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Address of editorial office:
Regional Association for Security and crisis management (RABEK) –
European centre for Operational research (ECOR)
Cerska 76a, 11000 Belgrade, Serbia
Tel. +38165-2503-213
e-mail: zeljkostevic88@yahoo.com, editor@oresta.rabek.org
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THE APPLICATION OF THE FUZZY AHP AND DEA FOR MEASURING THE EFFICIENCY OF FREIGHT TRANSPORT RAILWAY UNDERTAKINGS

Aleksandar Blagojević [1]*, Slavko Vesković [2], Sandra Kasalica [1], Aleksandra Gojić [3], Ahmet Allamani [4]

[1] Academy of Technical and Artistic Professional Studies Belgrade, Section: College of Railway Engineering, Belgrade, Serbia
[2] University of Belgrade, Faculty of Transport and Traffic Engineering, Vojvode Stepe 305, 11000, Belgrade, Serbia
[3] Railways of Republic of Srpska, Svetog Save 71, 74000 Doboj, B&H
[4] Railway Inspection Directorate, Albania

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Abstract: *Measuring the performance of railway undertakings is inevitably becoming a prerequisite for their survival on the market in today's dynamic and highly turbulent environment. Railway undertakings must find optimal solutions in order to efficiently and effectively operate, survive on the transport market, and develop and maintain their competitive advantages as well. The objective of this research is to define and evaluate the criteria that affect the efficiency of railway undertakings, increase their competitiveness and propose a DEA-based approach (i.e. a Data-Envelopment-Analysis-based approach) to the assessment of the efficiency of railway undertakings in increasing competitiveness. In order to solve the criteria selection problem, the Fuzzy Analytical Hierarchical Processes (FAHP) method was experimented with, which showed the priority of the assessment of the efficiency of railway undertakings, on the basis of the five groups of criteria. The criteria in a group that outperformed the other criteria in that group for their freight transport railway undertakings within a composite normalized range were used as the input and output indicators for the DEA. The evaluation of the efficiency of the railway undertakings was considered by using the DEA approach. The results show that the proposed approach successfully enables the consolidation of a set of criteria (resource, operational, financial, quality and safety) into a single assessment of the efficiency of the railway undertakings, while providing information on the corrective actions that can improve the efficiency of the railway undertakings.*

Key words: *railway undertaking, efficiency, DEA, fuzzy AHP*

*Corresponding author.
E-mail addresses: aleksandar.blagojevic23@gmail.com (Blagojević A.), veskos@sf.bg.ac.rs (Vesković S.), sandra.kasalica@gmail.com (Kasalica S.), gojic.aleksandra@gmail.com (Gojić A.), ahmet.allamani@dih.gov.al (Allamani A.)

1. Introduction

The twenty-first century can also be called a century of change, and the conditions under which organizations operate can be seen as very complex. The rapid changes in the business world and the increasing competition in the transport services market have imposed on all organizations, including transport companies, the need to harmonize their business with the requirements of the modern business environment. The market has become an arena in which product and service providers are ruthlessly battling for every promile of the market. Survival on the market can be ensured only by the fittest who are able to outperform competitors. New business conditions dictate new market demands and establish new competitive relationships on the market. The struggle for survival in the marketplace is becoming inevitable. In order to persist in this struggle, companies need to accept and adapt to new business conditions. The ever more intensive development of the transport market and the ever more complex demands of the users of transport services, with the growing pressure of competition, requires that the organization of the company should become the central determinant of business and the activities carried out should completely be harmonized and financially viable for both the provider and the user of services. In order to survive on the market, companies seek to find the optimal relationship between the resources invested and the goals achieved. The application of the new European transport policy at the end of the last century caused major changes in Europe's transport system. There is a major transformation of transport companies into the efficient companies that will be operating in a liberalized European transport market in the future. In a large number of European countries, as well as in the other countries of the world, standards have been adopted regarding the restructuring of the railway system. Appropriate legal acts were adopted for the transformation of railways. The previous restructuring stages had not allowed the complete liberalization of the railway transport market, the expected positive operation of the railway sector, the fulfilment of the requirements of the transport market, raising the quality of railway services to the required level, the interests of the community at the national, regional and local levels and others. The restructuring of the railway system only partially brought positive business results in the main railways or pan-European corridors, mainly in transit traffic (Stojić et al., 2012). Although the quality of the services of the railway system has slightly increased, it is still far from the quality required by the transport market. In providing an adequate quality of railway services, railway undertakings have a very important role, in addition to the railway infrastructure, in terms of: reliability, frequency, the timetable, traffic speed, safety, the organization of work in railway stations, competitive prices in the transport market, and so on. In a large number of countries in the present conditions, transport is mainly performed by the national operators that have emerged from the transformation, i.e. division of railway companies. Mostly, these companies are managed by the state. The liberalization of the railway transport market implies, above all, free and non-discriminatory access to the railway infrastructure, bearing in mind the fact that the transport function is performed by a larger number of operators on the appropriate national railway network. The efficiency of transport activities significantly affects the profitability of the business of all the entities involved in the process, but they cannot be provided without much effort in the process of quality management and transport activities. Given the fact that, in modern companies, it is necessary to constantly measure the causes of the achieved effect, it is quite clear that the system for performance/efficiency measurement of the railway operator must include all the criteria that affect it. In order for railway companies to successfully operate, it is very important for them to form a performance/efficiency measurement system appropriate to modern business conditions. Railway operators' operations in today's dynamic and competitive intensive environment require the precise and constant measurement of non-financial criteria, which are identified as the causes of the financial result, so that potential negative trends can be corrected before their effect negatively affects the final result of such operations, which, as a rule, is evaluated from a financial perspective. The subject matter of this research paper stems from the needs of the European countries, regardless of whether they are EU member states or the states applying for the membership, and its aim is to establish the market principles of business in the railway sector. Bearing in mind the fact that the efficiency of railway transport depends on the number of the services offered and the content of the services that have been implemented, it is necessary to determine the criteria that can define efficiency. Based on a detailed analysis of the situation in the research field, a fact was established that the methodological procedure for selecting the key criteria for the evaluation of the efficiency of railway undertakings is not sufficiently researched. For this reason, the objective of this research was to define and evaluate the criteria that affect the efficiency of railway undertakings and propose an approach based on the DEA method for the assessment of the efficiency of railway undertakings in order to increase competitiveness. The contribution of this paper reflects in the criteria selection approach and the evaluation of the efficiency of railway undertakings through the proposed DEA approach. Increasing the revenue, quality and scope of services and reducing the operating costs of the railway undertakings themselves can be improved by applying the proposed efficiency assessment approach.

2. Research Methodology

In addition to general scientific research methods (analysis, synthesis, induction, deduction and analogy), various methods and techniques were used to assess the efficiency of freight transport railway undertakings, such as the Fuzzy Analytical Hierarchical Process (FAHP) and Data Envelopment Analysis (DEA). The research itself was conducted in several phases (Figure 1). The first phase of the research was carried out through several mutually conditioned steps. The initial step in this paper was to identify the problem. Once the problem was identified and the importance of the efficiency of freight transport railway undertakings was determined, the subject matter of the research was defined, together with its objective. The second phase of the research covered an analysis of the literature, scientific and professional information on the railway system for the railway undertakings from the Western Balkans, Slovenia and Croatia, together with the aspects of efficiency measurement, as well as the criteria used. Based on the research done in the most frequently used criteria for the efficiency of railway undertakings from the available literature, the authors defined five groups of criteria. The additional difficulties in the implementation of these tasks imply the mutual influences and conditionality of the mentioned criteria. Thus, for example, the criteria selection problem, which is the initial problem, in a situation of conflicting goals, gives the level of measuring efficiency an additional importance. To select the priority criteria, the Fuzzy Analytical Hierarchical Process (FAHP) was used, which is supported by the literature fact that this method generates the results that are more precise than those obtained by the

AHP method. The third phase was the "core of the research study". In this phase, the previously defined problems related to the evaluation of the efficiency of railway undertakings were solved. A new DEA approach was proposed so as to assess the efficiency of a group of freight transport railway undertakings, which can greatly help in the function of increasing the competitiveness of the railway undertakings. In the fourth phase, the testing of the proposed DEA approach was performed on the selected/proposed (examples of) railway undertakings, with an analysis of the obtained results. This paper provides concluding remarks, as well as directions for future research.

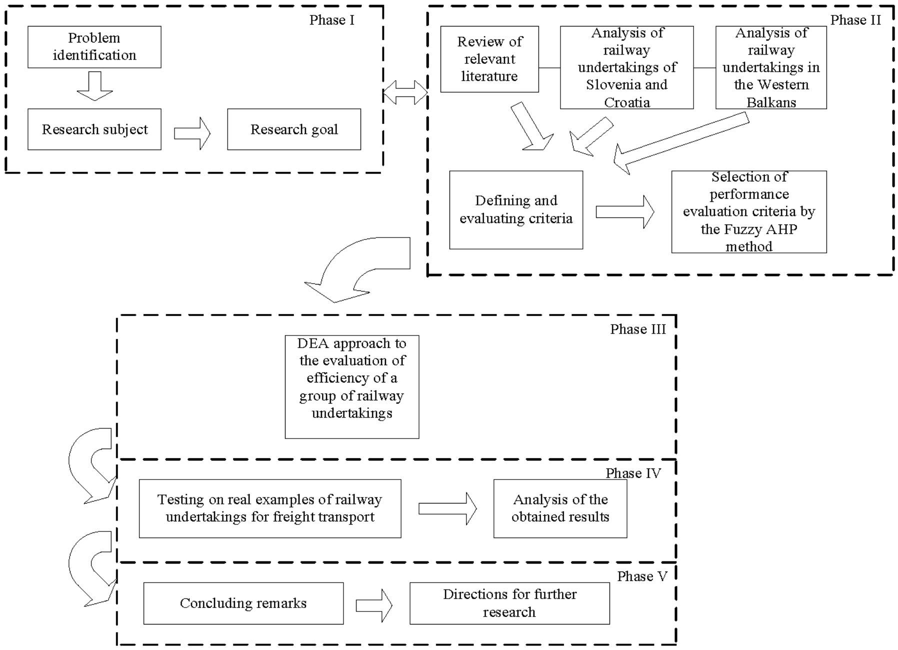

Figure 1 The research methodology

3. The Situation in the Research Area

In the conditions of the global market and increasingly intense competition, the European Union seeks to restructure railways and develop their competitiveness. The European Union is embarking on a comprehensive process for the restructuring and commercialization of rail transport, enabling the reaffirmation and improvement of rail quality and rail efficiency. The starting documents for the achievement of the objective are the Railway Plan, the Freight Charter, Directive 2004/51/EC, the European Technical Strategy for Railway Undertakings (White Paper 1996 and 2011). The main objective of the EU documents is to enable railways to be competitive in the transport market. According to the European Railway Technical Strategy, European Rail Infrastructure Management Managers (2008), the efficiency of rail passenger and freight traffic would increase even more than necessary if the overall costs of the company were reduced. The challenging scenario for railways is to facilitate major economic development in the future, which would generate greater demand for passenger and freight transport while maintaining a high level of the public awareness of the environment and reducing carbon dioxide emissions (increased energy efficiency). The scenario of large-scale economic development is the basis of the aforementioned strategy, as well as the need for the rail sector to be cost-effective and offer an attractive transport mode that will meet environmental standards, while introducing sustainable solutions. In order to be eligible, in line with the scenario presented above, the railway must be multi-functional and should reduce the total cost through: a high capacity (passenger and freight kilometers per kilometer of railway); the high reliability of services (an increased percentage of timely deliveries and fewer delays); the low levels of carbon dioxide emissions (tonnes per passenger and freight kilometers); noise reduction; increased comfort and adequate passenger space (the train station); the increased availability of the rolling stock; better information (before and during the trip); better safety (from the moment of entering the station to the moment of leaving it); a stable confidence level (the total equivalent of the lives lost as a result of the system operation). Garcia-Cebrian and Jorge-Moreno (1999) present the results of a study in which, on an example of 21 railway companies, they observed the impact of organizational change on business efficiency (increased revenue, reduced costs, increased productivity). Ehrma NN (2001) points out the fact that the deficit of state-owned railways is enormous and that the issue of the efficiency of companies has become an issue in economic and political debates. Permanent rail deficits also indicate the fact that an excess capacity throughout the industry, with a lack of state-run rail efficiency, could be a major reason for an insufficient or negative return on invested capital. In times when there is a large public debt throughout the world, the state has a natural interest in adjusting railway undertakings and making the capital allocated to them profitable. In the paper (Borenstein et al., 2004), a methodology is proposed to evaluate the performance of service providers. The goals of this paper were to identify the factors that could be used to evaluate the effectiveness of these decision-making units and identify the groups of similar units that develop the same functions and only differ in resource intensity. The analysis included the comparisons of the relative efficiency of several different units, including postal operators in Brazil, using the DEA. The authors indicated the fact that the proposed methodology could provide the useful information that might be helpful for managers in the decision-making process. Ming-Miin Yu and Erwin T.J. Lin (2008) evaluated the passenger and freight technical efficiency, service efficiency and technical efficiency of the 20 selected railways of other countries for 2002. The study found that those measures differed significantly. Because the data envelopment analysis of the multi-active network models the reality of rail operations, a further insight can be obtained and strategies for the improvement of operational performance can be proposed. In his study entitled "An Efficiency Analysis of European Countries' Railways", Pavlyuk Dmitry (2008) uses stochastic boundary analysis to evaluate the efficiency of the rail system in European countries. He views the rail as a system using its length of operating lines, a number of cars and wagons, employees and a market scale such as the population and tourists to carry passengers and freight. The result of the study showed that the rail systems show huge differences in technical efficiency between different countries, as well as between freight and passenger transport within the same country. Friebel, Ivaldi and Vibes (2010) attempted to measure the impact of reforms in European railways on the technical efficiency of the railways. To do this, they used input and output data analysis, applying the Cobb-Douglas function that implicitly assumes a separation between the input and the output. For the input data, they used the length of the lines on the network and the number of the employees, whereas as the output data, they used passenger km and tonne-kilometers, especially for passenger and freight transport. They worked on a sample of 11 European countries for the period 1980-2003. The three types of the reforms that have taken place in Europe (namely, separation, entry of other companies (competition) and the existence of an independent regulatory body) were added to the prior physical data. Their results indicate the fact that the rail reforms have increased rail transport efficiency, and that the reforms have been more successful when applied sequentially rather than all at once. Lan-bing Li and Jin-Li Hu (2011) model rail transport in their paper into the three processes: the production process (the input and the output), the consumption process (consumption/the output) and the earnings process (earnings/consumption), thus creating a unique multi-phase framework for measuring the Chinese railway performance from 1999 to 2008. First, they used the DEA model to evaluate productivity efficiency, consumption efficiency, and earnings efficiency from a statistical point of view. Then, they used the Malmquist TFP index to evaluate production productivity, consumption productivity, and earnings productivity from a dynamic point of view. They also used the average cumulative Malmquist TFP index to evaluate the impact of the management system reform of the Chinese rail system on rail transport in 2005. Jianjun (2012) analyzes the inefficiencies in production and points out the fact that rail transport has the need for the introduction of economical production by changing the way transport is organized by improving internal contractual relations and optimizing the business organizational structure, the rational use of resources, and an economically significant improvement of efficiency and effectiveness by creating a new way of economically organizing rail transport. Azadeh and Salehi (2014) define a methodology based on the DEA analysis in order to examine the efficiencies of infrastructure managers and railway undertakings and define deficiencies. The authors state that the level of the durability of the system depends on the amount of deficiencies. The smaller the operating deficiencies between the railway undertaking and the infrastructure manager (the smaller the gap between them), the more efficient the company will be in terms of challenges and difficulties in actual operations. Marchetti, D, & Wanke, P. (2017) use the DEA analysis to assess the efficiency of the Brazilian railway concessionaires between 2010 and 2014, when new competition regulations were introduced. The public policies designed so as to increase cluster efficiency are presented, and the options such as increasing, decreasing and magnitude inputs, restructuring, the best management practices, and infrastructure improvements are addressed. Kapetanovic, M. et al. (2017) use the DEA method to evaluate the efficiency of the railway undertakings of the majority of the European countries over the most recent period of time, analyzing the different input-output configurations of the model.

4. The Definition and Assessment of the Criteria for the Evaluation of the Efficiency of Freight Transport Railway Undertakings

Deciding on the selection of the criteria for the assessment of the efficiency of railway undertakings is a very complex process and belongs to the domain of strategic decisions. The adoption of this decision is in the function of managing a railway undertaking and, as such, this activity is complex, creative and permanent. In order to decide on the selection of the criteria for the assessment of the efficiency of railway undertakings, it is necessary to evaluate the proposed variant solutions of different criteria. How to evaluate them is the key issue in determining the method. There is a wide range of the criteria that can be studied when speaking about the efficiency of freight transport railway undertakings. In most cases, there are several criteria that are very often conflicting with one another. To select the best evaluation method or make the best decision when selecting criteria, previous experience and the literature in this field indicate the fact that the problem should be addressed by using multi-criteria decision-making methods. In this paper, one of today's most popular decision-making methods – Fuzzy Analytic Hierarchy Process (FAHP), is experimented with.

4.1. Fuzzy Analytic Hierarchy Process (FAHP)

The Analytic Hierarchy Process (AHP) method, developed by Tomas Saaty, is widely spread and has been in use for over 25 years, with a number of pieces of software developed to support its application. This method is a tool in decision- making, designed to enable decision-makers solve complex decision-making issues, involving a larger number of decision-makers, a greater number of criteria, and multiple time periods. The detailed explanations of this method are provided in many references dealing with decision theory. In this regard, the paper presents a new approach to the AHP method by using interval fuzzy numbers and the application of the modified fuzzy AHP method in defining and evaluating the criteria that influence the evaluation of the efficiency and effectiveness of railway undertakings. Different methods for transferring the previously mentioned AHP method into its fuzzy form are presented in the literature (Bottani, 2005). In addition, the paper (Van Laarhoven and Pedrcyz, 1983) proposes the first study that introduces the principles of fuzzy logic in the AHP method, using triangular fuzzy numbers. At the same time, a study by Buckley (1985) initiates the fact that trapezoidal fuzzy numbers express decision-makers' assessments, while the authors of the study (Boender et al., 1989) present a modification to the fuzzy multi-criteria method proposed in Chang's paper (1996). In the study (Chang, 1996), the severity of the criteria is calculated as the minimization of the logarithmic regression function. In this manner, weight alternatives are calculated by each criterion separately, while the aggregation of calculated weights can determine the fuzzy final result of the alternative. The study (Cebi and Bayraktar, 2003) presents a new approach to solving the AHP phase (FAHP) by using triangular fuzzy numbers. This approach is called an extended analytical method, which can be summarized as follows: define the association function for each attribute and sub-attribute, then calculate their degree of association, and ultimately apply the AHP phase for weight aggregation. Also, Vesković S., et al. (2015) apply the FAHP to evaluate the criteria for public transport obligations. Fuzzy sets generally use triangular, trapezoidal and Gaus fuzzy numbers, which convert uncertain numbers into fuzzy numbers. Using more complicated fuzzy numbers, such as trapezoidal or Gaus, allows a more precise description of the decision-making problem. To solve the problem of defining and evaluating the criteria for the assessment of the efficiency and effectiveness of railway undertakings, triangular fuzzy numbers (Chang, 1996) are used in this paper.

4.2. Criteria for the assessment of the efficiency of freight transport railway undertakings

In the process of defining the DEA approach to efficiency evaluation, it is necessary to consider and define the criteria that affect the efficiency of a railway undertaking. The criteria are chosen so as to allow for the evaluation of the efficiency of railway undertakings. For the purpose of defining and evaluating the criteria, research in the most frequently used literature criteria regarding the efficiency and effectiveness of railway companies was carried out. Based on the conducted research, it was concluded that the used criteria could be categorized into the following criteria groups: the resource criteria (capacity), the operational criteria, the financial criteria, the service quality criteria and the safety criteria. The management of railway undertakings can monitor partial activities and processes with the help of these criteria, but they cannot acquire a complete picture of how the whole system works. It is necessary to define an integrated measure that will somehow integrate all of these criteria. Such a measure would provide a much quicker and more comprehensive picture of how the system works and define appropriate corrective actions as well. The first phase involves the defining and grouping of the criteria. It is desirable at this stage that the information on how the analyzed system works should be used. It is also necessary to group the criteria by the type, by the subsystem they belong to, and by the decision level. Accordingly, a broader set of criteria need to be defined. There are different ways to group criteria in the railway system. In terms of the measurement level, it is possible to define criteria at the strategic, tactical and operational levels. Railway systems are complex systems with numerous interconnected subsystems, processes and activities. Each subsystem, process or activity is characterized by certain criteria. Based on the literature and knowledge, the following criteria of the freight transport operator are defined and shown in Table 1.

Table 1. The criteria for the assessment of the efficiency of freight transport railways undertakings

Group	Criteria
Resource criteria (capacity)	The network length The number of staff per km of the railway network The number of employees
Operational criteria	Commercial speed for freight trains The quantity of transported goods/freight Net tonne km Gross tonne km Train km
Financial criteria	Total income Profit per employee Electricity costs Fuel costs Railway infrastructure charges
Service quality criteria	The suitability of the available services The stability of services The reliability of services (the overdue delivery time) Available rolling stock
Safety criteria	The number of serious accidents per train km The number of accidents per train km The number of incidents per train km

The essence, meaning and reasons of each criteria group are explained further in the paper.

1) The Resource (Capacity) Group Criteria. The first group of the criteria was considered based on the network length, the number of staff per km of the railway network, and the total number of employees and the available number of the rolling stock of railway undertakings. The efficiency achieved by freight transport railway undertakings by carrying out their activities depends on the results of the work accomplished using resources (the capacity). There is a need to understand the state of the resources and the extent to which the resources have been used. The network length criterion relates to the characteristics of the network and greatly affects the efficiency of railway undertakings; namely, it is important for railway undertakings that railway networks should be branched and well connected. In addition, it is important that it should be well connected with international lines. Our railway networks are small and dense, with highly aligned timetables. The density of the network is significantly reflected through the accessibility of the rail service. *The number of employees* is one of the most sensitive segments of the railway sector restructuring process. The economic transition of the Central and Eastern European countries has resulted in very large differences between the individual systems of railway undertakings. It is actually easy to find the causes in some country-specific or group-of-counties-specific processes, for example: the successfulness of the restructuring of the extractive and heavy industries, the privatization and growth of road transport, the collapse of economic blocks (e.g. Yugoslavia), and the impact of military conflicts. In such circumstances, there is a simultaneous redundancy and shortage of labor. Railway companies' systems are burdened with a substantial excess of staff, which is increasingly evident due to the negative trend of rail transport, while on the other hand, there is a deficit of the labor force that has the knowledge and experience needed to meet new market demands. The number of employees is an important component of the efficient operation of railway undertakings, because low costs are the basis for the achievement of competitive advantages today. Fixed and operating costs of business are under increasing pressure and generally record growth trends. Railway undertakings are, by their very nature, a labor-intensive industry, which means that one of the main cost drivers is the cost of employees. This statement assumes an even greater weight given the fact that almost all transition countries, or their railway systems, have insufficient productivity in relation to the number of employees.

2) Operational Group Criteria. The second group of criteria was considered on the basis of the commercial speed of cargo transport trains, the quantity of the goods transported, net tonne and gross tonne kilometers, as well as driving kilometers. Commercial speed can be viewed as operational and as a quality service criterion. The efficiency of freight transport railway undertakings is indirectly dependent on commercial speed and the retention time in railway stations. Taking into consideration the fact that organizational measures cannot significantly affect the speed and time of travel during the circulation of the car, it can be concluded that, according to this criterion, the development of railway traffic depends on the retention time, i.e. on the criteria that can be influenced by organizational measures. In other words, lower retention times mean fewer circuits and more efficient transport. In the conditions of the further development of railway transport and the growing demands the economy and the population pose in terms of the speed of travel or the transport of goods, the speed of transportation means will play an increasingly important role in transport users' decision-making when selecting a type of transport. Therefore,

transport speed will certainly be one of the most important factors, which must be taken into consideration when conducting comparative analyzes of the efficiency of railway undertakings. The criteria of the production task, the transport of goods, as the main activities of railway undertakings, are expressed through the quantity of the goods transported. Railway undertakings generate certain revenues through the criteria that give the opportunity to see the amount of the work done. In the transport of goods, these are net tonne kilometers (the product of the mass of the goods transported in tonnes and transport distances). Railway undertakings do certain work in net tonne kilometers, which is considered to be a transport service for which the price for the net tonne kilometer is charged.

