3046
10	-3.49642	-3.62729	-0.33251	-1.4655

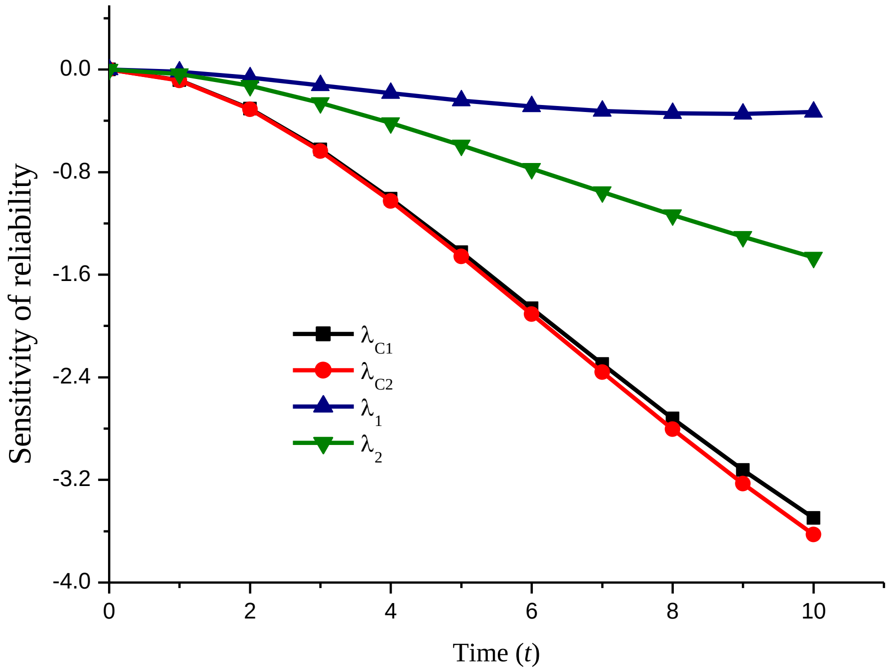

Figure 6. Sensitivity of reliability with respect to time

5. Result Discussion

In this paper, different reliability characteristics have been calculated and analysed for the railway communication system. Some results related to these reliability characteristics are given below:

(i) From Table 3 and Figure 3, one can see the behaviour of the availability of the proposed RCS. Availability of the RCS decreases with increases in the value of time *t*. At initially, i.e., time t = 0, availability is 1 and after 50 units of time, system availability is 0.9580. From Figure 3, one can see that the availability graph is constant for the time period 35 units to 50 units.

(ii) Table 4 and corresponding Figure 4 give an idea about the behaviour of the reliability of proposed railway communication system regarding time *t* for various system failure rates. From Table 4 and Figure 4, it is easily seen that the reliability of the proposed system decreases rapidly as increment in time *t*. At initially, reliability is one and after 10 units of time, reliability is 0.08341.

(iii) From Table 5, it is easily seen that the MTTF of the proposed system continuously decreases as all the failure rates λ_1, λc_1, λ_2, λc_2 increases. With respect to all stations failure rates MTTF is decreases in a uniform manner but with respect to control centre failure rates it decreases rapidly. Figure 5 demonstrates that the MTTF is high with variation in the failure rate of the first control centre and lowest regarding failure rates of path 1 stations which means failure rate of all stations of path 1 has more frequent downtime and disruption as compare to other failure rates.

(iv) Further, From Table 6 and Figure 6, one can see the behaviour of the failure rates on system reliability. The reliability of the RCS is more influenced by the variation in the second control centre failure rate which means second control centre failure rate causes the stronger change in reliability of the proposed system as time increases.

6. Conclusion

The present study discussed the performance of a railway communication system regarding sits component failure, and a procedure to evaluate the system's reliability measures. In order to numerically analyzed the performance of the composed system, Markov process has been applied. In the proposed railway communication system, the reliability is more influenced by failure rate of second control centre, thus the system reliability can be increased by a slight improvement in the failure rate of second control centre. In future work one can extend this investigation by using Link-Lk between switches so that communication is available even if there are multiple failures.

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