3) Financial Group Criteria. The third group of criteria was considered on the basis of the total revenue, earnings per employee, electricity costs, fuel costs and charges for the use of the railway infrastructure. Railway undertakings achieve income through the sale of products and services. The main activity carried out by railway undertakings is the transport of goods, and revenues from this activity are defined as transport revenues. In this sense, income is a reliable criterion of efficiency, as well as a precondition for the survival of the company. If a company generates no revenue, then it cannot survive on the market. Hence the obligation of railway undertakings to fully understand the function of demand for their services, because in this way they can assess the level of income they strive to achieve or they do achieve. A company's total income is realized as the product of the transport service and the price of the service. For the transport service as a specific product, the ratio of the consumed production factors (production costs, services) and the realized revenues is all the more significant, since production also simultaneously produces its final consumption, realizes the effects of investment in the transport process and achieves production goals (the financial result of the operations of railway undertakings). Transport costs are defined as the value of the factors consumed in the transport service production process or in the goods transport process. In this sense, according to the economic essence of the transport service production process, the basic structure of transport costs includes the costs of labor, which are a very heterogeneous group of investments in the transport process, which consist of the costs of electricity and the costs of fuel. The amount of these costs for a certain volume of production and the technological labor process is conditioned by objectively standardized consumption according to the quantity, the structure and values in a certain real time, and affects the evaluation of the efficiency of railway undertakings to a great extent. The costs of charges for the use of the railway infrastructure directly affect the situation on the transport market. Newly-introduced charges affect the position and role of domestic railway undertakings on the market. The survival of domestic railway undertakings depends on their conditions (the state of technical means, technology, organization, the commercial sector, etc.). When a domestic undertaking is/domestic undertakings are able to provide an appropriate level of the quality of the transport service, high charges will discourage competition on the railway market. If charges are high, the private sector will have no interest in introducing new railway undertakings. No foreign railway undertakings will come to the countries and railways where these charges are high, either. On the other hand, low charges increase the number of railway undertakings and win better-equipped, more capable, more competitive carriers on the free market. This is particularly true for countries in transition and countries where charges have just been introduced. In the countries and rail markets that are underdeveloped and where domestic railway undertakings cannot provide an

appropriate level of the service quality, the situation is just the opposite. There, high charges can only bear the bargain, which is usually a foreign railway undertaking, so it "chokes" domestic railway undertakings. Low charges stimulate competition, and in equal conditions, again, it will be difficult to "defend" domestic railway undertakings. To conclude, fees directly affect the evaluation of the efficiency of railway undertakings.

4) Service Quality Group Criteria. The fourth group of criteria was considered based on the suitability-ability of the offered services, the stability of services, the reliability of the service – exceeding the delivery deadline and the available number of the rolling stock of railway undertakings. The service quality is what constitutes the mirror of railway undertakings, what the customer sees as their image. The customer sees no business premises, no equipment, no technology, no management system and no organizational structure. Everything the customer sees is the quality of the transport service. The quality of the services rendered by railway undertakings lies in the key competences, i.e. sustainable competitive advantages, in relation to other railway undertakings, and significantly influences the assessment of the efficiency of railway undertakings. The convenience ability of the offered services is the criterion whose goal is to adapt railway undertakings to the requirements of service users in terms of the required capacity, mobility and elasticity in order to satisfy the requested service. Reliability is the core of the quality of a railway undertaking’s service, bearing in mind the fact that reliability appears as the most significant qualitative feature from the user's perspective. Research shows that there is a significantly higher reliability effect, as a measure of quality, on the satisfaction of service users than product users. This is particularly due to the specific nature of the transport service: the user's insolvency in the production process and the synchronization of the production and consumption processes, which makes it difficult at the same time to measure and maintain the default level of the service reliability. Thus, the level of the railway service reliability is very important for railway undertakings. The available number of the rolling stock is one of the key criteria for the competitiveness of railway undertakings in the open transport market. It can be seen as the service quality criterion and the operational criterion. The rolling stock is the fixed assets of railway undertakings that have the function of the means of work in the transport service manufacturing process. The rolling stock includes traction vehicles, i.e. locomotives, and hauled vehicles, i.e. all types of freight cars. It is of particular importance for a railway undertaking to achieve the optimal capacity, which implies such a use of the rolling stock which will establish the relatively most favorable relationship between the wearing of their useful properties, on the one hand, and their productivity, on the other. The rolling stock should be fast, energy-efficient and environmentally friendly and, above all, secure in order to achieve a higher quality of the service. With the liberalization of the market, there is growing competition between railway undertakings, both in terms of the scope and in terms of the quality of transport services, so it is very important to dispose of modern means of transport.

5) Safety Group Criteria. The fifth group of criteria was considered based on the number of serious accidents, accidents and incidents per driving kilometer. Safety is an important factor in determining the transport user for certain transport sectors, and therefore a significant factor in the size of transport and the volume of income. In addition to the impact on the transport size and the volume of income, traffic safety

affects the efficiency of railway undertakings as a result of railway accidents, damaging and destroying assets of high value, thus causing great material damage and traffic disruptions, which are also a cost for railway undertakings. Serious accidents mean any collision or slipping/derailing of trains resulting in the death of at least one person, or a serious injury to five or a larger number of persons, or a significant damage to the rolling stock (it implies the damage that may immediately be estimated by the railway investigating authority, the total value being at least EUR 2 million), the infrastructure or the living environment, as well as any other similar accident with an obvious impact on rail safety regulation or safety management. An accident means an unwanted or unintentional event or a special chain of events having severe consequences. Accidents are divided into the following categories: crashes, slipping from a rail track (derailing), accidents at a crossing, and accidents to persons caused by the rolling stock, fires and so forth. An incident means any event which is not an accident or a serious accident, which is related to the traffic of trains and which affects the safety of operation. In order to maintain high-level safety, the European Union has laid down the limit of common safety objectives in its documents.

The assessment of the criteria was based on the Fuzzy AHP (FAHP) method. Experts from the railway sector participated in the process of the evaluation of the relative importance of particular criteria for each group. Experts from the Ministry of Transport (E1), the Railway Directorate (E2), the Railway Safety Agency (E3), the Railway Infrastructure Manager (E4) and the Railway Undertaking (E5) were interviewed. They filled out a survey, in which they evaluated the importance of each criterion against the linguistic preference scale for each group. Table 2 shows the conversion of the linguistic variables into triangular fuzzy numbers (Chang, 1996.).

Table 2. The linguistic variables and their corresponding fuzzy numbers

Linguistic variable	Triangular fuzzy scales	Fuzzy reciprocal scale
Just equal	(1, 1, 1)	(1, 1, 1)
Equally important	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly important	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

Solving the highest-importance criteria selection problem for the purpose of assessing the efficiency of railway undertakings between the aforementioned groups was initiated by the application of the FAHP approach. For the illustrated example of the highest-importance criteria selection, an example of the selection of the criteria for the operational group is presented in this paper. In Table 3, a fuzzy matrix of the benchmarking criteria from the operational criteria group (Commercial speed for freight trains – B1, The quantity of transported goods/freight – B2, Net tonne km – B3, Gross tonne km – B4, Train km – B5), is given.

Table 3. The comparative matrix for the operational group criteria of freight railway undertakings

		B_1	B_2	B_3	B_4	B_5
	E1	(1,1,1)	(2/7,1/3,2/5)	(2/3,1,2)	(2/3,1,2)	(2/5,1/2,2/3)
	E2	(1,1,1)	(2/5,1/2,2/3)	(1/2,1,3/2)	(2/5,1/2,2/3)	(2/3,1,2)
B_1	E3	(1,1,1)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(2/3,1,2)
	E4	(1,1,1)	(2/3,1,2)	(2/3,1,2)	(2/5,1/2,2/3)	(2/3,1,2)
	E5	(1,1,1)	(2/7,1/3,2/5)	(1/2,1,3/2)	(2/3,1,2)	(1/2,1,3/2)
	E1	(5/2,3,7/2)	(1,1,1)	(1/2,1,3/2)	(1/2,1,3/2)	(3/2,2,5/2)
	E2	(3/2,2,5/2)	(1,1,1)	(1,1,1)	(1,1,1)	(1/2,1,3/2)
B_2	E3	(3/2,2,5/2)	(1,1,1)	(1/2,1,3/2)	(1,1,1)	(1/2,1,3/2)
	E4	(1/2,1,3/2)	(1,1,1)	(1/2,1,3/2)	(1/2,1,3/2)	(1/2,1,3/2)
	E5	(5/2,3,7/2)	(1,1,1)	(1/2,1,3/2)	(1,1,1)	(3/2,2,5/2)
	E1	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)	(1,1,1)	(1/2,1,3/2)
	E2	(2/3,1,2)	(1,1,1)	(1,1,1)	(1,1,1)	(2/3,1,2)
B_3	E3	(3/2,2,5/2)	(2/3,1,2)	(1,1,1)	(2/3,1,2)	(1,1,1)
	E4	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)	(1,1,1)	(1,1,1)
	E5	(2/3,1,2)	(2/3,1,2)	(1,1,1)	(2/3,1,2)	(1/2,1,3/2)
	E1	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)	(1,1,1)	(1/2,1,3/2)
	E2	(3/2,2,5/2)	(1,1,1)	(1,1,1)	(1,1,1)	(1/2,1,3/2)
B_4	E3	(3/2,2,5/2)	(1,1,1)	(1/2,1,3/2)	(1,1,1)	(1/2,1,3/2)
	E4	3/2,2,5/2)	(2/3,1,2)	(1,1,1)	(1,1,1)	(1/2,1,3/2)
	E5	(1/2,1,3/2)	(1,1,1)	(1/2,1,3/2)	(1,1,1)	(1/2,1,3/2)
	E1	(3/2,2,5/2)	(2/5,1/2,2/3)	(2/3,1,2)	(2/3,1,2)	(1,1,1)
	E2	(1/2,1,3/2)	(2/3,1,2)	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)
B_5	E3	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)	(2/3,1,2)	(1,1,1)
	E4	(1/2,1,3/2)	(2/3,1,2)	(1,1,1)	(2/3,1,2)	(1,1,1)
	E5	(2/3,1,2)	(2/5,1/2,2/3)	(2/3,1,2)	(2/3,1,2)	(1,1,1)

The fuzzy weight of the criteria is calculated by taking the geometric average of the expert's response (Lee, 2009). An example of the geometric mean calculation is only provided for B_{12}, while the other values shown in Table 4 are calculated analogously. An example of the calculation of the geometric mean for B_{12} reads as follows:

$n^- = (2/7x2/5x2/5x2/3x2/7)^{1/5} = 0.387$

$n^- = (1/3x1/2x1/2x1x1/3)^{1/5} = 0.488$

$n^+ = (2/5x2/3x2/3x2x2/5)^{1/5} = 0.677$

Table 4. The fuzzy comparative matrix for the operational criteria group

	B1	B2	B3	B4	B5
B1	(1,1,1)	(0.387, 0.488, 0.677)	(0.536, 0.870, 1.431)	(0.491, 0.660, 1.035)	(0.568, 0.871, 1.516)
B2	(1.477, 2.047, 2.252)	(1,1,1)	(0.574, 1, 1.383)	(0.758, 1, 1.176)	(0.776, 1.319, 1.840)
B3	(0.698, 1.149, 1.864)	(0.723, 1, 1.741)	(1,1,1)	(0.850, 1, 1.319)	(0.699, 1, 1.351)
B4	(0.967, 1.516, 2.038)	(0.850, 1, 1.319)	(0.758, 1, 1.176)	(1,1,1)	(0.500, 1, 1.500)
B5	(0.659, 1.149, 1.759)	(0.543, 0.758, 1.289)	(0.740, 1, 1.431)	(0.667, 1, 2)	(1,1,1)

In addition, the standard steps of the FAHP method are used in this paper (Stević, Ž. et al. 2015). The relative ranking of the importance of particular criteria based upon a criteria-pairwise comparison, for all the groups in freight transport is presented in Table 5.

Table 5. The relative ranking of the importance of particular criteria based upon the criteria-pairwise comparison of all the groups in freight transport

Group	**Criteria**	**W'** (fuzzy weight vector)	**W** (normalized weight vector)
Resource criteria (capacity)	The network length	0.101	0.065
	The number of staff per km of the railway network	1	**0.654**
	The number of employees	0.430	0.281
Operational criteria	Commercial speed for freight trains	0.632	0.151
	The quantity of transported goods/freight	**1**	**0.240**
	Net tonne km	0.841	0.202
	Gross tonne km	0.878	0.210
	Train km	0.821	0.197
Financial criteria	The total income	0.916	0.213
	Profit per employee	0.919	0.214
	Electricity costs	0.816	0.190
	Fuel costs	0.639	0.149
	Railway infrastructure charges	1	**0.233**
Service quality criteria	The suitability of the available services	0.738	0.256
	The stability of services	0.512	0.177
	The reliability of services (the overdue delivery time)	0.638	0.221
	The available rolling stock	1	**0.346**
Safety criteria	The number of serious accidents per train km	1	**0.558**
	The number of accidents per train km	0.473	0.264
	The number of incidents per train km	0.318	0.178

The comparative analysis carried out by using the FAHP method showed that, for each group, the priority criteria affected the assessment of the efficiency of the railway undertakings. Based on the results shown in the above Table 5, a conclusion can be drawn that, for the group of the resource criteria, the greatest relative weight is that of the *Number of staff per km of the railway network* (0.654), only to be followed by the *Operational criteria* group, *The quantity of goods transported* (0.240), the *Financial criteria* group, *The cost of charges for the use of the railway infrastructure* (0.233), the *Service quality* criteria, *The available rolling stock* (0.346) and the *Safety criteria* group, the highest relative weight being that of *The number of serious accidents* criterion (0.558), based on the railway experts' survey. The criteria that took precedence over the other criteria in their respective group(s) were used as the inputs and outputs for the evaluation of the efficiency of freight transport railway undertakings by applying the DEA method.

5. The Application of the DEA Method in Order to Assess the Efficiency of Freight Transport Railway Undertakings

Regardless of the system type, there is a need to monitor and quantify the effects of business. One of the basic criteria implies defining the relationship between the resources invested and the goals achieved. In the literature, that ratio is known as efficiency. Common to most approaches in the literature is the fact that the term "efficiency" pertains to the best utilization of resources while providing as many services as possible. In the literature, the problem of measuring the efficiency of railway undertakings has been emphasized as the problem of measuring the efficiency of multiphase (multistage) processes. The most commonly used method for the evaluation of the efficiency of multiphase processes is the Data Envelopment Analysis (DEA) method. There is a full range of models in the literature intended for the evaluation of the efficiency based on the DEA models. The DEA method makes it possible to compare the efficiency of comparable units, in this case a group of undertakings with a greater number of input and output variables. In this paper, a new approach to the assessment of the efficiency of freight transport railway undertakings is proposed. The proposed approach is based on the evaluation of efficiency by using the DEA method. The implementation/application of the proposed approach envisages several stages. First, it is necessary to define the inputs and outputs for the Decision-Making Unit (DMU), which requires an evaluation of efficiency and effectiveness (in this case, these are railway undertakings). Furthermore, the DEA approach is executed through two parallel processes. The first process implies the classifications of DMUs as either effective or ineffective, depending on the CCR grade (a model named after the initial letters of the surnames of the authors, Charnes, Cooper, Rhodes), and the BCC grade (a model named after initial letters of the surnames of the authors, Banker, Charnes, Cooper, 1984). The second process requires an RTS classification. This enables the identification of the DMUs that need rationalization. Finally, the optimal values for the inputs and outputs are derived by using the slack-based CRS (Constant Returns to Scale) model. The proposed DEA approach is shown in Figure 2.

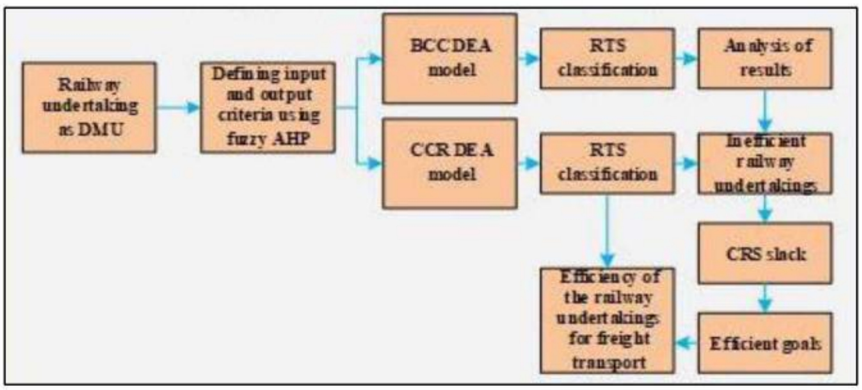

Figure 2. The application of the DEA method for the assessment of the efficiency of freight transport railway undertakings

The proposed DEA approach was tested and verified through a survey conducted on a sample of the national goods transportation railway undertakings in the Western Balkans, Slovenia and Croatia, which is accounted for in Table 6.

Table 6. The freight transport railways undertakings

Country	Railway undertakings	Abbreviation
Albania	Hekurudha Shqiptarë SH	HSH
Bosnia and Herzegovina	Railways of the Republic of Srpska	ŽRS
	Railways of the Federation of Bosnia and Herzegovina	ŽFBH
Montenegro	Montecargo	Montecargo
Croatia	Croatian Railways Cargo d.o.o.	HŽ-Cargo
North Macedonia	Railways of the Republic of North Macedonia Transportation Department J.S.C. Skopje	MŽT
Slovenia	Slovenian Railways-Freight Transport	SŽ-Freight Transport
Serbia	Serbia Cargo	SK

The freight transport railway undertakings as DMUs were designated with four inputs and one output, as previously determined by the Fuzzy AHP and as shown in Figure 3 below. The first input stands for the number of employees per kilometer of the railway network; the second input stands for the cost of the fees paid by the railway undertaking to the payment infrastructure manager; the third input stands for the available number of the vehicles of the rolling stock, and the fourth input stands for the number of serious accidents. The output of the model is the quantity of the goods transported.

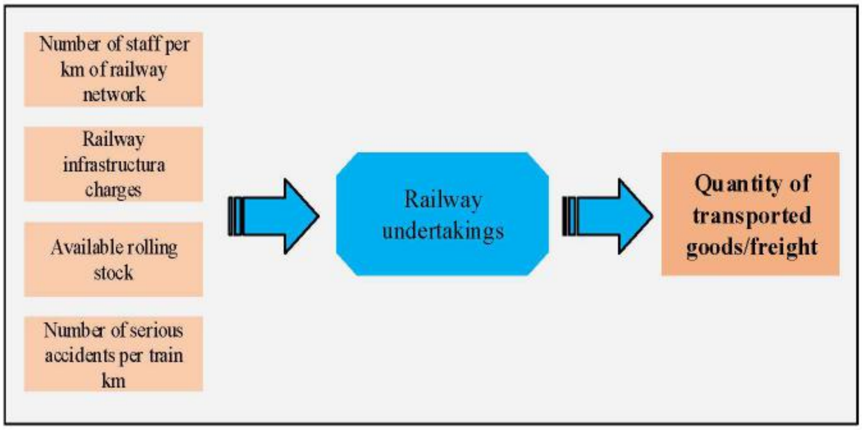

Figure 3. The railway undertaking as the DMU for efficiency assessment

The values for the input and output parameters for all the eight national railway undertakings are shown in Table 7. The data for the railway undertakings were obtained from the UIC statistics and the railway undertakings' annual reports for the year 2018 (https://www.uic.org/).

Table 7. The input and output parameters for the DEA model

Railway undertakings	Number of staff per km of the railway network	Railway infrastructure charges (Euro)	Available number of rolling stock	Number of serious accidents per train km	Quantity of the transported goods/freight (tons)
HSH	2.3	2.2	592	6	930000
ŽRS	4	2.1	2134	6	4568698
ŽFBH	2	2.1	2271	10	9120000
Montecargo	6	3.0	577	5	1000000
HŽ-Cargo	1.1	3.3	5513	7	6870000
MŽT	4	2.0	1353	3	1680000
SŽ Freight Transport	1	2.23	3142	13	20436000
SK	0.7	1.1	6901	6	10160000

The CCR is an original DEA model for the determination of relative efficiency for a DMU group. The one formulation of the CCR model aims to minimize inputs, while maintaining a given output level, i.e. the CCR input-oriented model (Model A1). The second formulation of the CCR model aims to maximize the outputs without increasing the value of any of the observed inputs, i.e. the CCR output-oriented model (Model A1'). The CCR models assume a constant CRS (Constant Returns to Scale), and CCR ratings measure overall efficiency.

Model A1 (primal)	Model A1' (dual)
$\theta^* = min\ \theta$	$\phi^* = max\phi$
With conditions:	With conditions:
$\sum_{j\in\{1,2,3,\dots,n\}} \lambda_j\, x_{ij} \le \theta_{xio}, i = 1,2,3,..,m;$	$\sum_{j\in\{1,2,3,\dots,n\}} \lambda_j\, x_{ij} \le x_{io}, i = 1,2,3,\dots,m;$
$\sum_{j\in\{1,2,3,\dots,n\}} \lambda_j\, y_{rj} \ge y_{ro}, r = 1,2,3,..,s;$	$\sum_{j\in\{1,2,3,\dots,n\}} \lambda_j\, y_{rj} \ge \phi y_{ro}, r = 1,2,3,.,s;$
$\lambda_j \ge 0, j = 1,2,3,\dots,n.$	$\lambda_j \ge 0, j = 1,2,3,\dots,n.$

(1)

If in the models A1 and A1' $\sum \lambda_j = 1$ is added, then the BCC input-oriented and the BCC output-oriented models are obtained, respectively. The BCC models assume the variable VRS (Variable Returns to Scale), and the BCC ratings measure pure technical efficiency.

In the paper (Seiford and Thrall, 1990), a connection was established between the solutions obtained by using the A1 and A1' models. $\lambda_j^*,\ j = 1,2,3,\dots,n$ and θ^* are the optimal solutions obtained with the model A1; then, there are the corresponding optimal solutions, $\lambda_j^{**},\ j = 1,2,3,\dots,n$ i ϕ^* obtained with the model A1', whereby $\lambda_j^* = \frac{\lambda_j^{**}}{\phi^*}$ i $\theta^* = \frac{1}{\phi^*}$. In this paper, the CCR and BCC models are used to investigate the sources of the inefficiency of the railway undertakings. In general, the sources of the inefficiency of railway undertakings may be caused by their inefficient operation or the noncompetitive conditions within which they operate. For this purpose, the Scale Efficiency Score $SS = \frac{Q_{CCR}}{Q_{BCC}}$ is used. This approach describes the sources of inefficiency, i.e. whether it is caused by inefficient work practices (BCC efficiency) or the noncompetitive conditions shown by a proportional efficiency assessment (SS) or both.

There are several approaches in the literature dedicated to the DEA that may be used to evaluate the RTS (Return to Scale) classification. The paper (Seiford and Zhu, 1999a) shows that there are at least three equivalent RTS methods. The first CCR RTS method is that introduced by Banker (Banker, 1984). The second BCC RTS method was developed by Banker et al. (1984), "Some models for estimating technical and scale inefficiencies in data envelopment analysis", Management Science, Vol. 30, No. 1-9, pp. 1078-1092), as an alternative approach to using free variables in the BCC dual model. The third RTS method is based on the Scale Efficiency Index, and the same is proposed in the paper (Fare et al., 1994a). The CCR RTS method is based on the sum of the values of the dual variables λ_j in the CCR model, and the same was used for the RTS classification of the observed railway undertakings.

The methods for the estimation of the RTS classification in the DEA provide important information about possible input and output data perturbations in a DMU analysis. This information may have a positive effect on the result achieved by the DMU. They allow ineffective DMUs to determine guidance in order to improve efficiency.

The problem of the determination of the optimum values for the inputs and outputs of those DMUs that demonstrate inefficiency can be solved by using additive DEA models. These models can simultaneously set effective goals to be pursued. This allows those DMUs that demonstrate inefficiency to achieve the optimum input/output ratio (Ralevic, 2014). The optimum values for each input and output separately can be calculated by determining input and output slots. The results and the analysis of the real-example model test results are presented further in this paper.

6. Analysis of the Research Results

Using the model A1, relative efficiency was developed for the observed group of the 8 freight transport railway undertakings. The CCR and BCC characteristics for each railway undertaking are given in Table 8.

Table 8. The evaluation of the efficiency of the freight transport railway undertakings

Railway undertakings	Efficiency evaluation by the CCR model	Benchmarks	Efficiency evaluation by the BCC model	RTS classification	Scale Score (SS)
HSH	0.242	SŽ- Freight Transport	1	Increasing	0.242
ŽRS	0.480	SŽ- Freight Transport, SK	0.933	Increasing	0.514
ŽFBH	0.617	SŽ- Freight Transport	0.988	Increasing	0.624
Montecargo	0.266	SŽ- Freight Transport	1	Increasing	0.266

HŽ-Cargo	0.597	SŽ- Freight Transport, SK	0.955	Increasing	0.625
MŽT	0.350	SŽ- Freight Transport, SK	1	Increasing	0.350
SŽ- Freight Transport	1		1	Constant	1
SK	1		1	Constant	1
Average	0.569		0.984		0.578

The results presented in the table show that there are two railway undertakings with the CCR ratings equal to 1. This rating measures the overall efficiency when a constant RTS is assumed. These are the railway undertakings of the Slovenian Railways – Freight Transport and Serbia Cargo. These railway undertakings can be seen as realistic and useful benchmarks for the other inefficient railway undertakings. Slovenian Railways – Freight Transport is one of the two undertakings with the best result. In addition, it is the undertaking that is apparently considered to be a benchmark. The railway undertakings rated below the average (0.569) are considered to be inefficient. Each railway undertaking is distinguished by its specific characteristics in rail transport; nonetheless, the railway undertakings should be open to improving performance and there should be one or more railway undertakings as an example for them to follow. The selection of the relevant benchmarks was derived from the calculation of the CCR DEA model by using the values obtained for the dual variables. The results shown in the fourth column of Table 9 show, for each inefficient railway undertaking, another railway undertaking suitable for comparison out of the set of the efficient ones. The BCC rating measures efficiency by assuming the variable RTS. In this empirical study, there are five railway undertakings awarded the BCC effective status, in addition to the two already retaining their previous effective status. For example, it can be concluded that the railway undertakings Hekurudha Shqiptarë SH., Montecargo and the Railways of the Republic of North Macedonia Transportation Department J.S.C. Skopje are efficiently operated, i.e. $(\theta_{BCC}^{*} = 1)$. In addition, it can be considered that the Railways of the Federation of Bosnia and Herzegovina have a BCC rating above the average, which means that they have good operating efficiency. Based on the results of the proportional efficiency evaluations, these are the railway undertakings with a good ratio of the achieved work result and the engaged resources (work in competitive conditions): SŽ-Tovorni promet, Srbija Cargo, Railways of the Federation of B&H and HŽ Cargo. Their relative efficiency scores are higher than the average value (0.578).

7. Conclusion

Measuring and improving the efficiency of the operations of railway undertaking are a precondition for their successful business and survival on the market. Measuring the efficiency of a company is one of the key managerial activities in modern companies that provides us with an insight into the current status of the company, the goals to be achieved in the future, as well as its current position on its way towards the achievement the set goals. Such a system is undoubtedly of strategic importance for every company that wants to survive and develop in today's conditions. Therefore, such a system must adequately be integrated into the strategic management system. Efficiency has a positive impact on a number of other important criteria pertaining to the work of railway undertakings, such as a better use of resources, a more rational use of energy, increased safety, an increased quality of service and so on. In order to evaluate the proper performance of operations in goods rail transport, i.e. the efficiency of railway operations, it was necessary to define and determine appropriate criteria. In this paper, group criteria are defined and evaluated, and priority criteria are selected for the purpose of evaluating the efficiency of freight transport railway undertakings based upon multi-criteria decision making and the fuzzy AHP method.

From each group, the used FAHP method revealed the priority criteria for the assessment of the efficiency of railway undertakings. The criteria that achieved an advantage within the composite normalized range over the other criteria from their respective group for the freight railway undertakings are as follows:

- from the resource criteria group, the number of employees per kilometer of the railway network has the highest relative weight;
- from the operational criteria group, it is the quantity of the transported goods;
- from the financial criteria group, it is the costs of the fees for the use of the railway infrastructure,
- from the service quality group, it is the available number of vehicles, and
- from the safety criteria group, it is the number of serious accidents that has the highest relative weight.

The DEA method was chosen so as to evaluate the efficiency of the railway undertakings, because it enables an analysis of mutually comparable units despite heterogeneous data, expressed by different units of measurement and affecting business efficiency in different ways.

An approach to the assessment of the efficiency of freight transport railway undertakings by using the DEA method is proposed, which enables the aggregation of all the groups of criteria into a single efficiency assessment, thus also providing information on the corrective actions that can improve the efficiency of railway undertakings. The paper evaluates the efficiency of the freight transport railway undertakings performed on the basis of the selected priority criteria and by using DEA excel solvers, using the CCR output-oriented model (the model assumes constant return in relation to the investment volume) and the BCC output-oriented model (the model assumes a variable return relative to the volume of investment/input). The output criterion on the basis of which the efficiency of railway undertakings was evaluated was the quantity of the transported goods. The output used in the analysis is a realistic one. The proposed approach based on the DEA method was tested and verified through a survey conducted on a sample of eight freight transport railway undertakings.

The model testing results show that there are two railway undertakings with the CCR ratings equal to 1, which is to say that this rating measures the overall efficiency when a constant RTS is assumed. These are the railway undertakings Slovenian Railways – Freight Transport and Serbia Cargo. These railway undertakings can be seen as realistic and useful benchmarks for the other inefficient railway undertakings.

Thus, the railway undertakings demonstrating good efficiency appear as benchmarks for those inefficient railway undertakings. The Slovenian Railways - Freight Transport has the best result. Also, it is the railway undertaking that appears most as a benchmark. The selection of relevant benchmarks was derived from the calculation of the CCR DEA model by using the values obtained for the dual variables.

Based upon the results of the BCC evaluation that measures efficiency under the assumption of the variable RTS in this research, it can be concluded that five railway undertakings out of the observed eight are efficiently operated.

These are the railway undertakings Slovenian Railways – Freight, Serbia Cargo, Hekurudha Shqiptarë SH., Montecargo and the Railways of the Republic of North Macedonia Transportation Department J.S.C. Skopje. The results show, for each inefficient railway undertaking, which railway undertaking is suitable for comparison with it from the set of the efficient ones. Each railway undertaking is characterized by its specific characteristics in rail transport; nonetheless, a railway undertaking should be open to improving performance and there should be one or more railway undertakings as an example for it to follow in order to improve its efficiency.

This paper opens a possibility of channeling the research into a narrower scientific field, which could be the identification of the new criteria that may affect the redefinition of the models and the development of the new models that would combine the proposed approach with other approaches, such as fuzzy logic, simulation, optimization models and others. In that manner, certain limitations would be overcome and the railway undertaking efficiency evaluation process would be improved.

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QUALITY IMPROVEMENT OF REMANUFACTURING LIFT ARM USING SIX SIGMA METHODS IN THE HEAVY-DUTY INDUSTRY IN INDONESIA: A CASE STUDY

Saryanto*, Humiras Hardi Purba, Aris Trimarjoko , Fuad Fatahillah

Department of Industrial Engineering, Mercu Buana University, Jakarta, Indonesia

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Abstract: *The high remanufacturing forecast reaching 160 billion dollars/year in the world of the equipment industry (heavy duty) is a promising business opportunity. However, the remanufacturing industry has a higher risk of product failure compared to original Equipment products. The remanufacturing of the heavy-duty industry in Indonesia in carrying out its production has a product failure rate of 834586,47 DPMO and is at 1.91 sigma with COPQ IDR 650,800,000.00 Six Sigma method is used in this research and is successful in reducing remanufacturing defective product for lift arm to 140762,5 DPMO, is at the level of 2.43 Sigma and COPQ IDR. 135,000,000.00 or decreased 78.71% from the previous condition.*

Key words: *Quality Improvement, Remanufacturing, Six Sigma, Product Failure*

1. Introduction

1.1. General

The Remanufacturing industry has been around for at least 28 year and provide significant economic, social and environment benefit. Strategy 3R (Reduce, Reuse and Recycle) ,system was founded in the USA and remanufacturing operation have grown substantially and become common practice in many Industries. In development countries, the remanufacturing engineering also developed rapidly and has been applied to industry. The United States Environment Protection Agency (EPA) implemented a Comprehensive Procurement Guidelines (CPG) program to enact waste reduction and resource conservation through the Reuse of used materials and ensuring recycling programs for certain materials can be made into materials to create new products. Reman world Magazine, March/April edition, 2018 states the remanufacturing industry is spreading in various countries with a total forecast of $ 160 billion/year with spread: the USA $ 100 billion, Europe $ 32, Asia $

* Corresponding author.
Saryanto.01@yahoo.com (Saryanto). humiras.hardi@mercubuana.ac.id (Purba, H. H.), aristrimarjoko@gmail.com (Trimarjoko, A) ah.fuadf@gmail.com (Fatahillah, F)

27 billion and Brazil $ 1.4 billion. The Asian region (including Indonesia) ranks third in the distribution of the remanufacturing industry.

The high remanufacturing forecast reaching 160 billion dollars/year in the remanufacturing industry is a promising business opportunity. However, the remanufacturing industry has a higher risk of product failure compared to original equipment products. In Indonesia, the remanufacturing industry, which is engaged in the Heavy-duty equipment industry, supports the repair of mining equipment and vehicles both in open pit and underground in running its production, experiencing a product failure rate of 834586.47 DPMO if the capability of the process is measured at the level of 1.91 sigma and the Cost of Poor Quality must be borne by USD. 43386.57 for January ~ May 2019. Valles et al., 2009 state that Six Sigma is a strategy of continuous organizational improvement to find and eliminate the causes of errors, damage, and delays in business organization processes. Gijo et al., 2014 with the application of the Six Sigma method resulted in a reduction intolerance related to problems and increased yield values from 85% to more than 99%. Get a total savings of US $70,000 per year. Hassan, 2013 shows that, for the calculation of the yield value of 95.75%, from this result, the sigma level was calculated and found an initial sigma value of 3.22 and a DPMO of 42,500. Using a target of a 2% defect rate, the target sigma value is calculated to be 3.55 and the DPMO value is 20.000. The results achieved 98.24%, according to the sigma level of 3.6 and the DPMO value of 17.600. Referring to various studies on problem-solving with the help of Six Sigma methods showing positive results that are marked by decreasing product failure rates and increasing sigma levels, then in this study, Six Sigma methods are expected to be used in the failure of remanufacturing lift arm products in the Heavy-duty industry in Indonesia to be reduced so that the process capability is getting better, COPQ can be suppressed and will certainly increase company profits.

1.2. Motivating of research

The companies engaged in manufacturing tools in Indonesia that support the repair of mining equipment and vehicles both in the open pit or underground. Companies are required to make innovation efforts so as not to lose their market share. Consumers always want innovative products, because their tastes and needs tend to change with the changing times. Products that consumers want are products that are not only able to meet their needs but are also able to provide satisfaction for their users. The activity carried out is to suppress as little as possible the name of the product defect to zero defect. In line with the principle of zero defects, Remanufacturing Companies engaged in heavy equipment, have full attention to this matter. Evidenced by improvement activities carried out by all company employees to reduce product defects. Lower-level employees (operators) to the top level of management, improvement activities carried out continuously. This research was conducted to examine the level of disability in the Machine Rebuild section with the Machining and Welding process in the company. The section is the final section of the process in the production process, where the level of disability is still high based on the 2018 Machining and Welding quality reports. Based on internal data of the 2018 Machining and Welding product defects, 1.99% with types of defective products as follows: Lift Arm (73.5%), Bucket (7.9%), Front Frame (3.3%), Rear Frame (3.3%), Tilt Lever (3.3%), Tilt Link (2.6%), Cabin (2.0%) and others (4.0%). Based on the 2018 defect product data, this study is motivated to reduce the Lift Arm product failure of the Machining and Welding process which has the highest accumulation of defective products by 73.5%, with the hope that Lift Arm quality can be improved to meet customer satisfaction and provide a more optimal company advantage.

2. Literature review

2.1. Quality Improvement

The purpose of quality control is to make the final product produced according to product specifications and standard sets (James, 2012). Speaking of quality, of course, there is no definite understanding of quality and quality has a broad scope and has a different understanding (Suwendra, 2014. Quality in terms of producers is the fulfillment of quality standards that have been owned (Purba, 2017). In addition there are several objectives for quality control, namely: (1) To improve the uncontrolled process, (2) To control the finished product, in this case it is done by taking the sample of the receipt, (3) To produce quality products, (4) Work for inspection or inspection cost to minimize, (5) strives to reduce the cost of product design and processes using certain production quality, and (6) Make sure the cost of production is minimized as low as possible (Cullison et al., 2013). For some of the widely used quality features, among others : (a) Quality is compliance with requirements or claims, (b) Quality is a match with use, (c) Quality is continuous improvement and improvement, (d) Quality is an effort to meet the needs of consumers from the beginning and at all times, (e) Quality is something that can satisfy the user (Chunxioa et al., 2013). Quality can generally be interpreted as a measure of quantity that indicates the stage of the good of a product, or can be interpreted as the best condition within certain limits in accordance with the will of the consumer. In general, the conditions required by consumers as the most important are product prices and product benefits. The two things are related: a. Specification of operating characteristics, b. Product age and reliability, c. Manufacture of product, d. The condition in which the product is made, e. Installation and maintenance of products and facilities in the field (Milln et al., 2013). So briefly the quality can be defined as satisfaction in the use of products that include aspects of: Product quality: The quality of the product or service Cost quality: Quality of cost, Delivery quality: Quality delivery products, Safety quality: Safety quality, utility of spirit: Quality in serving customers (Pylväs et al., 2015). Referring to the definition of quality, the improvement of product quality to increase customer satisfaction is an important attribute in a business organization (Nugroho, 2015).

2.2. Remanufacturing industry

In 2005 the remanufacturing industry began when the United States Environmental Protection Agency (EPA) implemented a comprehensive procurement Guidance program (CPG) to enact waste reduction and resource conservation guidelines through the Reuse of waste materials and ensuring recycling programs for certain materials can be made into materials to create new products. In 2004, the EPA established several remanufactured vehicle parts. Remanufacturing is to use a portion of its original form and replace or rebuild damaged parts. The testing process follows the same specification process as the new product manufacturing process (https://www.epa.gov/).

The remanufacturing industry is an industry that uses a portion of its original form/original equipment to rebuild damaged parts and replace it with new equipment through the testing process following the same specifications as the new product manufacturing process. The quality of Re-manufacturing products has the same standard as the manufacturing of new products (Ijomah, 2008; Ijomah and Childe, 2010). One of the complicating characteristics in remanufacturing is the stochastic and sporadic nature in the condition and quantity of the returned cores which impacts on many levels in the planning and control (Junior et al., 2012). Returned products can range from minor scratches to extensive damage and thus inspection and sorting procedures are required to filter the valuable cores. High quality returns are preferred as the quality of the returns determines the level of the remanufacturing effort required, the processing time, the rate of remanufacturing success, the process sequence used, the amount of cost savings, and the amount of cores being scrapped (Ortegon et al., 2013). The extent to which remanufacturing is done and the definition of sufficient quality depend on the type of remanufacturers and the business model; independent remanufactures try to repair as many parts as possible, whereas OEM remanufacturers can be more selective on the cores to accept. Reliable engineering expertise and capabilities is the backbone to a successful remanufacturing facility. Remanufacturing depends extensively on the skills of the technicians and the knowledge base related to the cores and their restoration (Ijomah. 2009).

2.3. Six sigma

Quality Six Sigma is a business strategic management which originally developed by Motorola in 1986 in order to enhance the quality of products through decreasing of product variations on manufacturing operations as they face compete in semiconductor industries. Through the application of the Six Sigma method, Motorolla has acknowledged an award of Malcolm Baldrige in 1988 as the first American's company which won its prestigious quality's award (Parsana et al. 2014). Quality in terms of producers is the fulfillment of quality standards that have been owned (Purba, 2017. According to these facts, by considering an obtained quality level for only 99 percent or 1 percent of defect levels on such cases in manufacturing industries or services can potentially lead to fatalities. Hence, for gaining the target of quality level of 99.9996 percent or free-defects, an organization requires both flexibility and discipline in solving problems using statistical approach rather than using simple intuition or by trial and error; wider usage of statistical treatments is one of the benefits of Six Sigma method (Pacheco, 2014). Application of Six Sigma's method is more valuable due to its contribution to the science and practice for particularly reduces waste and provides added values. Six Sigma allows users to identify waste and hidden costs, eliminates defects, increases profit margin, satisfies customers, encourages employee commitment and satisfaction as well as expands businesses (Patil et al, 2015). Six Sigma as a management system is applied to ensure that efforts and critical opportunities for improvement are well developed through metric methodology and an applied level is inline with its business strategy. Six Sigma enables an organization to improve quality process by identifying and eliminating the causes of defects and error terms through minimizing variability in manufacturing and business processes (Mittal, 2014). The stages for improve process ability (process capability) regarding Six Sigma method are specifically allowing the standard steps such as define, measure, analyze, improve, and control for interlinked statistical tests. For a particular project within organization of applied of Six Sigma the stage is typically consists of a step-by-step requires for obtain measurable target values i.e. reduces cycle time, decreases air pollution, reduces costs, improves customer satisfaction, and increases profits (Mittal, 2014). It is inevitable, in order to gain benefits, as a results of Six Sigma's application in an organization or company, would require relatively high of initial investment, but might be offer benefits in long terms including cost savings, generated profits, improved consistency of quality processes, better employee performance, and better service quality and products. Those elements particularly would lead an organization or company to provide a higher customer satisfaction as well as to gain the ultimate goal of organization (Mittal, 2014). By applying DMAIC using a statistical approach, the root causes of the problem can be found and can improve the production process. The results of the six sigma improvement show the process capability increased from 2.2 to 3.1 sigma, saving $18,394.2 per month (Syafwiratama et al., 2017). Six sigma is a systematic, flexible, measurable and effective method in solving various problems in the industrial world (Trimarjoko et al., 2019). Seeing the results of the studies mentioned above, the Six Sigma method is used in improving the quality of Remanufacturing Lift Arm in the Heavy-duty equipment industry in Indonesia to be better to meet customer satisfaction and provide better company benefits.

3. Research methodology

The research methodology is a systematic description of the steps taken by the author from the beginning to the end of the study so that the implementation of the research becomes clear and focused following the research objectives. Through the following principal steps: (1) Describe the issue that is happening (2) Measure baseline performance or sigma level as an initial standard Re-manufacturing the Lift Arm process. (3) Analyzing the cause of product failure factors in the Lift Arm Remanufacturing process. (4) Determine the improvement efforts that can be done to improve the quality of the Re-manufacturing Lift Arm process, (5) Evaluate and control the results of repairs. The 5 stages are following the rules of problem-solving using the Six Sigma method namely the DMAIC phases (Define, Measure, Analyze, Improvement and Control). The research methodology used in this study is shown in Figure 1.

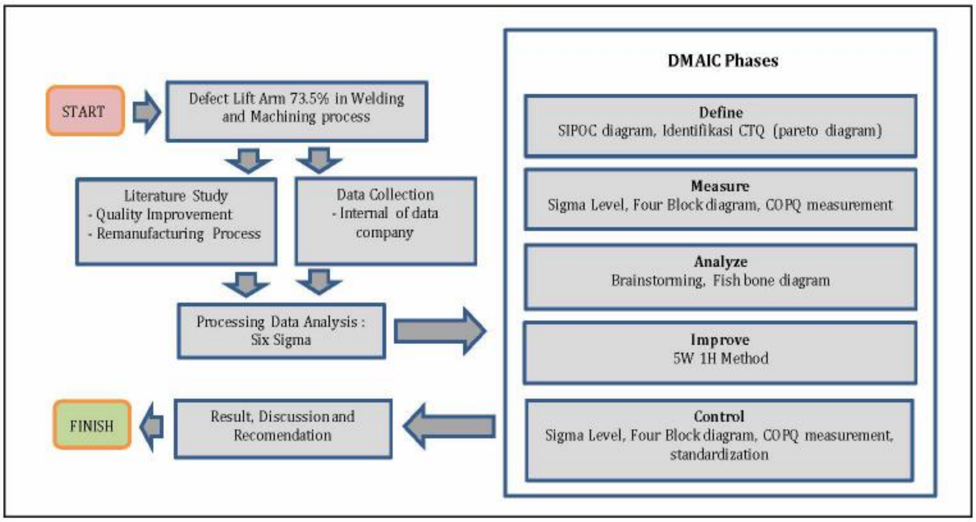


Figure 1. Research Methodology of Problem Solving Lift Arm in Welding and Machining Process

4. Processing and analysis

Processing and Analysis in this study using the Six Sigma (DMAIC) method based on previous research studies Six Sigma is a systemic, flexible, measurable and effective method, with a combination of methods and other tools proven to be able to reduce defective products, reduce errors, reduce customer complaints and improve process capability in maintaining company sustainability and can improve company competitiveness. Six Sigma has structured steps known as DMAIC phases (define, measure, analyze, improve, control).

4.1. Define phase

At the stage of defining activities carried out to identify problems that occur based on consumer needs and determination of goals (reduction of product failure). The initial step of the define stage is to identify the sequence of activities that occur in the welding and machining process that aims to find out at which stage the problem is. As for the sequence of activities intended in the SIPOC Diagram as in Table 1.

Table 1 SIPOC Diagram of Remanufacturing Lift Arm.

Supplier	Input	Process	Output	Customer
Disassembly Area	Damage part	Welding Process	Part finish process	Assembly Area
Scope Of Work Schedule	Consumable Welding	Machining Process	Remanufacturing	
Material	Consumable Machining		OK tag from Quality Control	

The Welding and Machining process is critical in this research. The next step is to find out the Critical to Quality in the Lift Arm welding dam machining process is carried out the production data collection and Welding and Machining Lift Arm product failure in January ~ May 2019 with the percentage and types of product failure as in Pareto diagram Figure 2.

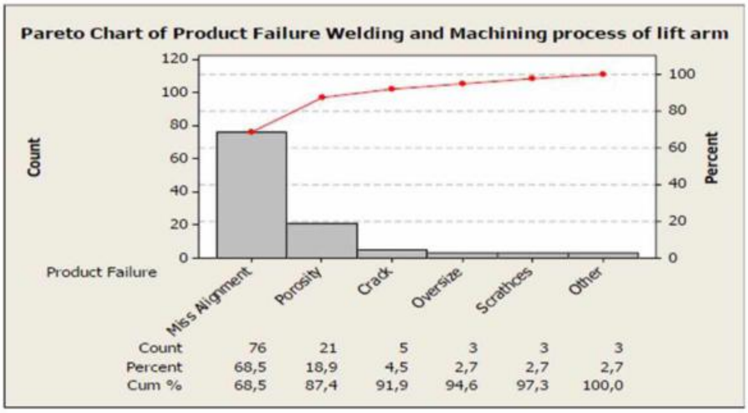


Figure 2 Pareto Chart of product failure Welding and Machining process Lift Arm

Refer to the Pareto diagram as in Figure 3. Can be interpreted that 6 types of defective products occur in the welding process and Machining Lift Arm, namely: Miss Alignment (68.5%), Porosity (18.9%), Crack (4.5%), Oversize (2.7%) Scratches (2.7%) and Others (2.7%). Based on the concept of Pareto product failure that has an accumulation of 80% into the improvement priority in problem-solving, then CTQ in this study there are 2 types are Miss Alignment and Porosity product failure with cumulative 68.5% + 18.9% = 87.4%. Research focuses on solutions to eliminate these two defects

4.2. Measure phase

The measuring stage is the second stage in the quality improvement program with the Six Sigma method in this stage, the capability process/sigma level measurement is used to determine the ability of the process before improvement, plotting the ability of the process into 4 block diagrams to determine the improvement direction from the control side of technology and also carried out measurements cost of poor quality (COPQ) to determine the financial losses caused by defective products.

4.2.1. Capability process/Sigma level measurement

Based on the collection of production data and product failure (defects) in the Remanufacturing Lift Arm process from January to May 2019 obtained from the report of the production department and Quality control total production 266 part, and total defect 111 part, the calculation of the process capability/sigma level is shown in Table 2.

Table 2. Measurement of Level Sigma Current Condition that Representative of Before Improvement

Item	Value
Total Production	266
Total Product Failure (defect)	111
CTQ (Control to Quality)	2
DPMO (Defect per Million Opportunity)	834586,47
Level of Sigma	1,91

4.2.2. Four block diagram

Four block diagram is a description of a process and states improvement direction that leads to two sides of improvement, namely technology and control which is a description of the ability of the process (Z) of an ongoing process. Based on Sigma Level 1.91, it can be calculated Zshif value as a reflection of control ability and Zst value which reflects the ability of technology and then plots it in Four block diagrams that show the capability of the ongoing process (Z). The Zshif and Zbench.lt calculations in the four-block diagram are as follows:

Zst = Zbench.lt + 1.5
1.91 = Zbench.lt + 1.5
Z bench.lt = 1.91 – 1.5
= 0.41
Zshift = Zbench.st – Zbench.lt
= 1.91– 0.41
= 1.50

The next step is after knowing the value of Z shif (control ability) and Zst (sigma level) then it can be done by making four block diagrams to illustrate the current process condition (current condition), as for the Four block diagrams referred as in Figure 3.

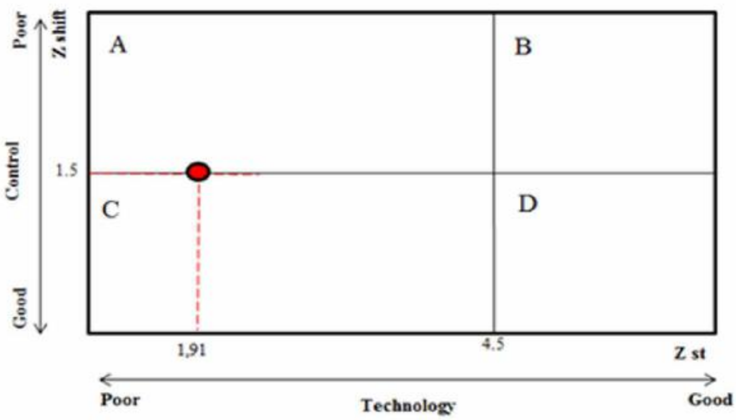


Figure 3. Four Block Diagram Product Failure (Defect) Welding and Machining process Lift Arm

Looking at the Four block diagram above(figure 4), it is known that from the control side it is good and still lacking in technology, meaning that improvements are needed so that both sides are expected in the category of proper control and technology.

4.2.3. Cost of poor quality (COPQ) mesurement

In addition to measuring the baseline performance of the Remanufacturing Lift Arm process, a cost analysis is also carried out due to poor quality, in this case, the cost of losses caused by product failure. The company's internal source costs must be borne due to product failure resulting in rework. Cost of Poor Quality US$ 390,87 per pcs. As a result, the costs of losses due to product failure in January - May 2019 are as follows:

Table 3. Calculation of Cost of Poor Quality January - May 2019

No	Month	Product Failure (Pcs)	COPQ (USD)
1	January 2019	22	8599.14
2	February 2019	18	7035.66
3	March 2019	22	8599.14
4	April. 2019	23	8990.01
5	May 2019	26	10162.62
	Total	111	43386.57

Table 3. shows the cost of losses resulting from product failure from January to May 2019 Cost of Poor Quality of USD 43386.57.

4.3. Analyze phase

Analyze Phase is the third stage of the DMAIC method. In this stage, what needs to be done is to analyze why deviations or product failures occur by looking for the causes that cause these product failures. In this case, a defect analysis arises in the Remanufacturing Lift Arm process which consists of 2 types of product failure eg alignment and porosity Fishbone diagram (Cause and Effect diagram) as in Figures 4 and 5.

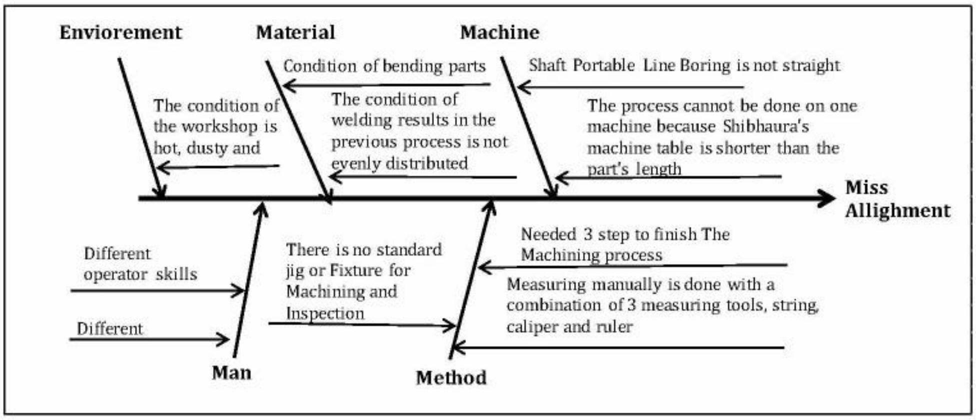


Figure 4. Cause and Effect diagram of product failure Miss Alignment

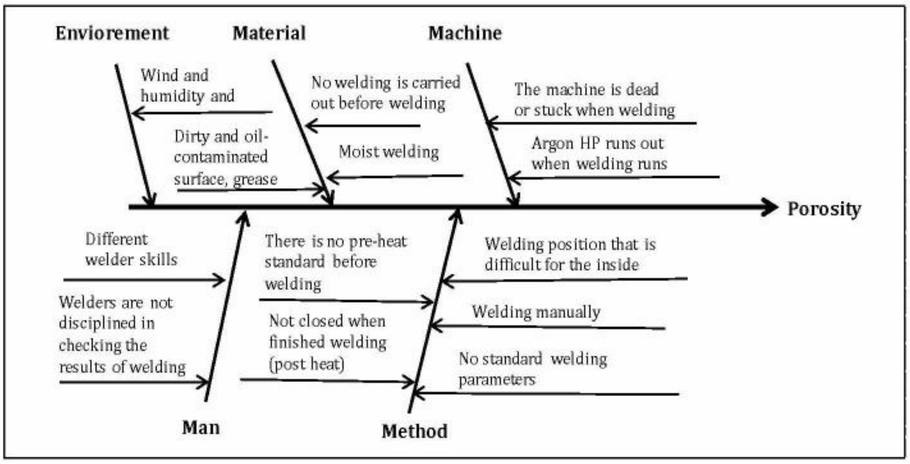


Figure 5. Cause and Effect diagram of product failure Porosity

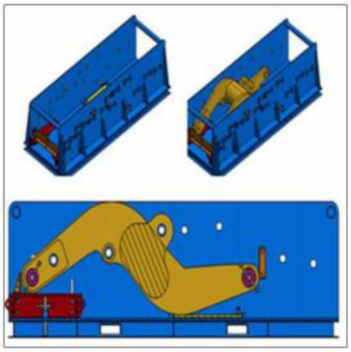
From the cause and effect diagram, in Figure 4 the root cause of Miss Alignment in the Remanufacturing Lift Arm process is: (1) The process cannot be done in a CNC machine because parts are longer than the machine table. (2) If done with a Portable Line boring machine, the machining and inspection process is done manually so that it depends on the operator's skill. (3) Different parts condition when received such as bending, crack, and welding results from the previous process are uneven. (4) There is no standard process using jigs and fixtures. (5) The absence of standard parameters, methods for the machining process. From the cause and effect diagram, in Figure 5. the root cause of Porosity in the Remanufacturing Lift Arm process is (1) Dirty, oil-contaminated, grease and inconsistent surface cleaning by grinding before welding. (2) The welding process is carried out without Jigs, fixtures, parameters, and procedures as well as the inconsistency of checking after welding. (3) Humid winds and conditions in the welding area. (4) Difficult welding for inner diameter. (5) Uneven Welder Skill.

4.4. Improve phase

Improve stage is determining the proposed improvement of the root causes that have been done at the Analyze stage. The improvement plan is carried out using the 5W + 1H method that contains plans and corrective actions for each of the factors causing product failure Miss Alignment and porosity that have accumulated 80% of the largest product failures from the overall product failures that occur in the welding and machining process.

4.4.1. Improve plan product failure of Miss Alignment

Miss Alignment product failure that occurs in the machining process is a major problem in the Machining process based on joint discussion with the 5w + 1H method. Miss alignment problem is reconditioned: The Machining process changes from a manual process with 3 settings to 1 time setting with "Jig". This changes from the previous process of series per 1-2 holes into 10 holes directly in 5 different places. The inspection or checking process can also be reduced by eliminating the alignment checking/alignment from using the meter, caliper, ruler, thread and pendulum become unnecessary because the Jig hole size has been adjusted to the part specifications. Quality Control focuses on checking dimensions with bore gauge and caliper and visual smoothing of machining results. The process is faster than the previous 4 days to 2 days so that it can increase the capacity of workshops that were previously 13-14 units to 24-25 units per month. The engine parameters with a speed of 200 rpm, feeding rate. 1,2 and feeding 0.5 - 1 mm per step. From the operator side, this process will change from grade 4 or multi-skill CNC operator to grade 2 semi-automatics. The Jig referred to above is as in Figure 6.



Fiaure 6. lia Machinina process desian

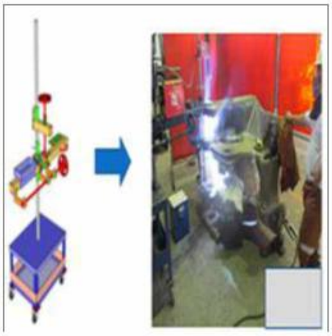


Figure 7. Jig Semi Automatic Welding process design

4.4.2. Improve plan product failure of Porosity

Product Porosity failure that occurs in the Welding process is a major problem in the Machining process based on joint discussion with the 5w + 1H method. Miss alignment problem is reconditioned: The welding process for Inside diameter or the inner hole is replaced from before the manual process becomes semi-automatic by modifying the machine and making Jigs and fixtures. With this process, a change occurred before Welder did welding to just run the machine correlated with Jigs and fixtures so that the welding results will be standard, even and the operator can prepare other parts in the queue. The making of Welding Procedure Standard starts from the cleaning process with chemical and grinding, preheat up to a temperature of 120º Celsius, welding and PWHT with glass wool after finishing. In the surrounding area of welding made a cover or screen to keep the wind and humidity. The improvements to the welding process in question are shown in Figure 7.

4.4. Control phase

This stage is the final phase of the DMAIC phase. What is done at this stage is monitoring and controlling the results after improvement. Process capability/sigma level, mapping sigma level into four blocks are salted and the calculation of the cost of poor quality (COPQ) is again carried out to determine the effectiveness of the results of improvements, in addition to the process of standardization of new processes that are also carried out to avoid similar failure products occurring in the future.

4.5.1. Capability process/sigma level measurement (after improvement)

Based on data taken from the production and quality control department with a duration from the first week of July 2019 to the fourth week of November 2019, the total production for Lift Arm is 341 units with a total of 24 units of product failure, with a percentage of 7.03%. The calculation of process capability/sigma level is presented in Table 4.

Table 4. Calculation of process capability/sigma level after improvement

Item	Value
Total Production	341
Total Product failure (defect)	24
CTQ (Control to Quality)	2
DPMO (Defect per Million Opportunity)	140762,5
Sigma level	2,43

Table 4. Shows that the capability of the welding and machining process after Improvement is at 2.4 sigma with DPMO 140762.50 better than the conditions before Improvement 1.91 sigma with DPMO 834586.47.

4.5.2. Four block diagram (after improvement)

Referring to Table 3 above and the same calculation as in the measure phase, the sigma level after improvement (2.43) can be mapped in the four-block diagram as in Figure 8.

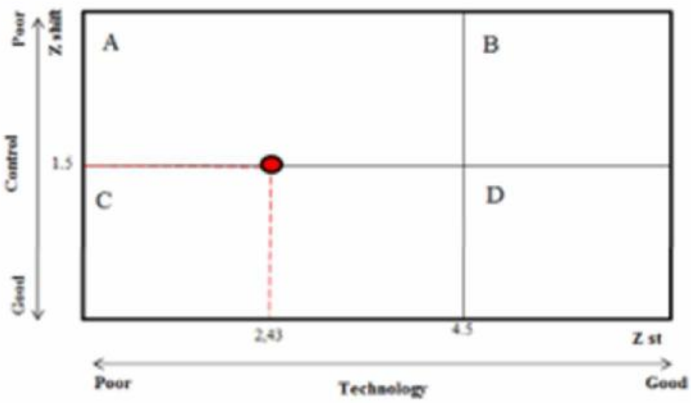


Figure 8. Four Block Diagram Product Failure (Defect) Welding and Machining process Lift Arm (after improvement)

4.5.3. Calculation of cost of poor quality (COPQ) after improvement

Just as in the measuring stage, the calculation of the cost of poor quality (COPQ) is a calculation of the company's losses that must be borne by the product failure that occurs. Cost of Poor Quality US$ 390,87 per pcs. The COPQ calculations after improvement can be seen in Table 5.

Table 5. Calculation of Cost of Poor Quality After Improvement

No	Month	Product Failure (Pcs)	COPQ (USD)
1	July 2019	5	1954.35
2	August 2019	5	1954.35
3	September 2019	5	1954.35
4	October 2019	5	1954.35
5	November 2019	4	1563.48
	Total	24	9380.88

Table 5. Above can be interpreted that the loss that must be borne by the company due to product failure after improvement as much as USD 9380.88 decreased from before improvement USD 43386.57 equivalent to 78.37%.

4.5.4. Standardization

To avoid similar failure products, namely Miss Alignment and Porosity Lift Arm in the process of welding and machining in the Indonesian remanufacturing industry, socialization of the results of improvement to all relevant levels and the creation of new standards in the form of Operational Procedure Standards (SOP) related to welding and Machining processes.

5. Conclusion

Referring to the entire stages of this research, it can be concluded that improving quality by using the Six Sigma method in this study can reduce Lift Arm failure products in the welding and machining process and can increase company profits due to decreased product failure. This study generally strengthens previous studies that the Six Sigma method is effective in identifying and analyzing product failures, and can improve the capability/level of sigma to get better quality products. Seeing the positive results contained in this study, it is recommended for further studies the use of the Six Sigma method in combination with other tools of quality can be used in improving quality in the other remanufacturing industries. To increase the repertoire of research using the Six Sigma method becomes more varied.

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MODELLING PROCEDURE FOR THE SELECTION OF STEEL PIPE SUPPLIER BY APPLYING THE FUZZY AHP METHOD

Edmundas Kazimieras Zavadskas [1], Zenonas Turskis [1], Željko Stević [2]*, Abbas Mardani [3,4]

[1] Vilnius Gediminas Technical University, Institute of Sustainable Construction, Faculty of Civil Engineering, Lithuania
[2] University of East Sarajevo, Faculty of Transport and Traffic Engineering Doboj, Bosnia and Herzegovina
[3] Informetrics Research Group, Ton Duc Thang University, Vietnam
[4] Faculty of Business Administration, Ton Duc Thang University, Vietnam

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Abstract: *The objective of this study is the supplier evaluation and selection by applying the fuzzy multi-criteria analysis. The study used the fuzzy Analytic Hierarchy Process (FAHP) to choose the most suitable supplier for the purchase of materials necessary for the production of pre-insulated pipes. Decision-makers selected among five suppliers based on nine criteria. Effective execution of procurement, in this case, the procurement of material needed for the production logistics subsystem, influences the overall efficiency of the business. Results show that it is very important to perform the right ranking in the process of supplier selection. Good decisions can ensure lower costs and higher quality of the production and therefore a better position in the market. Also, applied methodology and the rank show that supplier A is the most suitable solution.*

Keywords: *fuzzy AHP, optimization, supplier selection*

1. Introduction

Lately, the area of multi-criteria analysis is rapidly developing, thanks to the large number of publications dealing with the adoption of individual decisions based on the applied methods that belong to the specified field. For example, Fallahpour et al. (2020) introduced a new integrated MCDM approach under uncertainty by integrating Fuzzy Preference Programming as a modification of Fuzzy Analytic Hierarchy Process, with Fuzzy Inference System as a fuzzy rule-based expert system.

* Corresponding author.
edmundas.zavadskas@vgtu.lt (E. K. Zavadskas), zenonas.turskis@vgtu.lt (Z. Turskis), zeljkostevic88@yahoo.com and zeljko.stevic@sf.ues.rs (Ž. Stević), mabbas3@liveutm.onmicrosoft.com (A. Mardani)

The AHP method is one of the most common methods, among the dozens of approaches proposed, to solve complex multi-level decision-making problems. The importance of this method is the fact that there are conferences dedicated only to the method of the Analytic Hierarchy Process. However, despite this fact, there is a constant strive to create better opportunities and more accurate problem-solving. For this reason, there is an enlargement of the AHP method, and creation of a fuzzy approach that allows a more precise definition of the most favourable alternative, or decision. AHP is often used in integration with other approaches, as can be seen in the study (Stević et al., 2015) where this method is integrated with TOPSIS.

For this study, the extended fuzzy AHP method based on triangular fuzzy numbers (TFN) was used, where extended analysis of the target was performed for each object. In addition to this one, some other methods can be applied as well, such as Maxmax, Maxmin, SAW, ELECTRE, PROMETHEE, TOPSIS, but for the issues addressed in this paper, it is much better to apply the AHP method. Methods Minmax, Maxmin and SAW are straightforward methods of multi-criteria analysis, of which only the SAW method takes the importance of criteria into account, and as such are not applied frequently in solving complex problems. Methods ELECTRE and PROMETHEE have several versions and based on the authors' knowledge stemming from the extensive review of the literature, we cannot say that these methods are not applied, they are, but to a much lesser extent than the AHP method, especially when it comes to the field of choice of suppliers. Due to its simple concept, the TOPSIS method has become very popular and is applied in many areas of decision-making procedures (Zavadskas et al., 2016). However, despite that, this method is often criticized because there is no possibility for adequate handling of uncertainty and imprecision at the moment when the decision-maker wants accurate results. When compared to other methods, the AHP method has frequently shown features that are more practical, which is of great importance. Some of the advantages of this method are outstanding problem structuring from the highest to the lowest level, pointing to the subjectivity that exists with the decision-makers, less susceptibility to errors in assessing thanks to the redundancy of comparison in pairs, use for complex decision-making and the like. As the most common shortcomings of this method stand out that there are not enough measures in the Saaty scale to compare pairs of elements of specific decision-making problems with quite many criteria.

However, a combination with fuzzy logic can somehow eliminate or reduce the disadvantage. Chapter 3 presents it in detail. Applying the AHP method enables more accurate interpretation of results because all values are the sum of an alternative one as a contrast to other methods where it is not the case, and thus it is possible to see how exactly the optimum solution is better than other estimated solutions. The primary reason for the AHP method application would be its ability to handle quantitative and qualitative criteria equally.

In current business conditions, for one company to achieve the market position that makes it competitive, and to keep it, continuous measuring and monitoring of performance is necessary. If there is a deviation (which is often the case) from the planned values, it is necessary to undertake specific corrective measures to ensure the achievement of higher values. However, a better route by which it is possible to achieve efficient business management is a proactive way, where business results are not expected, but they are managed instead. Thanks to the constant changes to which the market is exposed and to increasingly stringent requirements placed on the market, it is undoubtedly a challenge to maintain a competitive position. It can be achieved if there is an adequate production, which means as low product cost as possible, as higher product quality as possible, high accuracy of delivery to final users, reliability, response to specific requirements set by users, i.e. flexibility and cooperation that can be accomplished with both – customers and suppliers. The research carried out in this paper connects the first two logistics stages: the procurement and production, which, with their consolidation, are making logistics of materials-effective execution of activities related to the system. The inclusion of the selection of the best suppliers significantly affects the price and quality of the final product. These are some of the most important factors determining success in the market. The correct choice of suppliers from the start provides the ability for timely, continuous and efficient production, which enables achieving the above-described benefits and makes that production competitive.

The researched company is engaged in the production of pre-insulated pipes for heating and their application and installation in all heating systems. To be able to carry out the production smoothly, one of the necessary materials that need to be procured is steel pipes. In the market, there are many potential suppliers of the material mentioned above, and it is necessary to set aside those who particularly stand out based on their characteristics and based on criteria of the company that is the subject of research. After a thorough market analysis carried out by experts from the commercial service, the choice was reduced to five suppliers representing variants of which three are located in the domestic and the other two on the international market, which includes the territory of neighbouring countries.

A similar issue is treated in (Bronja and Bronja, 2015; Chatterjee and Stević, 2019), where it can be seen the exact significance of the expert team, which, in addition to the selection of potential solutions, created a total of nine quantitative and qualitative criteria based on which it is necessary to evaluate potential suppliers. Based on the current market needs and demands, as well as on the knowledge, skills and abilities acquired over the years in the same business, an expert team has evaluated criteria, bringing out the different weight value. The most significant aim and the contribution of this study is performing the optimization of the purchasing process through the proposed model for the application of fuzzy AHP method to this problem, and the possibility of establishing a long-term collaboration with the chosen supplier, which would enable additional benefits for the company.

The paper is structured in several sections. In the introduction, aims and motivation for research are described. The second section shows a brief literature review with an emphasis on the fuzzy AHP method and the problem of supplier selection. The third section shows steps used in the fuzzy AHP method, while in the fourth section, an empirical study is shown. The paper ends with a conclusion and future tasks.

2. Brief literature review

There are many criteria to evaluate suppliers but the question is how to choose the right one from a given set, which will help to choose the best option. Some authors have tried to answer this question, so Webber (1991) investigated the criteria for the selection of suppliers in the manufacturing and retail environment in the 74 documents published from 1966 to 1991. He concluded that quality, delivery and price are prevailing as the dominant criteria. Besides, geographical location, financial position and production capacity fall to the second group of factors.

Verma and Pullman (1998) conducted a study among 139 managers whose aim was to examine how to make a compromise when selecting suppliers. Their work indicated that the managers are paying the greatest attention to quality as the most critical attribute of suppliers, followed by delivery and price. Karpak et al. (2001) took delivery reliability as a criterion for selection, while Bhutta and Huq (2002) used four criteria to evaluate suppliers: price, quality, technology and service.

Many researchers use Multi-Criteria Decision-Making (MCDM) methods and differently control target alternatives (Turskis et al., 2019). Different researchers developed different models to select the best supplier in a competitive market environment. Keshavarz Ghorabaee et al. (2016) extended the EDAS method for fuzzy multi-criteria decision-making. Later, Keshavarz Ghorabaee et al. (2017) presented a novel model based on interval type-2 fuzzy sets and EDAS method. Aouadni et al. (2017) presented a model based on the Meaningful Mixed Data TOPSIS (TOPSIS-MMD) method. Recently, Yazdani et al. (2019) developed a grey combined compromise solution (CoCoSo-G) method for supplier selection.

The criteria that are a base for the choice of suppliers were selected based on two factors: the most commonly used criteria in the same or similar research, and current needs of the company and demands that it might face in the market. As mentioned in the introduction, the company's expert team selected the criteria set.

The AHP method was addressed to the problem of supplier selection, in many types of research (Galankashi et al., 2016; Chen et al., 2006; Jain et al., 2018; Stević et al., 2016) the choice of supplier in the industry, where the general purpose of the model is applied to the leading electro motor manufacturer of Turkey (Barbarosoglu and Yazgac, 1997), the choice of supplier in textile company (Ertugrul and Karakasoglu, 2006), where the focus is on the identification and discussion of criteria which make up an essential part of the decision. It is the price, quality, service level and profile of suppliers (Chan and Kumar, 2007). Then, the choice of supplier among manufacturers of TFT-LCD, where the applied model can identify strengths, opportunities on the one, and the cost and risk on the other hand (Lee 2009).

The AHP method has a wide application in practical research in a wide range of areas, thus contributing to the improvement of the entire management system (Erdogan et al., 2017; Hashemkhani Zolfani et al., 2011). In the supply chain, decisions based on this method provide the proper choice of suppliers that affects the formation of a more efficient flow of further chain. Decision-makers used many methods, which do not belong to the field of multi-criteria analysis, to solve similar problems. However, they frequently combined them with the AHP method.

3. The fuzzy AHP

The creator of the AHP method is Thomas Saaty (1977; 1980). According to (Saaty, 1990), the AHP is a measurement theory, which is dealing with comparing pairs and which relies on expert opinion in order to perform the priority scale. Fuzzy ratings and scales provide decision-makers possibilities to express better the level of their knowledge accuracy (Zemlickienė & Turskis, 2020; Turskis et al., 2015). Various approaches of fuzzy AHP method were developed as an extended fuzzy AHP method based on triangular fuzzy numbers (Setyohadi & Suyoto, 1977; Saaty, 1978, 1982; Van Laarhoven & Pedrycz, 1983; Chang, 1996; Zhu et al., 1999). Zadeh (1965) introduced the theory of fuzzy sets. Its application enables DMs to deal with uncertainties effectively. Fuzzy sets used generally triangular, trapezoidal and Gaussian fuzzy numbers, which convert uncertain fuzzy numbers. More details can be found in (Xu and Liao, 2014). The authors use Chang's (1996) extent analysis method in this study. Steps of the approach application are relatively simple and easy, requiring less time than many other fuzzy extensions of the AHP method. At the same time, it can eliminate the shortcomings of the classical AHP method.

Assume that $X = \{x_1, x_2, ..., x_n\}$ is a number of objects, and $U = \{u_1, u_2,...,u_m\}$ is a number of aims.

For each object, an extended goal analysis is performed. Values of the extended analysis "m" for each object can be shown:

$$M_{gi,}^{1} M_{gi,}^{2} M_{gi,}^{m} i = 1,2, \dots n., \tag{1}$$

where $M_{g,}^{j}\ j = 1,2, \dots m.,$ are TFNs.

Steps of fuzzy AHP are:

Step 1: The value of fuzzy synthetic extent Si for the i-th criterion is as follows:

$$S_i = \sum_{j=1}^{n} M_{gi}^{j} \times \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \tag{2}$$

To obtain equation (3),

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \tag{3}$$

we need to perform additional fuzzy operations with "m" values:

$$\sum_{j=1}^{n} M_{gi}^{j} = \left(\sum_{j=1}^{m} l_j , \sum_{j=1}^{m} m_j , \sum_{j=1}^{m} u_j \right) \tag{4}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} M_{gi}^{j} = \left(\sum_{i=1}^{n} l_i , \sum_{i=1}^{n} m_i , \sum_{i=1}^{n} u_i \right) \tag{5}$$

Then it is necessary to calculate:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right]^{-1} \left[\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right] \tag{6}$$

Step 2: The sets of weight values for each criterion are calculated by decision-makers according to the principle of comparing fuzzy numbers. For example, for two fuzzy numbers S_b and S_a,, the decision-makers define the possibility degree of $S_b \geq S_a$ as follows:

$$V(Sb \geq Sa) = \sup \geq [\min \mu_{Sb}(x), \mu_{Sa}(y)] \tag{7}$$

where *sup* represents the supremum and when a pair (x, y) exists such that x ≥ y and $\mu_{Sb}(x) = \mu_{Sa}(y) = 1$, it follows that V(Sb ≥ Sa) =1 and V(Sa ≥ Sb) = 0. Since S_b and S_a are convex fuzzy numbers defined by the TFNs (l_1, m_1, u_1) and (l_2, m_2, u_2) respectively, it follows that:

$$V(Sb \geq Sa) = 1 \text{ iff } Sb \geq Sa; V(Sa \geq Sb) = hgt(Sb \cap Sa = \mu_{Sb}(x_d) \tag{8}$$

where *iff* represents "if and only if" and *d* is the ordinate of the highest intersection point between the μ_{Sb} and μ_{Sa} TFNs and x_d is the point on the domain of μ_{Sb} and μ_{Sa} where the ordinate *d* is found. The term *hgt* is the height of fuzzy numbers on the intersection of S_b and S_a.

$$V(S_b \geq S_a) = \begin{cases} 1, & if\ m_b \geq m_a \\ 0, & if\ l_a \geq u_b \\ \frac{l_a - u_b}{(m_b - u_b) - (m_a - l_a)}, & otherwise \end{cases} \tag{9}$$

where "*d*" is the ordinate of the largest cross-section in point *D* between μS_a and μS_b, as shown in Figure 1.

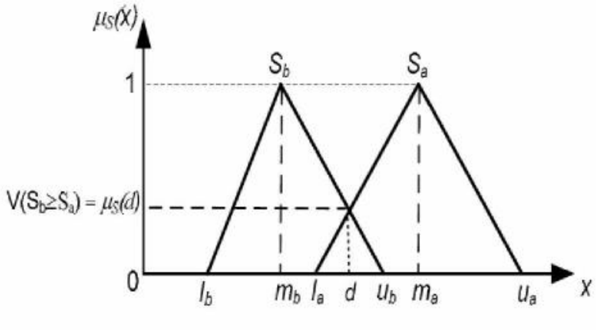

Figure 1. Intersection between S_b and S_a

The decision-makers need both values V($S_1 \geq S_2$) and V($S_2 \geq S_1$) to compare S_1 and S_2.

Step 3: The level of possibility for a convex fuzzy number to be greater than "*k*" convex number S_i (*i* =1, 2, ..., k) can be defined as follows:

$$V(S_i \geq S_1, S_2, \dots, S_k) = \min V(S_i \geq S_k), = w'(S_i) \tag{10}$$

$$d'(A_i) = \min V(S_i \geq S_k), k \neq i, k = 1,2, \dots, n \tag{11}$$

The following expression gives the weight vector:

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T, \tag{12}$$

Step 4: Through normalization, the weight vector is reduced to the expression:

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^T, \tag{13}$$

where W represents a crisp number.

Through its application, the fuzzy AHP method alleviates the main disadvantage of the classical AHP method, and that is, as previously mentioned, an insufficiently big comparison scale. To this end, various scales have been developed based on comparing the fuzzy triangular numbers, where the decision-maker can evaluate the significance of criteria or alternatives much closer and easier, and thus reducing his/her subjectivity that is present in solving these problems to a minimum.

4. Numerical example

The following criteria are applied in this study: the price of materials, pipe length, delivery time, way of payment, transport distance, quality, reliability, flexibility and relationship with customers that are still in operation, which are marked with C_1-C_9 respectively. Therefore, there are four criteria, quantitatively expressed and five qualitative criteria, as shown in Figure 2. Detailed explanation of used criteria in this study can be found in (Stević et al. 2016).

As mentioned in the introduction, the panel selected a set of criteria for evaluating suppliers. These selected critical criteria are the same as those most widely used in practice. Consideration of them and their share in the selection of suppliers achieves a significant overall business performance. The level of meeting the needs of end-users and the requirements of strict standards and norms is well reflected in the company's profit.

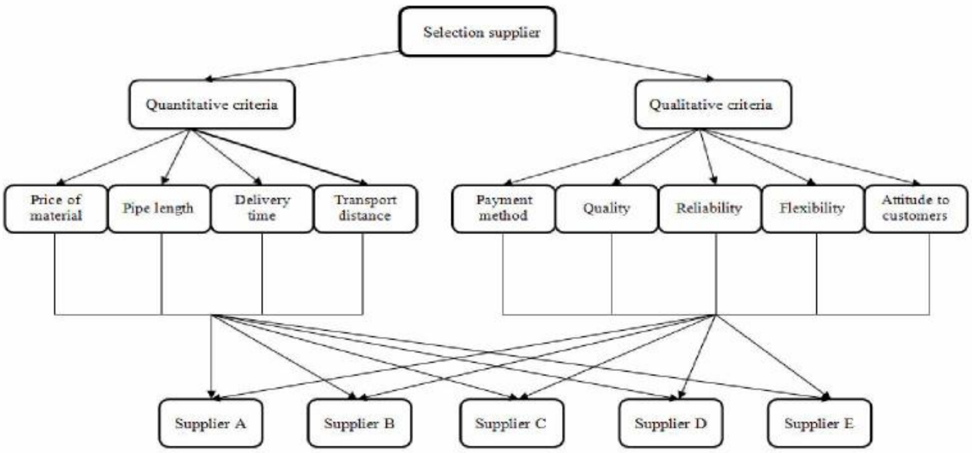

Figure 2. The hierarchical structure of the proposed model

Table 1. Values of triangular fuzzy scale

Linguistic Scale	Triangular Fuzzy	Triangular Fuzzy

	Scale	Reciprocal Scale
Just equal	(1, 1, 1)	(1, 1, 1)
Equally important	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly more important	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

One of the main features of MCDM process is that the different criteria cannot have the same significance, so following the methodology described for decision making which applies the extended AHP method, i.e. fuzzy AHP to get the required results, it is necessary to perform criteria comparison based on TFNs, as shown in Table 2. The comparison was made based on the scale shown in Table 1 (Chang, 1996).

By comparing them, weight values of criteria are determined. The criteria weights have a large significance in the further application of methods because, on the base of these values, the best solution is determined. If a variant is better according to criteria that are very important when deciding, it increases the possibility to have exactly this variant as an optimum.

Table 2. Comparison of criteria on the base of triangular numbers

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(2/3,1,2)	(1/2,2/3,1)	(1/2,1,3/2)	(1/2,1,3/2)
C_2	(1/2,1,3/2)	(1,1,1)	(2/3,1,2)	(1,3/2,2)	(1,3/2,2)
C_3	(1,3/2,2)	(1/2,1,3/2)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)
C_4	(2/3,1,2)	(1/2,2/3,1)	(2/5,1/2,2/3)	(1,1,1)	(1,1,1)
C_5	(2/3,1,2)	(1/2,2/3,1)	(2/5,1/2,2/3)	(1,1,1)	(1,1,1)
C_6	(1/2,1,3/2)	(1,1,1)	(2/3,1,2)	(1/2,1,3/2)	(1,3/2,2)
C_7	(1,1,1)	(2/3,1,2)	(1/2,2/3,1)	(1/2,1,3/2)	(1/2,1,3/2)
C_8	(1,1,1)	(2/3,1,2)	(1/2,3/2,1)	(2/3,1,2)	(1/2,1,3/2)
C_9	(2/3,1,2)	(2/3,1,2)	(2/5,1/2,2/3)	(1,1,1)	(2/3,1,2)

	C_6	C_7	C_8	C_9
C_1	(2/3,1,2)	(1,1,1)	(1,1,1)	(1/2,1,3/2)
C_2	(1,1,1)	(1/2,1,3/2)	(1/2,1,3/2)	(1/2,1,3/2)
C_3	(1/2,1,3/2)	(1,3/2,2)	(1,3/2,2)	(3/2,2,5/2)
C_4	(2/3,1,2)	(2/3,1,2)	(1/2,1,3/2)	(1,1,1)
C_5	(1/2,2/3,1)	(2/3,1,2)	(2/3,1,2)	(1/2,1,3/2)
C_6	(1,1,1)	(1/2,1,3/2)	(1/2,1,3/2)	(1,3/2,2)
C_7	(2/3,1,2)	(1,1,1)	(1,1,1)	(1,1,1)
C_8	(2/3,1,2)	(1,1,1)	(1,1,1)	(2/3,1,2)
C_9	(1/2,2/3,1)	(1,1,1)	(1/2,1,3/2)	(1,1,1)

By applying the equation (4), (5) and (6), the following values are obtained:

S_1=(6.333;8.667;12.5)x(1/120;1/84.5;1/61.364)=(0.053; 0.103;0.204)

S_2=(6.667;10;14) x(1/120;1/84.5;1/61.364)=(0.056;0.118; 0.228)

S_3=(9.5;13.5;17.5)x(1/120;1/84.5;1/61.364)=(0.079;0.160;0.285)

S_4=(6.4;8.167;12.167) x(1/120;1/84.5;1/61.364)=(0.053;0.097; 0.198)

S_5=(5.9;7.833;12.167) x(1/120;1/84.5;1/61.364)=(0.049; 0.093; 0.198)

S_6=(6.667;10;14)x(1/120;1/84.5;1/61.364)=(0.056;0.118;0.228)

S_7=(6.833;8.667;12)x(1/120;1/84.5;1/61.364)=(0.057;0.103;0.196)

S_8=(6.667;9.5;13.5)x(1/120;1/84.5;1/61.364)=(0.056;0.112;0.220)

S_9=(6.4;8.167;12.167)x(1/120;1/84.5;1/61.364)=(0.053;0.097. 0.198)

After completion of the calculation using the equation (9), values are obtained as described in step three to the amounts:

$V(S_1 \geq S_2)=V(S_1 \geq S_6)=0.908; V(S_1 \geq S_3)=0.687; V(S_1 \geq S_4)=(S_1 \geq S_5)=V(S_1 \geq S_7)=V(S_1 \geq S_9)=1; V(S_1 \geq S_8)=0.943$

$V(S_2 \geq S_1)=V(S_2 \geq S_4)=V(S_2 \geq S_5)=V(S_2 \geq S_6)=V(S_2 \geq S_7)=V(S_2 \geq S_8)=V(S_2 \geq S_9)=1; V(S_2 \geq S_3)=0.78$

$V(S_3 \geq S_1)=V(S_3 \geq S_2)=V(S_3 \geq S_4)=V(S_3 \geq S_5)=V(S_3 \geq S_6)=V(S_3 \geq S_7)= V(S_3 \geq S_8)= V(S_3 \geq S_9)=1$

$V(S_4 \geq S_1)=0.960$; $V(S_4 \geq S_2)=V(S_4 \geq S_6)=0.871$; $V(S_4 \geq S_3)= 0.654$; $V(S_4 \geq S_5)=V(S_4 \geq S_9)=1$; $V(S_4 \geq S_7)=0.959$; $V(S_4 \geq S_8) =0.904$

$V(S_5 \geq S_1)=0.935$; $V(S_5 \geq S_2)= V(S_5 \geq S_6)=0.850$; $V(S_5 \geq S_3)= 0.640$; $V(S_5 \geq S_4)= V(S_5 \geq S_9)=0.973$; $V(S_5 \geq S_7)=0.934$; $V(S_5 \geq S_8)=0.882$

$V(S_6 \geq S_1)=V(S_6 \geq S_2)=V(S_6 \geq S_4)=V(S_6 \geq S_5)=V(S_6 \geq S_7)=V(S_6 \geq S_8)=V(S_6 \geq S_9)=1; V(S_6 \geq S_3)=0.78$

$V(S_7 \geq S_1)=V(S_7 \geq S_4)=V(S_7 \geq S_5)=V(S_7 \geq S_9)=1$; $V(S_7 \geq S_2)= V(S_7 \geq S_6)=0.903$; $V(S_7 \geq S_3)=0.672$; $V(S_7 \geq S_8)=0.934$

$V(S_8 \geq S_1)=V(S_8 \geq S_4)=V(S_8 \geq S_5)=V(S_8 \geq S_7)=V(S_8 \geq S_9)=1$; $V(S_8 \geq S_2)= V(S_8 \geq S_6)=0.965$; $V(S_8 \geq S_3)=0.746$

$V(S_9 \geq S_1)=0.960$; $V(S_9 \geq S_2)=0.871$; $V(S_9 \geq S_3)=0.654$; $V(S_9 \geq S_4)=V(S_9 \geq S_5)=1$; $V(S_9 \geq S_6)=0.871$; $V(S_9 \geq S_7)=0.959$; $V(S_9 \geq S_8)=0.904$.

Then, using the equation (10) and (11), the values shown below are obtained.

$d'(A_1)=\min V(S_1 \geq S_2,S_3,S_4,S_5,S_6,S_7,S_8,S_9)=0.687$

$d'(A_2)=\min V(S_2 \geq S_1,S_3,S_4,S_5,S_6,S_7,S_8,S_9)=0.780$

$d'(A_3)=\min V(S_3 \geq S_1,S_2,S_4,S_5,S_6,S_7,S_8,S_9)=1$

$d'(A_4)=\min V(S_4 \geq S_1,S_2,S_3,S_5,S_6,S_7,S_8,S_9)=0.654$

$d'(A_5)=\min V(S_5 \geq S_1,S_2,S_3,S_4,S_6,S_7,S_8,S_9)=0.640$

$d'(A_6)=\min V(S_6 \geq S_1,S_2,S_3,S_4,S_5,S_7,S_8,S_9)=0.780$

$d'(A_7)=\min V(S_7 \geq S_1,S_2,S_3,S_4,S_5,S_6,S_8,S_9)=0,672$

$d'(A_8)=\min V(S_8 \geq S_1,S_2,S_3,S_4,S_5,S_6,S_7,S_9)=0,746$

$d'(A_9)=\min V(S_9 \geq S_1,S_2,S_3,S_4,S_5,S_6,S_7,S_8)=0,654$

After the equation (12) is applied, criteria weights are obtained, and from the equation (13), normalized weights of criteria are determined:

W'=(0.687;0.780;1;0.654;0.640;0.780;0.672;0.746;0.654)

W=(0.104;0.118;0.151;0.099;0.097;0.118;0.102;0.113;0.099)

On the basis of the procedure and obtained results, the most significant criterion in this study is the third criterion, delivery time, which has importance of 15.1%, then the quality and pipe length have the same importance with a share of 11.8 %, while the other criteria have lower values. These three most significant criteria in a large number of practical examples dealing with similar tasks have large importance.

A crisp value from Table 2 is taken to calculate the level of consistency CR = 0.01.

After these values were obtained in order to reach a ranking, and then after making the choice of the suitable variant, it is necessary to compare suppliers in relation to each criterion individually as already described, depending on whether the criteria are quantitative or qualitative. Comparison of suppliers with respect to the first criterion, the cost of materials, is shown in Table 3.

Table 3. Comparison of suppliers with respect to the first criterion

C_1	S_A	S_B	S_C	S_D	S_E
S_A	(1,1,1)	(1,3/2,2)	(3/2,2,5/2)	(2,5/2,3)	(5/2,3,7/2)
S_B	(1/2,2/3,1)	(1,1,1)	(1/2,1,3/2)	(1,3/2,2)	(3/2,2,5/2)
S_C	(2/5,1/2,2/3)	(2/3,1,2)	(1,1,1)	(1/2,1,3/2)	(1,3/2,2)
S_D	(1/3,2/5,1/2)	(1/2,2/3,1)	(2/3,1,2)	(1,1,1)	(1/2,1,3/2)
S_E	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(1/2,2/3,1)	(2/3,1,2)	(1,1,1)

By applying the equation (4), (5) and (6), the following values are obtained:

S_A=(8;10;12)x(1/38.234;1/28.734;1/21.919)=(0.209;0.348;0.547)

S_B=(4.5;6.617;8)x(1/38.234;1/28.734;1/21.919)=(0.118; 0.215; 0.365)

S_C=(3.567;5;7.167)x(1/38.234;1/28.734;1/21.919)=(0.093;0.174;0.327)

S_D=(3;4.067;6) x(1/38.234;1/28.734;1/21.919) =(0.078; 0.142; 0.274)

S_E=(2.852;3.5;5.067) x(1/38.234;1/28.734;1/21.919)=(0. 075;0.122;0.231)

After the application of Eq. (7), values are obtained as described in step three to the amounts:

$V(S_A \geq S_B)=V(S_A \geq S_C)=V(S_A \geq S_D)=V(S_A \geq S_E)=1$

$V(S_B \geq S_A)=0.540$; $V(S_B \geq S_C)=V(S_B \geq S_D)=V(S_B \geq S_E)=1$

$V(S_C \geq S_A)=0.404$;$V(S_C \geq S_B)=0.836$; $V(S_C \geq S_D)=V(S_C \geq S_E) =1$

$V(S_D \geq S_A)=0.240$;$V(S_D \geq S_B)=0.681$;$V(S_D \geq S_C)=0.850$;$V(S_D \geq S_E)=1$

$V(S_E \geq S_A)=0.089$;$V(S_E \geq S_B)=0.549$; $V(S_E \geq S_C)=0.726$;$V(S_E \geq S_D)=0.884$

Then, by applying the equation (8), the following values are obtained:

$d'(A_A)=\min V(S_A \geq S_B,S_C,S_D,S_E)=1$

$d'(A_B)=\min V(S_B \geq S_A,S_C,S_D,S_E,)=0.540$

$d'(A_C)=\min V(S_C \geq S_A,S_B,S_D,S_E)=0.404$

$d'(A_D)=\min V(S_D \geq S_A,S_B,S_C,S_E)=0.240$

$d'(A_E)=minV(S_D \geq S_A,S_B,S_C,S_D,)=0.089$

By applying the equation (10), criteria weight values are computed, and applying equation (11), normalized weight values of criteria are as follow:

W'=(1; 0.540; 0.404; 0.240; 0.089)

W=(0.404; 0.237; 0.178; 0.106; 0.039)

It shows that, according to the first criterion, material prices, the best solution is supplier A. In the same way, values are obtained for suppliers for the remaining eight criteria, whose final values are shown in the table below.

Table 4. Final values for suppliers according to each criterion

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
	0.104	**0.118**	**0.151**	**0.099**	**0.097**	**0.118**	**0.102**	**0.113**	**0.099**
S_A	0.440	0.241	0.280	0.090	0.169	0.285	0.242	0.214	0.264
S_B	0.237	0.241	0.280	0.436	0.133	0.236	0.169	0.170	0.121
S_C	0.178	0.132	0.194	0.161	0.242	0.146	0.242	0.258	0.264
S_D	0.106	0.187	0.246	0.090	0.242	0.146	0.214	0.179	0.264
S_E	0.039	0.199	0	0.223	0.214	0.186	0.133	0.179	0.087

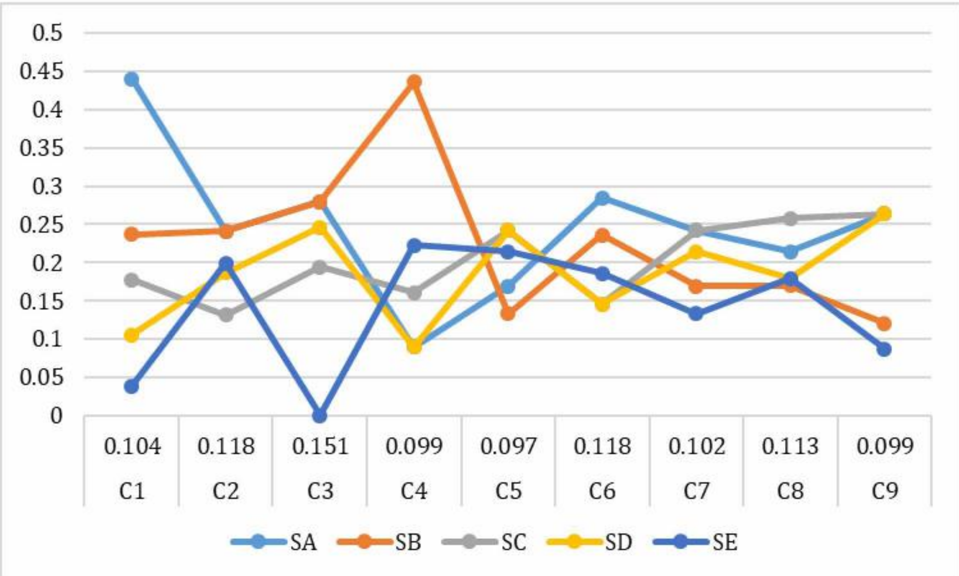

Figure 3. Values of suppliers in relation to criteria

Suppliers' values based on each criterion are shown in Fig. 3, where it is clearly visible which supplier is the best solution, and based on which criteria. Supplier A according to the first criterion and supplier B according to the fourth criterion reach the greatest values.

In order to choose the best solution and the best supplier, obtained values for suppliers from Table 4 should be multiplied by the values of the criteria in the following way:

$A_A=W_{K1}*WA_A+W_{K2}*WA_A+W_{K3}*WA_A+W_{K4}*WA_A+W_{K5}*WA_A+W_{K6}*WA_A+W_{K7}*WA_A+W_{K8}*WA_A+W_{K9}*WA_A$

$A_B=W_{K1}*WA_B+W_{K2}*WA_B+W_{K3}*WA_B+W_{K4}*WA_B+W_{K5}*WA_B+W_{K6}*WA_B+W_{K7}*WA_B+W_{K8}*WA_B+W_{K9}*WA_B$

$A_C=W_{K1}*WA_C+W_{K2}*WA_C+W_{K3}*WA_C+W_{K4}*WA_C+W_{K5}*WA_C+W_{K6}*WA_C+W_{K7}*WA_C+W_{K8}*WA_C+W_{K9}*WA_C$

$A_D=W_{K1}*WA_D+W_{K2}*WA_D+W_{K3}*WA_D+W_{K4}*WA_D+W_{K5}*WA_D+W_{K6}*WA_D+W_{K7}*WA_D+W_{K8}*WA_D+W_{K9}*WA_D$

$A_E=W_{K1}*WA_E+W_{K2}*WA_E+W_{K3}*WA_E+W_{K4}*WA_E+W_{K5}*WA_E+W_{K6}*WA_E+W_{K7}*WA_E+W_{K8}*WA_E+W_{K9}*WA_E$

Application of previously described methodology leads to results shown in Figure 4.

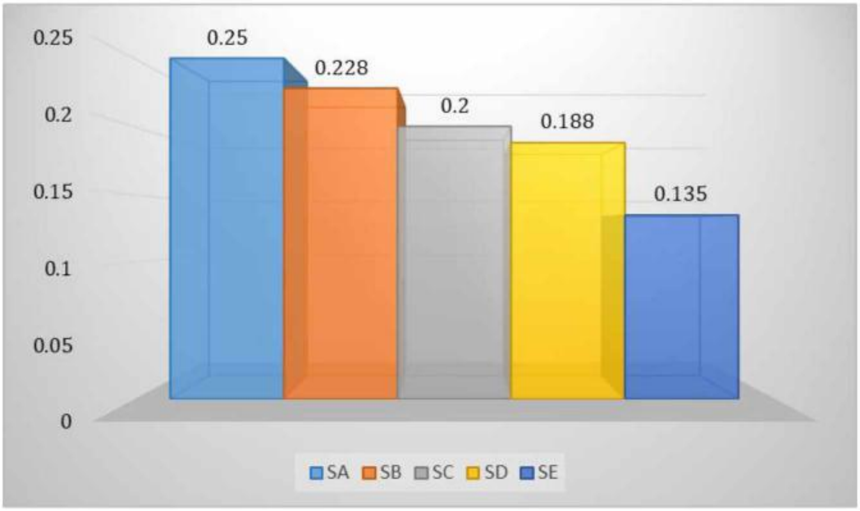

Figure 4. Ranking alternatives

Since suppliers A and B have the maximum values according to the first and fourth criterion, when it comes to comparison of alternatives against the criteria, modeling the results in the best way possible can be done by a change of their values. Obtained results in which supplier A is the most suitable solution are valid if the lower limit of the first criterion is 0.063 and the upper limit of the fourth criterion is 0.140. Given the fact that, based on the great number of criteria, the selected supplier stands out as the best or equally best solution, which can be seen in Table 4, to change the obtained results, except for the previous modeling, it is necessary to change the weight value largely.

5. Conclusion

Making decisions based on overview of great number of different criteria that largely are influencing efficiency of day–to–day business is certainly a challenge because multiple criteria are to be satisfied, which sometimes may be opposed. Procurement logistics in today's modern age is a very important factor in a complete supply chain, so its optimization can ascertain a certain effect on the entire logistics system. It is necessary to take into account a number of criteria that could affect the formation of the final price of the product, and therefore the position company achieves in the market. Application of fuzzy AHP Method makes decisions possible, by taking into account the importance of criteria and their relative priority that reflects market demands and needs. By using the fuzzy AHP Method in this study, it can be concluded that the purchase of steel pipes for the production of pre-insulted pipes should be done from supplier A.

After the sensitivity analysis, it can be concluded that the model is stable because, in the case of changing the importance of the criteria up to 30%, results remain the same and the chosen solution remains first ranked. This means that a change of results obtained requires large turbulences in the market, both in terms of suppliers and their characteristics, as well as from the aspect of user requirements.

Depending on market trends where demands and needs change frequently, it is necessary to apply the methods of multi-criteria analysis more often to adopt appropriate decisions that ensure efficient operations which can be one way of future research.

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THE APPLICATION OF THE HYBRID INTERVAL ROUGH WEIGHTED POWER HERONIAN OPERATOR IN MULTI-CRITERIA DECISION-MAKING

Dragan Pamučar [1*], Aleksandar Janković [2]

[1] Department of logistics, Military Academy, University of Defence in Belgrade, Belgrade, Serbia
[2] Ministry of Transport and Maritime Affairs, Directorate for Transport, Montenegro

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Abstract: *In this paper, a new multi-criteria model which enables the processing of uncertainty and inaccuracy data through the application of interval rough numbers (IRN) is presented. The multi-criteria model represents the integration of the Power Aggregator (PA) and the Weighted Heronian Mean (WHM) operators. The goal of the forming of a hybrid Weighted Power Heronian Mean (WPHM) is to integrate the advantages of both operators into a single multi-criteria model, which has the following advantages: 1) it eliminates the influence of unreasonable arguments; 2) it takes into account the degree of support between input arguments and 3) it takes into account the interconnectedness of input arguments. Based on the mathematical concept of the IRN, the hybrid WPHM operator was extended and the IRNWPHM multi-criteria model was created. The IRNWPHA multi-criteria model enables objective decision-making in the case of imprecise input parameters in the initial decision matrix. Also, the IRNWPHA model allows flexible decision-making and the verification of the robustness of results through a variation of the* p *and* q *parameters. The IRNWPHM model was tested on a real-world multi-criteria example. The results showed that the IRNWPHM operator enabled a successful transformation of the uncertainties and inaccuracies that exist in group decision-making.*

Key words: *interval rough numbers; Heronian mean; multi-criteria decision-making; power operator.*

1. Introduction

The information that appears in real-world problems is often very difficult to quantify, since many facts, such as the complexity of phenomena and the ambiguity of human reasoning, represent significant limitations. In the multi-criteria modeling of decisions, different decision-makers are likely to use the linguistic expressions of a

* Corresponding author.
dpamucar@gmail.com (D. Pamucar), aleksandar.jankovic@gmail.com (A. Janković)

different precision to express their preferences (Herrera and Martínez, 2000). In such situations, uncertainty theories, such as fuzzy sets (Zadeh, 1965), rough sets (Pawlak, 1982) and the other generalizations of the mentioned theories, are a good tool for presenting uncertainty.

In order to reach the best solution in group multi-criteria models, operators for the aggregation of group preferences and the calculation of the criterion functions of alternatives have been developed. In general, aggregation operators are important tools for the fusion of information into multi-criteria problems, which can also be used to evaluate alternatives. To date, many information fusion operators that can be used in decision-making models have been developed (Beliakov et al., 2007; Xu et al., 2012; Liu et al., 2015), including: the Bonferroni mean (Bonferroni, 1950), the Hamy mean (Hara et al., 1998), the Dombi operators (Dombi, 1982), the Maclaurin mean (Maclaurin, 1729), the Heronian mean (Beliakov, 2007), the Muirhead mean (Muirhead, 1902), Power aggregation (Yager , 2001) and numerous hybrid forms of aggregation operators (Pamučar et al., 2020; Sinani et al., 2020).

A better understanding of correlations between attributes can be very important for making objective decisions, so it is necessary to take into account the fact that relationships between attributes can be a significant determinant of an aggregated outcome. Therefore, the operators that have this feature have attracted significant attention in multi-criteria decision-making. Based on the analysis presented by Liu et al. (2016), it can be concluded that some aggregation operators only take into account the significance of the information presented in a decision matrix, while the interrelationships between data are neglected. Although there are aggregation operators which respect interrelationships between data, there are still significant shortcomings of some aggregation operators that need to be highlighted. For example, when aggregating data, power aggregation (Yager, 2001) only takes into account the influence of a change in the vector of the weight coefficients of criteria on aggregated values. At the same time, Power Aggregation (PA) does not take into account the relationships between aggregated arguments. On the other hand, Bonferroni mean (BM) operators respect the correlation between the attributes C_i and C_j ($i \neq j$, $C = \{C_1, C_2, ..., C_n\}$), but ignore the relationship between the attributes C_i and itself. Considering the correlations between the attributes using the BM may also lead to redundancy (Liu et al., 2016). BM operators consider the correlation between C_i and C_j ($i \neq j$) and the simultaneous correlation between C_j and C_i ($i \neq j$), which may result in potential redundancy (Dutta et al., 2015).

Some of the requirements for decision-making in real-world systems include flexible decision-making, respect for the mutual influence between decision attributes and the elimination of the influence of awkward data. To achieve this goal, the integration of the PA and the weighted Heronian mean (WHM) operators into a hybrid WPHA operator is presented in this paper. The HM operator is a very useful tool, which takes into consideration the relationships between the attributes being aggregated. The WPHA operator combines the advantages of the PA and HM operators, and is a powerful tool with the following features: 1) it eliminates the influence of unreasonable arguments; 2) it takes into account the degree of support between input arguments, and 3) it takes into account the interconnectedness of input arguments. In recent years, the advantages of aggregation operators have been implemented through multi-criteria models in a number of uncertainty theories: fuzzy sets (Pamucar et al., 2020; Ecer and Pamucar, 2020), intuitionistic fuzzy sets (Xu and Yager, 2011; He and He, 2016), interval- valued intuitionistic fuzzy sets (Liu and Li, 2017), hesitant fuzzy sets (He et al., 2015), rough sets (Sremac et al., 2018; Pamucar et al., 2018; Yazdani et al., 2020) and so on. To the authors' knowledge, no study considering the fusion of the PA and WHM aggregators in an interval rough environment has been carried out to date. Therefore, the logical goal and motivation for this study imply the presentation of a hybrid IRNWPHM operator. In this paper, Interval rough numbers were used to exploit uncertainties and inaccuracies, as they have certain advantages over traditional fuzzy sets (Yazdani et al., 2020). These advantages are especially evident when IRNs are applied in group decision-making.

The rest of the paper is organized into the next six sections. After the Introduction, the preliminaries of IRNs are presented in the second section of the paper. In the third section, the mathematical integration of the WHM and PA operators in an IRN environment is presented. In the fourth section of the paper, the structure of the multi-criteria IRN WPHM model is presented. In the fifth section, the model was tested on a real-world example, and the results were validated through the variation of the *p* and *q* parameters. Finally, the concluding remarks are given in the sixth section of the paper.

2. Interval Rough Numbers

Assume that *U* is the universe containing all the objects registered in an information table. Assume that there is a set of the k classes representing the DM's preferences $R=(J_1,J_2,...,J_k)$ provided that they belong to the row that satisfies the condition $J_1<J_2<...<J_k$ and another set of the k classes that also represent the DM's preferences $R^*=(I_1,I_2,...,I_k)$. Assuming that all the objects are defined in the universe and related to the DM's preferences. In R^*, every class of objects is represented by the interval $I_i=\{I_{li},I_{ui}\}$, provided that $I_{li}\le I_{ui}$ $(1\le i\le m)$, and $I_{li},I_{ui}\in R$ are satisfied. Then, I_{li} denotes the lower interval limit, while I_{ui} denotes the upper interval limit of the i class. If both class limits (the lower and the upper limits) presented so as $I^*_{l1}<I^*_{l2}<,...,<I^*_{lj},I^*_{u1}<I^*_{u2}<,...,<I^*_{uk}$ $(1\le j,k\le m)$ are satisfied, respectively, then the two new sets containing the lower class $R^*_l=(I^*_{l1},I^*_{l2},...,I^*_{lj})$ and the upper class $R^*_u=(I^*_{u1},I^*_{u2},...,I^*_{uk})$ can be defined, respectively. If that is the case, then for any class $I^*_{li}\in R$ $(1\le i\le j)$ and $I^*_{ui}\in R$ $(1\le i\le k)$, the lower approximation of I^*_{li} and I^*_{ui} can be defined as follows (Pamucar et al., 2018):

$$\underline{Apr}(I^*_{li})=\cup\left\{Y\in U/R^*_l(Y)\le I^*_{li}\right\} \tag{1}$$

$$\underline{Apr}(I^*_{ui})=\cup\left\{Y\in U/R^*_u(Y)\le I^*_{ui}\right\} \tag{2}$$

The above-mentioned approximations of I^*_{li} and I^*_{ui} are defined by applying the following equation:

$$\overline{Apr}(I^*_{li})=\cup\left\{Y\in U/R^*_l(Y)\ge I^*_{li}\right\} \tag{3}$$

$$\overline{Apr}(I_{ui}^{*}) = \cup\left\{Y \in U / R_{u}^{*}(Y) \geq I_{ui}^{*}\right\} \tag{4}$$

Both object classes (the upper and the lower classes (I_{li}^{*} and I_{ui}^{*}, respectively)) are defined by their lower limits $\underline{Lim}(I_{li}^{*})$ and $\underline{Lim}(I_{ui}^{*})$, and by their upper limits $\overline{Lim}(I_{li}^{*})$ and $\overline{Lim}(I_{ui}^{*})$, respectively:

$$\underline{Lim}(I_{li}^{*}) = \frac{1}{M_L}\sum R_{l}^{*}(Y)\left|Y \in \underline{Apr}(I_{li}^{*})\right. \tag{5}$$

$$\underline{Lim}(I_{ui}^{*}) = \frac{1}{M_L^{*}}\sum R_{u}^{*}(Y)\left|Y \in \underline{Apr}(I_{ui}^{*})\right. \tag{6}$$

where M_L and M_L^{*} denote the number of the objects contained in the lower approximations I_{li}^{*} and I_{ui}^{*}, respectively. The upper limits $\overline{Lim}(I_{li}^{*})$ and $\overline{Lim}(I_{ui}^{*})$ are defined by the equations (7) and (8), as follows:

$$\overline{Lim}(I_{li}^{*}) = \frac{1}{M_U}\sum R_{l}^{*}(Y)\left|Y \in \overline{Apr}(I_{li}^{*})\right. \tag{7}$$

$$\overline{Lim}(I_{ui}^{*}) = \frac{1}{M_U^{*}}\sum R_{u}^{*}(Y)\left|Y \in \overline{Apr}(I_{ui}^{*})\right. \tag{8}$$

where M_U and M_U^{*} denote the number of the objects contained in the upper approximations I_{li}^{*} and I_{ui}^{*}, respectively.

For the lower class of objects, the rough boundary interval from I_{li}^{*} is represented as $RB(I_{li}^{*})$ and denotes the interval between the lower and the upper limits:

$$RB(I_{li}^{*}) = \overline{Lim}(I_{li}^{*}) - \underline{Lim}(I_{li}^{*}), \tag{9}$$

while for the upper object class, the rough boundary interval I_{ui}^{*} is obtained based on the following equation:

$$RB(I_{ui}^{*}) = \overline{Lim}(I_{ui}^{*}) - \underline{Lim}(I_{ui}^{*}) \tag{10}$$

Then, the uncertain class of the objects I_{li}^{*} and I_{ui}^{*} can be expressed by using their lower and upper limits, as follows:

$$RN(I_{li}^{*}) = \left[\overline{Lim}(I_{li}^{*}), \underline{Lim}(I_{li}^{*})\right] \tag{11}$$

$$RN(I_{ui}^{*}) = \left[\overline{Lim}(I_{ui}^{*}), \underline{Lim}(I_{ui}^{*})\right] \tag{12}$$

It can be seen that every class of objects is defined by its lower and upper limits that create the interval rough number that can be defined as:

$$IRN(I_{i}^{*}) = \left[RN(I_{li}^{*}), RN(I_{ui}^{*})\right] \tag{13}$$

Interval rough numbers are characterized by specific arithmetic operations, which differ from those dealing with typical rough numbers. Arithmetic operations between two interval rough numbers $IRN(A)=\left(\left[a_1,a_3\right],\left[a_3,a_4\right]\right)$ and $IRN(B)=\left(\left[b_1,b_2\right],\left[b_3,b_4\right]\right)$ are done by applying the following expressions (14), (15), (16), (17) and (18) (Pamučar et al., 2019):

(1) The addition of interval rough numbers "+"

$$IRN(A)+IRN(B)=\left(\left[a_1,a_2\right],\left[a_3,a_4\right]\right)+\left(\left[b_1,b_2\right],\left[b_3,b_4\right]\right)$$
$$=\left(\left[a_1+b_1,a_2+b_2\right],\left[a_3+b_3,a_4+b_4\right]\right) \tag{14}$$

(2) The subtraction of interval rough numbers "-"

$$IRN(A)-IRN(B)=\left(\left[a_1,a_2\right],\left[a_3,a_4\right]\right)-\left(\left[b_1,b_2\right],\left[b_3,b_4\right]\right)$$
$$=\left(\left[a_1-b_4,a_2-b_3\right],\left[a_3-b_2,a_4-b_1\right]\right) \tag{15}$$

(3) The multiplication of interval rough numbers "×"

$$IRN(A)\times IRN(B)=\left(\left[a_1,a_2\right],\left[a_3,a_4\right]\right)\times\left(\left[b_1,b_2\right],\left[b_3,b_4\right]\right)$$
$$=\left(\left[a_1\times b_1,a_2\times b_2\right],\left[a_3\times b_3,a_4\times b_4\right]\right) \tag{16}$$

(4) The division of interval rough numbers "/"

$$IRN(A)/IRN(B)=\left(\left[a_1,a_2\right],\left[a_3,a_4\right]\right)/\left(\left[b_1,b_2\right],\left[b_3,b_4\right]\right)$$
$$=\left(\left[a_1/b_4,a_2/b_3\right],\left[a_3/b_2,a_4/b_1\right]\right) \tag{17}$$

and

(5) The scalar multiplication of interval rough numbers, where $k>0$

$$k\times IRN(A)=k\times\left(\left[a_1,a_2\right],\left[a_3,a_4\right]\right)=\left(\left[k\times a_1,k\times a_3\right],\left[k\times a_3,k\times a_4\right]\right) \tag{18}$$

3. Interval Rough Weight Power Heronian Operator

The Power Aggregation (PA) operator proposed by Yager (2001) is a very significant aggregation operator, which eliminates the impact of unreasonable arguments. The traditional PA operator can be defined as follows:

Definition 1 (Yager, 2001): Let ($\xi_1,\xi_2,...,\xi_n$) be a set of non-negative numbers and *p,q* ≥ 0. If

$$PA(\xi_1,\xi_2,...,\xi_n)=\frac{\sum_{i=1}^{n}\left(1+T(\xi_i)\right)\xi_i}{\sum_{i=1}^{n}\left(1+T(\xi_i)\right)} \tag{19}$$

where $T(\xi_i)=\sum_{j=1,j\neq i}^{n}Sup(\xi_i,\xi_j)$ and $Sup(\xi_i,\xi_j)$ denote the degree of the support that ξ_i received from ξ_j, where $Sup(\xi_i,\xi_j)$ satisfies the following axioms:

1) $Sup(\xi_i,\xi_j)=Sup(\xi_j,\xi_i)$;
2) $Sup(\xi_i,\xi_j)=\left[0,1\right]$;
3) $Sup(\xi_i,\xi_j)>Sup(\xi_i,\xi_k),\ if\ \left|\xi_i-\xi_j\right|<\left|\xi_i-\xi_k\right|$.

The Heronian Mean (HM) operator was proposed by Beliakov (2007). The HM takes into account the interconnectedness between input arguments (Liu and Pei, 2012). The HM operator can be defined as follows:

Definition 2 (Yu, 2013): Let $p,q \geq 0$, $(\xi_1, \xi_2, ..., \xi_n)$ be a set of non-negative numbers. If

$$HM^{p,q}(\xi_1, \xi_2, ..., \xi_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \xi_i^p \xi_j^q \right)^{\frac{1}{p+q}} \tag{20}$$

then $HM^{p,q}$ is called the Heronian Mean (HM) operator.

Definition 3 (Zhao, 2019): Let $p,q \geq 0$ and $(\xi_1, \xi_2, ..., \xi_n)$ represent a set of non-negative numbers. Then, the Weight Heronian Mean (WHM) operator for averaging can be defined as follows:

$$WHM^{p,q}(\xi_1, \xi_2, ..., \xi_n) = \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \left((nw_i \xi_i)^p \cdot (nw_j \xi_j)^q \right) \right)^{\frac{1}{p+q}} \tag{21}$$

where $WHM^{p,q}$ is called the Weighted Heronian Mean (WHM) operator.

Based on the defined settings of the PA and WHM operators, Eqs. (19) and (21), in the following part a hybrid Interval Rough Weighted Power Heronian Aggregation (IRWPHA) operator was developed.

Definition 4: Set $\xi_i = \left(\left[\xi_i^L, \xi_i^U \right], \left[\xi_i'^L, \xi_i'^U \right] \right)$ (*i = 1,2,..,n*) as a collection of IRNs in Ω ; then the IRWPHA can be defined as follows:

$$IRNWPHA^{p,q}(\xi_1, \xi_2, ..., \xi_n) = \left(\frac{2}{n(n+1)} \sum_{i=1,j=i}^{n} \left(\left(n \frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t} \xi_1 \right)^p \cdot \left(n \frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t} \xi_j \right)^q \right) \right)^{\frac{1}{p+q}} \tag{22}$$

where $w_t = \dfrac{(1+T(\xi_i))}{\sum_{i=1}^{n} (1+T(\xi_i))}$, $T(\xi_i) = \sum_{j=1,j\neq i}^{n} Sup(\xi_i, \xi_j)$ and $\sum_{i=1}^{n} w_i = 1$,

where $Sup(\xi_i, \xi_j)$ denote the degree of the support that ξ_i received from ξ_j, where $Sup(\xi_i, \xi_j)$ satisfies the following axioms (Đordević et al., 2019):

1) $Sup(f(\xi_i), f(\xi_j)) = Sup(f(\xi_j), f(\xi_i))$;
2) $Sup(f(\xi_i), f(\xi_j)) = [0,1]$;
3) $Sup(f(\xi_i), f(\xi_j)) > Sup(f(\xi_i), f(\xi_k)), \; if \; d(\xi_i, \xi_j) < d(\xi_i, \xi_k)$, where $d(\xi_i, \xi_j)$ represents the distance between the numbers ξ_i and ξ_j.

Then $IRNWPHA^{p,q}$ represents the IRN weight power Heronian aggregation operator.

Theorem 1: Set $\xi_i = \left(\left[\xi_i^L, \xi_i^U \right], \left[\xi_i'^L, \xi_i'^U \right] \right)$ as a collection of IRNs in Ω ; then, according to Eq. (22), aggregation results are obtained for IRNs, and the following aggregation formula can be developed:

$$IRNWPHA^{p,q}(\xi_1,\xi_2,...,\xi_n)=\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i\right)^p\cdot\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j\right)^q\right)\right)^{\frac{1}{p+q}}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i^L\right)^p\cdot\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j^L\right)^q\right)\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i^U\right)^p\cdot\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j^U\right)^q\right)\right)^{\frac{1}{p+q}}\end{array}\right],\\ \left[\begin{array}{c}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i'^L\right)^p\cdot\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j'^L\right)^q\right)\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i'^U\right)^p\cdot\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j'^U\right)^q\right)\right)^{\frac{1}{p+q}}\end{array}\right]\end{array}\right) \tag{23}$$

Proof:

The proof for *Theorem 1* is presented in the following section. Based on the equations (19) and (22), the following is obtained:

a) $nw_i\xi_i=n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i$ and $nw_j\xi_j=n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j$;

b) $$\left(n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i\right)^p=\left(\begin{array}{c}\left[n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i^L,n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i^U\right],\\ \left[n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i'^L,n\frac{nw_iw_i}{\sum_{t=1}^{n}w_tw_t}\xi_i'^U\right]\end{array}\right)^p;$$

c) $$\left(n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j\right)^q=\left(\begin{array}{c}\left[n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j^L,n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j^U\right],\\ \left[n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j'^L,n\frac{nw_iw_j}{\sum_{t=1}^{n}w_tw_t}\xi_j'^U\right]\end{array}\right)^q;$$

d)

$$\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j\right)^q=\left(\begin{bmatrix}\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^L\right)^p\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^L\right)^q,\\ \left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^U\right)^p\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^U\right)^q\end{bmatrix},\begin{bmatrix}\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i'^L\right)^p\left(n\frac{n\overline{w_i w_j}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_j'^L\right)^q,\\ \left(n\frac{n\overline{w_i w_i}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_i'^U\right)^p\left(n\frac{n\overline{w_i w_j}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_j'^U\right)^q\end{bmatrix}\right)$$

Finally, the equation for IRN is obtained by means of the weight power Heronian operator ($IRNWPHA^{p,q}$) for aggregation, as follows:

$$IRNWPHA^{p,q}(\xi_1,\xi_2,\ldots,\xi_n)=\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j\right)^q\right)\right)^{\frac{1}{p+q}}$$

$$=\left(\begin{bmatrix}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^L\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^L\right)^q\right)\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{nw_i w_i}{\sum_{t=1}^{n} w_t w_t}\xi_i^U\right)^p\cdot\left(n\frac{nw_i w_j}{\sum_{t=1}^{n} w_t w_t}\xi_j^U\right)^q\right)\right)^{\frac{1}{p+q}}\end{bmatrix},\begin{bmatrix}\left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{n\overline{w_i w_i}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_i'^L\right)^p\cdot\left(n\frac{n\overline{w_i w_j}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_j'^L\right)^q\right)\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1,j=i}^{n}\left(\left(n\frac{n\overline{w_i w_i}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_i'^U\right)^p\cdot\left(n\frac{n\overline{w_i w_j}}{\sum_{t=1}^{n} \overline{w_t w_t}}\xi_j'^U\right)^q\right)\right)^{\frac{1}{p+q}}\end{bmatrix}\right)$$

So, *Theorem* 1 is true.

Theorem 2 (Idempotency): Set $\xi_i=\left(\left[\xi_i^L,\xi_i^U\right],\left[\xi_i'^L,\xi_i'^U\right]\right)$ as a collection of IRNs in Ω ; if $\xi_i=\xi$, then $IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n)=IRNWPHA^{p,q}(\xi,\xi,..,\xi)$.

Proof:

Since $\xi_i=\xi$, i.e. $\xi_i^L=\xi^L$, $\xi_i^U=\xi^U$, $\xi_i'^L=\xi'^L$, $\xi_i'^U=\xi'^U$, then

$$IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n)=IRNWPHA^{p,q}(\xi,\xi,..,\xi)$$

$$=\left(\begin{array}{l}\left[\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi_t^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{L}\right)\right)}\xi_i^{L}\right)^{p}\left(\frac{n\left(1+T\left(\xi_j^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{L}\right)\right)}\xi_j^{L}\right)^{q}\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi_i^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{U}\right)\right)}\xi_i^{U}\right)^{p}\left(\frac{n\left(1+T\left(\xi_j^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{U}\right)\right)}\xi_j^{U}\right)^{q}\right)^{\frac{1}{p+q}}\end{array}\right],\\ \left[\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi_i'^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{L}\right)\right)}\xi_i'^{L}\right)^{p}\left(\frac{n\left(1+T\left(\xi_j'^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{L}\right)\right)}\xi_j'^{L}\right)^{q}\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi'^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{U}\right)\right)}\xi_i'^{U}\right)^{p}\left(\frac{n\left(1+T\left(\xi'^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{U}\right)\right)}\xi_j'^{U}\right)^{q}\right)^{\frac{1}{p+q}}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{l}\left[\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{L}\right)\right)}\xi^{L}\right)^{p}\left(\frac{n\left(1+T\left(\xi^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{L}\right)\right)}\xi^{L}\right)^{q}\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{U}\right)\right)}\xi^{U}\right)^{p}\left(\frac{n\left(1+T\left(\xi^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{U}\right)\right)}\xi^{U}\right)^{q}\right)^{\frac{1}{p+q}}\end{array}\right],\\ \left[\begin{array}{l}\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi_t'^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t^{U}\right)\right)}\xi'^{L}\right)^{p}\left(\frac{n\left(1+T\left(\xi'^{L}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{L}\right)\right)}\xi'^{L}\right)^{q}\right)^{\frac{1}{p+q}},\\ \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{n\left(1+T\left(\xi'^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{U}\right)\right)}\xi'^{U}\right)^{p}\left(\frac{n\left(1+T\left(\xi'^{U}\right)\right)}{\sum_{t=1}^{n}\left(1+T\left(\xi_t'^{U}\right)\right)}\xi'^{U}\right)^{q}\right)^{\frac{1}{p+q}}\end{array}\right]\end{array}\right)$$

$$=\left(\begin{array}{l}\left[\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(n\frac{1}{n}\xi^{L}\right)^{p}\left(n\frac{1}{n}\xi^{L}\right)^{q}\right)^{\frac{1}{p+q}},\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(n\frac{1}{n}\xi^{U}\right)^{p}\left(n\frac{1}{n}\xi^{U}\right)^{q}\right)^{\frac{1}{p+q}}\right],\\ \left[\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(n\frac{1}{n}\xi^{L}\right)^{p}\left(n\frac{1}{n}\xi''^{L}\right)^{q}\right)^{\frac{1}{p+q}},\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(n\frac{1}{n}\xi^{U}\right)^{p}\left(n\frac{1}{n}\xi''^{U}\right)^{q}\right)^{\frac{1}{p+q}}\right]\end{array}\right)$$

$$=\left(\begin{array}{c}\left[\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi^{L}\right)^{p+q}\right)^{\frac{1}{p+q}},\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi^{U}\right)^{p+q}\right)^{\frac{1}{p+q}}\right],\\ \left[\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi'^{L}\right)^{p+q}\right)^{\frac{1}{p+q}},\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\xi'^{U}\right)^{p+q}\right)^{\frac{1}{p+q}}\right]\end{array}\right)=\xi$$

The *Theorem 2* proof is completed.

Theorem 3 (Boundedness): Set $\xi_i=\left(\left[\xi_i^L,\xi_i^U\right],\left[\xi_i'^L,\xi_i'^U\right]\right)$ as a collection of RNs in Ω; let $\xi^-=\left(\left[\min(\xi_i^L),\min(\xi_i^U)\right],\left[\min(\xi_i'^L),\min(\xi_i'^U)\right]\right)$ and $\xi^+=\left(\left[\max(\xi_i^L),\max(\xi_i^U)\right],\left[\max(\xi_i'^L),\max(\xi_i'^U)\right]\right)$, then $\xi^-\leq IRNWPHA^{p,q}(\xi_1,\xi_2,...,\xi_n)\leq\xi^+$.

Proof:

Let $\xi^-=\min(\xi_1,\xi_2,...,\xi_n)=\left(\left[\min(\xi_i^L),\min(\xi_i^U)\right],\left[\min(\xi_i'^L),\min(\xi_i'^U)\right]\right)$ and

$\xi^+=\max(\xi_1,\xi_2,...,\xi_n)=\left(\left[\max(\xi_i^L),\max(\xi_i^U)\right],\left[\max(\xi_i'^L),\max(\xi_i'^U)\right]\right)$; then, it can be said that $\xi_i^{L-}=\min_i(\xi_1^L)$, $\xi_i^{U-}=\min_i(\xi_i^U)$, $\xi_i'^{L-}=\min_i(\xi_i'^L)$, $\xi_i'^{U-}=\min_i(\xi_i'^U)$, $\xi_i^{L+}=\max_i(\xi_i^L)$, $\xi_i^{U+}=\max_i(\xi_i^U)$, $\xi_i'^{L+}=\max_i(\xi_i'^L)$, $\xi_i'^{U+}=\max_i(\xi_i'^U)$. Based on that, the following inequalities can be formulated:

$\xi^-\leq\xi_i\leq\xi^+$;

$\min_i(\xi_j^L)\leq\xi_j^L\leq\max_i(\xi_j^L)$;

$\min_i(\xi_j^U)\leq\xi_j^U\leq\max_i(\xi_j^U)$;

$\min_i(\xi_j'^L)\leq\xi_j'^L\leq\max_i(\xi_j'^L)$;

$\min_i(\xi_j'^U)\leq\xi_j'^U\leq\max_i(\xi_j'^U)$.

According to the above-shown inequalities, it can be concluded that $\xi^-\leq IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n)\leq\xi^+$ holds.

Theorem 4 (Commutativity): Let the interval rough set $(\xi_1',\xi_2',...,\xi_n')$ be any permutation of $(\xi_1,\xi_2,..,\xi_n)$. Then, $IRNWPHA^{p,q}(\xi_1,\xi_2,..,\xi_n)=IRNWPHA^{p,q}(\xi_1',\xi_2',..,\xi_n')$. *Proof*: This property is obvious.

4. The IRNWPHA Model for Multi-Criteria Decision-Making

Based on the IRNWPHA operator, a model for the group multi-criteria evaluation of alternatives that includes the following steps can be defined:

Step 1. The formation of the initial decision matrix. Defining a set of the experts E_e ($1 \le e \le t$, t represents the number of the experts) who evaluate alternatives and form expert correspondent matrices $X^e = \left[x_{ij}^e; x_{ij}^e \right]_{m \times n}$ ($1 \le e \le t$). The aggregation of the expert matrices $X = \left[x_{ij} \right]_{m \times n}$ into the initial decision matrix was performed by using the IRN power Heronian aggregator (Đordević et al., 2019).

Step 2. The initial decision matrix normalization. The normalization of the initial decision matrix is performed by applying the equation (24). Thus, the normalized matrix $N = \left[IRN(n_{ij}) \right]_{m \times n}$ is formed.

$$n_{ij} = \begin{cases} x_{ij} / \sum_{i=1}^{n} x_{ij} & if \quad j \in B \\ 1 - x_{ij} / \sum_{i=1}^{n} x_{ij} & if \quad j \in C \end{cases} \tag{24}$$

Step 3. The determination of the criterion function alternatives. By using IRNWPHA (23), the score function values $H(n_i) = IRNWPHA^{p,q,r}\left(IRN(n_1), IRN(n_2), ..., IRN(n_m) \right)$ are obtained, representing the final values of the preferences by the alternatives.

Step 4. Ranking alternatives. The ranking of the alternatives $\{A_1, A_1, ..., A_m\}$ is done based upon the value of the criterion function $H(n_i)$, where the alternative that has a higher value $H(n_i)$ is preferable.

5. Case Study

In the following section, the application of the IRNWPHA multi-criteria model for solving real-world problems is discussed. The IRNWPHA model was applied to evaluate the work of the advisors in dangerous goods transport. The criteria accounted for in Table 1 were taken from a study by Pamucar et al. (2019), in which the application of a linguistic neutrosophic methodology in order to evaluate advisors' work was considered.

Table 1. The criteria for the evaluation of advisors' work (Pamucar et al. 2019)

Number	Criteria	Type
1.	The knowledge of regulations and professional development	Benefit
2.	The analytical processing of the established requirements	Benefit
3.	The quality of the proposed measures	Benefit
4.	The level of implementation of the proposed measures	Benefit
5.	The quality of the professional training of the employees	Benefit
6.	A response to situations of emergency	Benefit
7.	The preparation of documents	Benefit
8.	The method for solving professional questions	Benefit
9.	Activity in professional bodies	Benefit

A total of eight experts participated in the research (e_i, $i = 1,2,...,8$). The experts used the following nine-point scale to evaluate the work of the ten advisors (A_i, $i = 1,2,...,10$): 1 – Very low (VL); 2 – Medium low (ML); 3 – Low (L); ... ; 8 – High (H); 9 – Very high (VH). The weighting coefficients of the criteria were taken from the Pamucar et al. (2019) study:

$w_j = (0.1178, 0.0875, 0.1020, 0.1087, 0.1302, 0.0904, 0.0838, 0.1163, 0.1632)^T$.

In the following section, the application of the IRNWPHA is presented through the steps defined in the previous section of the paper:

Step 1 - The formation of the initial decision matrix:

Eight experts evaluated the advisors using a nine-point scale. Expert correspondent matrices with evaluation of advisors are shown in Table 2.

Table 2. The expert correspondent matrices

Expert 1								
Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A1	(3;3)	(5;6)	(7;8)	(1;2)	(5;6)	(3;4)	(3;4)	(5;6)
A2	(8;9)	(7;8)	(5;6)	(9;9)	(5;6)	(5;6)	(7;8)	(5;6)
A3	(6;5)	(3;4)	(1;2)	(3;4)	(3;3)	(5;6)	(9;9)	(5;6)
A4	(4;5)	(3;3)	(3;4)	(7;8)	(5;5)	(5;6)	(9;9)	(5;6)
A5	(7;7)	(7;8)	(9;9)	(5;6)	(7;7)	(5;6)	(5;6)	(5;6)
A6	(5;5)	(3;4)	(5;6)	(1;2)	(3;3)	(3;4)	(3;4)	(5;6)
A7	(5;5)	(5;5)	(3;4)	(1;1)	(7;7)	(5;6)	(1;1)	(3;4)
A8	(6;7)	(9;9)	(5;6)	(1;1)	(7;8)	(5;6)	(5;6)	(5;5)
A9	(5;5)	(3;4)	(3;4)	(1;2)	(3;4)	(5;6)	(3;4)	(5;6)
A10	(4;5)	(5;5)	(5;6)	(3;3)	(5;5)	(3;4)	(5;6)	(7;7)
...								
Expert 8								
Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A1	(5;6)	(9;9)	(7;8)	(7;8)	(1;2)	(3;3)	(8;9)	(7;8)
A2	(9;9)	(9;9)	(9;9)	(8;9)	(9;9)	(9;9)	(9;9)	(7;7)
A3	(7;8)	(3;4)	(5;6)	(7;8)	(8;9)	(8;9)	(7;8)	(7;8)
A4	(9;9)	(8;9)	(8;9)	(9;9)	(8;9)	(8;9)	(9;9)	(5;6)
A5	(7;8)	(7;8)	(5;6)	(7;8)	(8;9)	(5;6)	(7;8)	(5;6)
A6	(7;8)	(3;4)	(5;5)	(7;8)	(8;9)	(5;5)	(8;9)	(8;9)
A7	(7;8)	(5;6)	(5;5)	(1;1)	(8;9)	(8;9)	(8;9)	(8;9)
A8	(7;8)	(7;8)	(5;6)	(1;2)	(9;9)	(9;9)	(7;8)	(9;9)
A9	(5;6)	(1;2)	(1;1)	(7;8)	(3;4)	(5;6)	(7;7)	(8;9)
A10	(5;5)	(5;6)	(5;6)	(7;8)	(1;2)	(7;8)	(7;8)	(5;6)

The dilemmas and uncertainties that exist during the expert evaluation of the advisors are shown by using the values given in Table 2. Thus, for example, for the expert E8 in the position A1-C1, the value is (5;6). This means that, during the evaluation of the advisor A1 (for the criterion C1), the E8 expert had a dilemma between the two values from the nine-point scale, i.e. there was a dilemma between the values 5 and 6 from the scale. Also, with the expert E8 in the position A1-C2, it is possible to notice that there was no dilemma about the choice of the values from the composition, so the value (9;9) was assigned.

In the next part, the transformation of uncertainty into an IRN was performed by using the equations (1) – (13). After the aggregation of the IRN expert correspondence matrices, an aggregated IRN initial decision matrix was obtained, as in Table 3. The aggregation was performed by using the equation (21).

Table 3. The aggregate initial decision matrix

Alt.	C1	C2	C3	...	C9
A1	([4.00,6.00],[4.63,6.70])	([5.63,7.41],[6.58,8.06])	([3.87,6.60],[4.86,7.33])		([5.85,7.50],[6.58,8.28])
A2	([8.35,8.95],[8.35,8.95])	([7.58,8.63],[8.44,8.89])	([5.09,7.75],[5.85,8.28])		([5.16,7.61],[5.91,8.59])
A3	([6.20,7.77],[7.19,8.33])	([4.36,7.38],[5.31,7.91])	([3.70,4.90],[4.70,5.90])		([6.26,7.90],[7.29,8.63])
A4	([6.49,8.37],[6.55,8.61])	([5.81,8.38],[6.54,8.77])	([5.77,8.22],[6.65,8.71])		([5.85,7.50],[6.58,8.25])
A5	([6.62,7.82],[7.35,8.38])	([5.86,7.67],[6.89,8.39])	([5.21,6.63],[5.75,7.42])		([6.62,7.82],[7.39,8.55])
A6	([4.49,6.37],[4.84,7.28])	([3.86,6.33],[4.79,7.26])	([4.68,7.40],[5.29,7.98])		([5.15,7.65],[6.20,8.38])
A7	([6.64,7.80],[6.70,8.16])	([5.90,7.67],[6.08,8.10])	([3.97,6.56],[4.63,6.84])		([4.24,6.30],[5.01,7.04])
A8	([5.85,7.50],[6.11,7.88])	([5.78,8.10],[6.22,8.41])	([4.71,6.40],[5.47,7.43])		([4.33,6.10],[5.00,7.06])
A9	([4.20,5.36],[4.84,6.30])	([2.38,5.04],[3.34,5.92])	([3.33,5.97],[3.85,6.96])		([5.06,6.88],[6.06,7.88])
A10	([6.17,8.26],[5.81,8.29])	([5.87,7.26],[5.97,7.59])	([4.58,6.40],[5.47,7.43])		([6.18,7.60],[6.84,8.26])

Step 2 – The initial decision matrix normalization:

The normalization of the elements of the initial decision matrix is a logical step in the multi-criteria models in which criteria are represented by the different units of measurement and/or in which there are two types of criteria (Benefit and Cost). In this paper, the normalization of the value of the initial decision matrix is omitted because: 1) a nine-point scale was used to evaluate all the alternatives, i.e., all the criteria are presented by the same units, and 2) all the criteria belong to the group of the benefit criteria, i.e. there are no two types of the criteria.

Steps 3 and 4 - The determination of the criterion function $H(n_i)$ of alternatives and the ranking of alternatives

By using the IRNWPHA, Equation (23), the alternatives of the criterion functions are obtained as in Table 4.

Table 4. The ranking of the alternatives

Alt.	IRN $H(n_i)$	Crisp $H(n_i)$	Rank
A1	([4.12,6.18],[4.96,7.05])	5.58	9
A2	([6.52,8.04],[7.13,8.54])	7.57	1
A3	([5.46,7.34],[6.38,8.03])	6.83	4
A4	([6.38,8.12],[7.04,8.59])	7.55	2
A5	([6.16,7.52],[6.93,8.22])	7.22	3
A6	([4.43,6.7],[5.19,7.54])	5.96	8
A7	([4.97,6.93],[5.55,7.6])	6.26	7
A8	([5.15,7.2],[5.8,7.84])	6.50	6
A9	([3.91,6.1],[4.67,6.93])	5.40	10
A10	([5.21,7.39],[5.73,7.97])	6.57	5

The calculation of the IRN value from Table 4 is shown in the next section. Table 5 shows the values of the alternative A1 according to the criteria C1-C9.

Table 5. The values of the alternative A1

Criterion	IRN value
C1	([4.00,6.00],[4.63,6.70])
C2	([5.63,7.41],[6.58,8.06])
C3	([3.87,6.60],[4.86,7.33])
C4	([2.04,4.96],[3.04,5.96])
C5	([3.07,5.13],[4.07,6.13])
C6	([2.87,4.93],[3.50,5.90])
C7	([5.32,7.03],[6.32,8.03])
C8	([3.63,5.51],[4.36,6.57])
C9	([5.85,7.50],[6.58,8.28])

Since $IRN\ H(n_1)=\left(\left[n_1^L,n_1^U\right],\left[n_1^{'L},n_1^{'U}\right]\right)$ consists of the four segments, $IRN\ H(n_1)$ aggregation will be performed separately for each of the segments, i.e. $IRNWPHA^{1,1}\left(n_1^L(C1);n_1^L(C2),...,n_1^L(C9)\right)$, $IRNWPHA^{1,1}\left(n_1^U(C1);n_1^U(C2),...,n_1^U(C9)\right)$, $IRNWPHA^{1,1}\left(n_1^{'L}(C1);n_1^{'L}(C2),...,n_1^{'L}(C9)\right)$ and

$IRNWPHA^{1,1}\left(n_1^{'U}(C1);n_1^{'U}(C2),...,n_1^{'U}(C9)\right)$. The segment calculation $IRNWPHA^{1,1}\left(n_1^{L}(C1);n_1^{L}(C2),...,n_1^{L}(C9)\right)$ is shown in detail in the next section:

Step 1: Normalized number functions are calculated:

$f\left(n_1^L(C1)\right)=\frac{4}{4+5.63+...+5.85}=0.11$; $f\left(n_1^L(C2)\right)=\frac{5.63}{4+5.63+...+5.85}=0.16$, ..., $f\left(n_1^L(C9)\right)=\frac{5.85}{4+5.63+...+5.85}=0.16$.

Step 2: The calculation of the degree of support for numbers:

$Sup(f(n_1^L(C1)),f(n_1^L(C1)))=0.045$, $Sup(f(n_1^L(C1)),f(n_1^L(C3)))=0.003$, $Sup(f(n_1^L(C1)),f(n_1^L(C4)))=0.054$, ..., $Sup(f(n_1^L(C8)),f(n_1^L(C9)))=0.061$

Step 3: By applying Equation (23), $IRNWPHA^{p=1,q=1}\left(n_1^{L}(C1);n_1^{L}(C2),...,n_1^{L}(C9)\right)$ is calculated as follows:

$$IRNWPHA^{p=1,q=1}(4.00;5.63;3.87;2.04;3.07;2.87;5.32;3.63;5.85)=$$

$$=\left(\frac{2}{9(9+1)}\left(\begin{array}{l}\left(9\cdot\frac{0.118\cdot(1+0.257)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 4.00\right)^1\cdot\\ \left(9\cdot\frac{0.118\cdot(1+0.257)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 4.00\right)^1+\\ \left(9\cdot\frac{0.088\cdot(1+0.409)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.63\right)^1\cdot\\ \left(9\cdot\frac{0.088\cdot(1+0.409)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.63\right)^1\\ +...+\\ \left(9\cdot\frac{0.163\cdot(1+0.450)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.85\right)^1\cdot\\ \left(9\cdot\frac{0.163\cdot(1+0.450)}{\begin{array}{l}0.118(1+0.257)+0.088\cdot(1+0.409)+0.102\cdot(1+0.254)+\\0.109\cdot(1+0.495)+...+0.163\cdot(1+0.450)\end{array}}\cdot 5.85\right)^1\end{array}\right)\right)^{\frac{1}{1+1}}$$

$$=4.124$$

The remaining segments are calculated in the same manner: $IRNWPHA^{1,1}\left(n_1^{U}(C1);n_1^{U}(C2),...,n_1^{U}(C9)\right)$, $IRNWPHA^{1,1}\left(n_1^{'L}(C1);n_1^{'L}(C2),...,n_1^{'L}(C9)\right)$

and $IRNWPHA^{1,1}\left(n_1^{\prime U}(C1);n_1^{\prime U}(C2),...,n_1^{\prime U}(C9)\right)$, so $IRN\,H(n_1)=([4.12,6.18],[4.96,7.05])$ is obtained. Thus, the remaining values $IRN\,H(n_i)$ from Table 4 are obtained. For an easier ranking of the alternatives, the IRN values $IRN\,H(n_i)$ were transformed into crisp $H(n_i)$ values, and the following rank of the advisors was defined: A2> A4> A5> A3> A10> A8> A7> A6> A1> A9.

The previous research (Pamucar et al., 2018) showed that changes in the *p* and *q* parameters had led to changes in the structure of the Heronian function, which further led to changes in the values of the decision model. Since it is inevitable that there is an influence of the parameters *p* and *q* on the results of the functions, it is necessary to check their influence on the results of the model. The initial rank shown in Table 4 was obtained based upon the values of the parameters *p*=*q*=1. In the next part, two scenarios were formed. In the first scenario, the influence of changing the parameter *p* in the interval $p\in[1,\ 100]$ was considered, while the value of the parameter *q* did not change (*q*=1), see Figure 1.

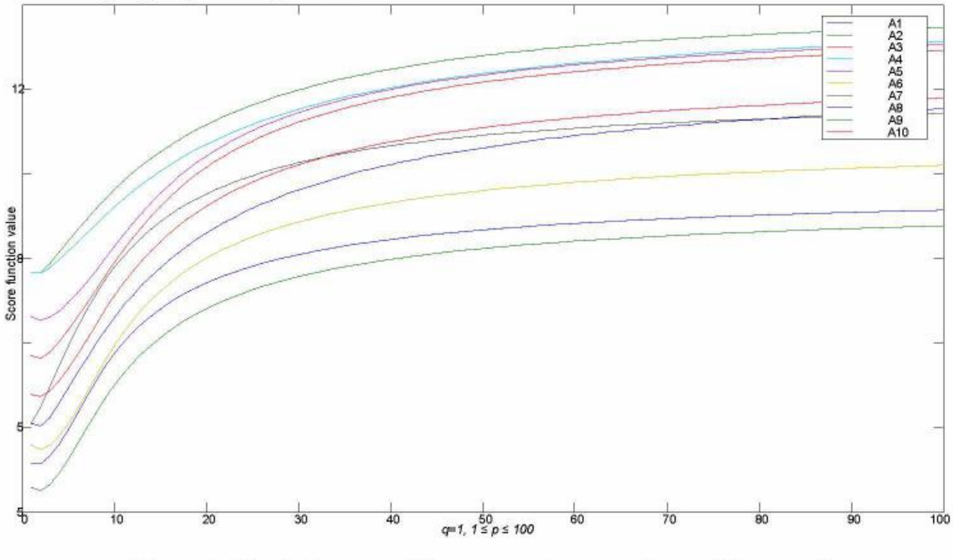

Figure 1. The influence of the parameter p on the ranking results

In the second scenario, the influence of changing the parameter *q* in the interval $q\in[1,\ 100]$ was considered, while the value of the parameter *p* did not change (*p*=1), see Figure 2.

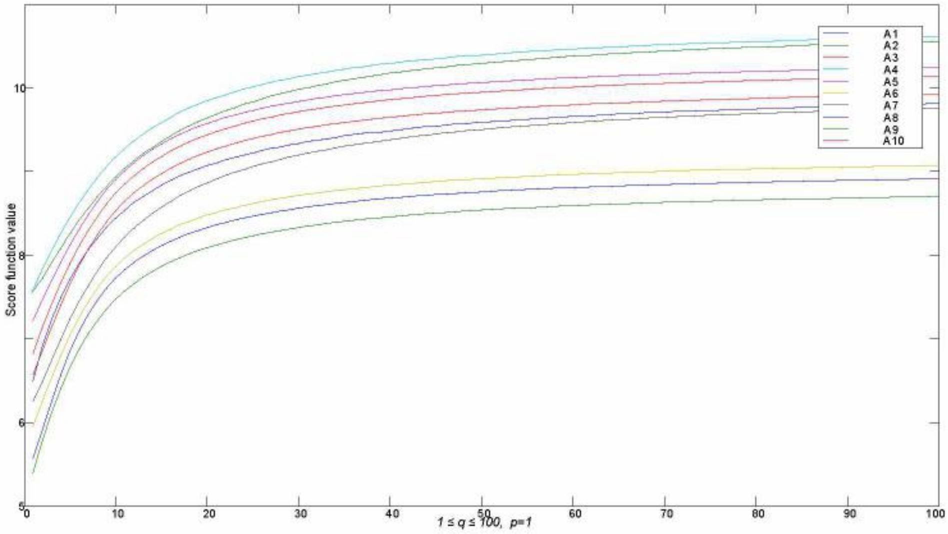

Figure 2. The influence of the parameter q on the ranking results

Both scenarios confirmed the expectations that a change in the values of the parameters p and q leads to a change in the values of criterion functions. Also, with a change in parameter values, the calculation of criterion functions becomes more complicated, since a larger number of mutual connections between criteria are simultaneously considered. Both scenarios showed that, when the values of the parameters p and q changed, there were minor changes in the ranks of the considered alternatives. According to Figures 1 and 2, it is also clear that there are no changes in the ranks of the first four ranked alternatives (A2, A4, A5 and A3). From this, it can be concluded that there is a satisfactory advantage between the considered alternatives, and that the alternatives A2 and A4 stand out as dominant from the considered set. Based on all the above-said, it is possible to conclude that the obtained rank A2> A4> A5> A3> A10> A8> A7> A6> A1> A9 is both confirmed and credible.

6. Conclusions

The application of the original IRNWPHA multi-criteria model for the evaluation of advisors in dangerous goods transport is presented. The model modified the weighted Heronian aggregator by using a power aggregator in an interval rough environment. The IRNWPHA multi-criteria model enables objective decision-making in the case of uncertain and imprecise input parameters in the initial decision matrix. Also, the IRNWPHA model allows flexible decision-making and the verification of the robustness of the results through the variation of p and q parameters. The IRNWPHA combines the advantages of the PA and WHM operators, and is a powerful decision-making tool characterized by the following features: 1) it eliminates the impact of unreasonable arguments; 2) it takes into account the degree of support between input arguments. and 3) it takes into account the interconnectedness of input arguments.

Since this is a new multi-criteria model, whose application has successfully been demonstrated in real research, it can be concluded that there is justification for the development of the presented methodology. Future research may be based upon combining the IRNWPHA methodology with other MCDM techniques so as to improve the characteristics of the traditional MCMD methods. Future research may also focus on integrating IRNs with D numbers, which would allow for the creation of reasoning algorithms in uncertainty conditions. At the same time, an approach based upon IRN and D numbers would be a concept for the intelligent management of decision-making processes. In addition, improving the model by using neutrosophic fuzzy values might be an option for further research work.

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HIGHLY EFFICIENT FE SIMULATIONS BY MEANS OF SIMPLIFIED COROTATIONAL FORMULATION

Dragan Marinković [1]*, Manfred Zehn [1], Ana Pavlović [2]

[1] Berlin Institute of Technology, Department of Structural Analysis, Germany
[2] Department Industrial Engineering, Alma Mater Studiorum University of Bologna, Italy

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Abstract: *Finite Element Method (FEM) has deservedly gained the reputation of the most powerful numerical method in the field of structural analysis. It offers tools to perform various kinds of simulations in this field, ranging from static linear to nonlinear dynamic analyses. In recent years, a particular challenge is development of FE formulations that enable highly efficient simulations, aiming at real-time dynamic simulations as a final objective while keeping high simulation fidelity such as nonlinear effects. The authors of this paper propose a simplified corotational FE formulation as a possible solution to this challenge. The basic idea is to keep the linear behavior of each element in the FE assemblage, but to extract the rigid-body motion on the element level and include it in the formulation to cover geometric nonlinearities. This paper elaborates the idea and demonstrates it on static cases with three different finite element types. The objective is to check the achievable accuracy based on such a simplified geometrically nonlinear FE formulation. In the considered examples, the difference between the results obtained with the present formulation and those by rigorous formulations is less than 3% although fairly large deformations are induced.*

Key words: *Structural analysis, Co-rotational FEM, Geometric nonlinearity, Solid, Shell*

1. Introduction

Structural analysis is an important engineering discipline encountered in various fields of mechanical and civil engineering. Reliable, accurate and efficient predictions of structural behavior in general, and deformations in particular, are of crucial importance for successful design and optimization of structures, testing their functionality, prediction of their load-carrying capacity and life-time, etc. Recently, this aspect started gaining in importance in some modern fields as well, such as

* Corresponding author.
dragan.marinkovic@tu-berlin.de (D. Marinković), manfred.zehn@tu-berlin.de (M. Zehn), ana.pavlovic@unibo.it (A. Pavlović)

interactive simulations, where physics-based simulation are supposed to increase the realism of various applications.

Until several decades ago, computations in the field of structural analysis were performed mainly analytically by implementing significant simplifications. Those simplifications have made the models mathematically tractable, but seriously affected the accuracy of the obtained results and therewith their reliability. However, the development of modern hardware tools has set the ambient for development of modern numerical methods for this purpose and the development of computer aided engineering (CAE) software packages skyrocketed. Those programs offered a great assistance to engineers in the previous decades.

Among different methods, the Finite Element Method (FEM) has established itself as the method of choice for all problems characterized by complex domains, arbitrary boundary conditions and described by partial differential equations. Problems in the field of structural analysis fit perfectly well into this description and this is why structural analysists have initiated the development of this powerful method and made the very first steps (Turner et al., 1956). Its general applicability to many other engineering fields was later recognized and richly used.

Initial developments of FEM were done for the least complicated but quite often encountered problems of structural analysis, namely the linear static problems. Those problems are characterized by slowly increasing loads of constant direction, constant geometric boundary conditions and quite small deformations, so that the initial and deformed structural configurations are almost identical. Those assumptions imply that the balance can be considered over the initial configuration (Bathe, 1996). It is a straightforward task to extend the FE formulations from linear static to linear dynamic cases and it basically comes down to extending the equations by including inertial and damping effects.

However, high level of structural optimization implies exploitation of structures to the levels quite close to their limits. In such cases, structural deformations are not small any more and more sophisticated FE formulations were needed to meet the objectives. Total Lagrange and updated Lagrange formulations have set the standards in commercially available FE codes. The essential difference between the two lies in the choice of the reference configuration. Principally it could be any configuration between the initial one and the last determined one, but the common sense choice would be to use either the initial one (total Lagrange formulation) or the last determined one (updated Lagrange formulation). Different strain and stress measures are used in those two formulations and building the tangential stiffness matrix also reflects those differences, but numerics of the two formulations is essentially the same and the choice between the two is basically a matter of taste.

Another interesting formulation, namely the corotational FE formulation, appeared several decades ago. Related to FEM the term ‘corotational’ was used for the first time in a paper by Belytschko and Hseih (1979). The idea to cover geometric nonlinearities by attaching a corotational frame to single elements was introduced by Horrigmoe and Bergan (1978). The work in this direction continued under the supervision of Bergan and the developments were summarized in a survey article by Nygard and Bergan (1989). Crisfield (1990, 1997) and Crisfield & Moita (1996) introduced “consistent CR formulation” by developing the stiffness matrix as the actual variation of the internal force. Rankin and Brogan (1986) proposed the concept of element independent corotational formulation. A high-quality survey of these developments and a detailed analysis of their properties was provided by Felippa and Haugen (2005).

This paper suggests a corotational (CR) FE approach that offers a trade-off between numerical efficiency and achievable accuracy by simplifying the rigorous corotational FE formulation. The idea is to offer a FE formulation that would be of high interest for specific applications such as multi-body system dynamics, where parts exhibit large rigid-body deformations but small strains, or applications involving real-time simulations. In this paper, the achievable accuracy will be tested with a few basic solid and shell elements in cases involving large local rigid-body rotations.

2. Simplified CR formulation and implemented elements

2.1. Basic principles of the simplified CR formulation

While the linear FE formulations offer very efficient and stable computations, the nonlinear formulations are very time consuming and prone to computational stability issues, as they might not necessarily produce converged solutions. On the other hand, linear formulations are accurate only for small deformations, but geometrically nonlinear formulations offer engineering accuracy for large deformations involving arbitrarily large rigid-body rotations. While one would wish to have advantages of both formulations in one formulation, it is certainly not possible to have all the advantages to the full extent. But a formulation may offer a kind of trade-off or a compromise between those.

The formulation that will be explained here follows the idea of element-based CR formulation. Hence, the basic concept is to attach a coordinate system to an element and keep the linear FE formulation of the element with respect to this coordinate system. The attached coordinate system follows the element in its rigid-body motion. As the elastic behavior of the element remains linear with respect to the attached coordinate system, this implies that the element matrices are computed only once for this coordinate system. As deformation proceeds, it is necessary to determine the motion of the attached coordinate system, or, in other words, to determine the element rigid-body motion. Once this is described by the element rotational matrix, Re, the related element matrices and vectors can be rotated to the current configuration and the assemblage of the global matrices and vectors can be done for further computation.

Hence, the element elastic behavior is described as linear with respect to the attached corotational frame and the element stiffness matrix with respect to this frame is not updated. In this manner, the local element deformation and the stress state is neglected from the consideration of geometrically nonlinear effects, thus simplifying the formulation significantly compared to the rigorous nonlinear formulations. The formulation keeps the very important aspect of geometric nonlinearity, namely the rigid-body rotation that is accounted for on the element level. In continuum every point exhibits its own rigid-body rotation, generally speaking. Obviously, this aspect is described here in a coarser way, as it is always the case with methods that apply discretization. But one may arbitrarily adjust the 'resolution' of accounting for rigid-body rotation by performing finer or coarser FE meshing.

Hence, assuming the element rigid-body rotation between the initial and the current configuration is known and given as the rotation matrix Re, the element stiffness matrix, Ke, is update with respect to the global coordinate system in a straightforward manner:

$$ {}^{t}\mathbf{K}_{\mathbf{e}} = {}^{t}\mathbf{R}_{\mathbf{e}}\, {}^{0}\mathbf{K}_{\mathbf{e}}\, {}^{t}\mathbf{R}_{\mathbf{e}}^{\mathrm{T}}, \qquad (1)$$

where the left superscript denotes the moment in time at which the term is given. This simple way of updating the element stiffness matrix is where the efficiency of the method resides. Not only is the tangential stiffness matrix efficiently updated, but also its condition number does not change dramatically in this manner, so that the stability of computation is kept to a large extent. This is not always the case with rigorous nonlinear FE formulations in which single elements may suffer significant deformations, and, as a consequence, the solution may not converge.

In order to perform nonlinear computations, one needs the tangential stiffness matrix, the update of which was elaborated above, and the internal forces. In order to determine the internal forces, deformational displacements and rotations are required. Those are obtained when the rigid-body rotation is removed from the overall element displacements. This procedure is best explained using Figure 1. In this figure, a tetrahedron element is shown in its original and an arbitrarily deformed configuration. Again, it is assumed that the rigid-body rotation of the element is known. It is sufficient to rotate the deformed element back to the original element configuration. By comparing so obtained element configuration with the initial one, one obtains deformational displacements.

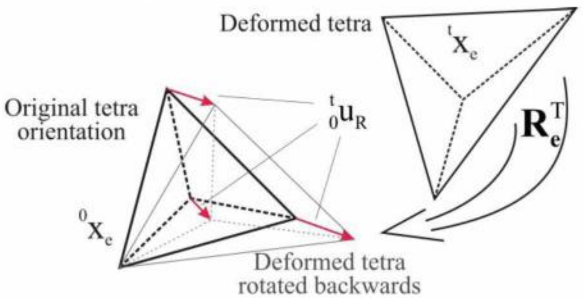

Figure 1. Extraction of deformational displacements for a tetrahedron element

Hence, the expression for the deformational displacements reads:

$$ {}^{t}_{0}\mathbf{u}_{\mathrm{R}} = {}^{t}\mathbf{R}_{\mathbf{e}}^{\mathrm{T}}\, {}^{t}\mathbf{x}_{\mathbf{e}} - {}^{0}\mathbf{x}_{\mathbf{e}}, \qquad (2)$$

where $\mathbf{x}_{\mathbf{e}}$ denotes the element configuration, i.e. those are the nodal coordinates of all element nodes. With the known deformational displacements, one may simply multiply those with the element stiffness matrix for the initial configuration to obtain the internal forces with respect to the initial configuration. The internal forces are the rotated to the current configuration. Those steps are summarized in the following expression:

$$ {}_{t}^{t}\mathbf{F}_{e} = {}^{t}\mathbf{R}_{e}\, {}_{0}^{t}\mathbf{F}_{e} = {}^{t}\mathbf{R}_{e}\, {}^{0}\mathbf{K}_{e}\, {}^{t}\mathbf{R}_{e}^{T}\, {}^{t}\mathbf{x}_{e} - {}^{t}\mathbf{R}_{e}\, {}^{0}\mathbf{K}_{e}\, {}^{0}\mathbf{x}_{e}, \tag{3} $$

If finite elements contain rotational degrees of freedom, the procedure is essentially the same for rotations, as illustrated in Figure 2.

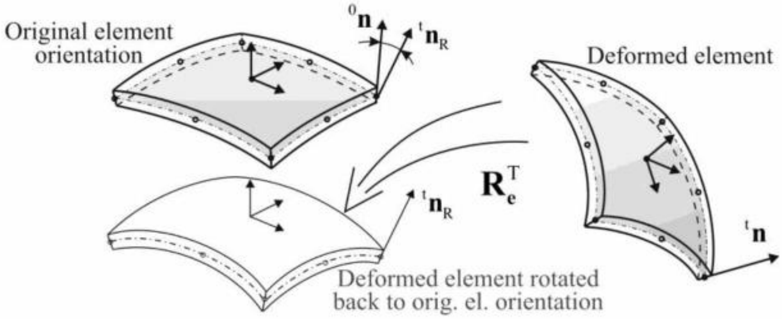

Figure 2. Extraction of deformational rotations for a shell element

With the incremental nodal rotations, $\Delta^{t-\Delta t}\theta_{i1}$, $\Delta^{t-\Delta t}\theta_{i2}$ and $\Delta^{t-\Delta t}\theta_{i3}$, the element nodal normals are updated by means of the rotation matrix $\mathbf{Q}_i$ (Argyris, 1982):

$$ {}^{t-\Delta t}\mathbf{Q}_{i} = \mathbf{I} + \frac{\sin {}^{t-\Delta t}\gamma_{i}}{{}^{t-\Delta t}\gamma_{i}}\, {}^{t}\mathbf{S} + \frac{1}{2}\left(\frac{\sin\left({}^{t-\Delta t}\gamma_{i}/2\right)}{\left({}^{t-\Delta t}\gamma_{i}/2\right)}\right)^{2} {}^{t-\Delta t}\mathbf{S}_{i}^{2}, \tag{4} $$

where

$$ {}^{t-\Delta t}\gamma_{i} = \sqrt{\Delta^{t-\Delta t}\theta_{i1}^{2} + \Delta^{t-\Delta t}\theta_{i2}^{2} + \Delta^{t-\Delta t}\theta_{i3}^{2}}, \tag{5} $$

$$ {}^{t-\Delta t}\mathbf{S}_{i} = \begin{bmatrix} 0 & -\Delta^{t-\Delta t}\theta_{i3} & \Delta^{t-\Delta t}\theta_{i2} \\ \Delta^{t-\Delta t}\theta_{i3} & 0 & -\Delta^{t-\Delta t}\theta_{i1} \\ -\Delta^{t-\Delta t}\theta_{i2} & \Delta^{t-\Delta t}\theta_{i1} & 0 \end{bmatrix}, \tag{6} $$

so that

$$ {}^{t}\mathbf{n}_{i} = {}^{t-\Delta t}\mathbf{Q}\, {}^{t-\Delta t}\mathbf{n}_{i}. \tag{7} $$

After rotating the element from the current to initial orientation, it is a straightforward task to compute the internal moments and rotate them again to the current configuration in an analogous manner as done above with the forces.

2.2. Finite elements implemented into the formulation

So far three finite elements have been implemented into the proposed simplified CR formulation – two solid elements and one shell element.

The solid elements are the linear tetrahedron element and the quadratic hexahedron element. The linear tetrahedron element is notorious for its too stiff behavior and is, therefore, often avoided in modeling. However, it has two nice properties. It is numerically very efficient and can discretize any geometry. Actually, the second advantage makes it often inevitable in FE models in order to model some

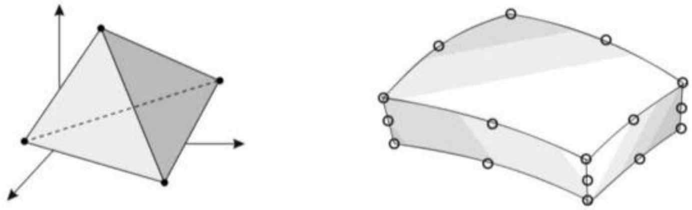

Figure 3. Linear tetrahedron element (left) and quadratic hexahedron element (right)

areas of the model that are otherwise too difficult to discretize. In addition, this element is characterized by unambiguity of the rotation matrix. There is a single, unique rotation matrix describing the rigid-body rotation of this element, which is not the case with most of the finite elements. For any two given configurations of the element there is a unique matrix that transforms the element from one configuration to the other one. This is due to the fact that the element employs the linear shape functions, so that the deformation gradient has a constant value over the whole element domain. Polar decomposition of this transformation matrix yields the rotation matrix. In order to obtain reasonable results with this element, a quite fine discretization is required. But this also increases the "resolution" of accounting for the rigid-body rotation, which is a positive aspect regarding the corotational FE approach. Nguyen et al. (2016) have used this element in combination with a corotational FE approach that implements the projector matrix for the sake of better result convergence.

The quadratic hexahedron element is in most aspects the opposite of the linear tetrahedron element. It offers the best accuracy among solid elements (apart from those that use special techniques), but is numerically very demanding and requires partitioning of complex geometries for successful meshing, whereby the 'corners of the geometry' will still require tetrahedron elements. The rotational matrix is not unique for the element, i.e. it differs for different points within the element domain. Hence, it is ambiguous and one has to decide what strategy to use in order to determine it. It may be determined by local coordinate systems defined in a special ways by using the current nodal positions. A better option would be to use the deformational gradient at some point of the element, whereby the element centroid appears to be a natural choice – exactly this option was applied in this work. The best, but also the most demanding option would be to obtain some kind of average rotational matrix of the element. It is so far, however, an open question with respect to what criteria the averaging is to be performed.

The implemented shell element is a linear triangular shell element (Figure 4) recently developed (Rama et al., 2018, 2018a, 2018b, Marinkovic et al., 2019). Essentially, the element is a combination of a plate element and a membrane element. It implements the Mindlin-Reissner kinematics and uses the Discrete Shear

Gap (DSG) method (Li et al., 2019) in combination with the strain smoothing technique to alleviate the notorious shear locking. The membrane part of the element is actually the ANDES membrane formulation developed by Felippa and Militello (1992). Similarly to the linear tetrahedron element, this one also has a unique rotational matrix, can discretize any surface geometry and is numerically highly efficient. Due to the flat shape of the element, the discretized geometry is actually faceted, which affects the achievable accuracy.

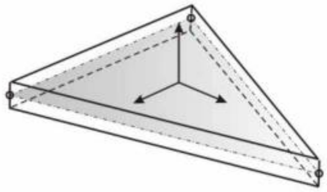

Figure 4. Linear triangular shell element

3. Numerical examples

In what follows, three examples of large deformations, each for one type of implemented elements, will be considered in order to investigate the achievable accuracy by means of the proposed corotational FE formulation. The major purpose is the comparison of computed displacements obtained by rigorous geometrically nonlinear FE formulation (computed in Abaqus) and those obtained by the presented development. In accordance with this objective, all quantities will be given as dimensionless. The selected examples are of academic nature involving structures of rather simple geometry. Sufficiently large loading will be chosen to produce geometrically nonlinear deformations, i.e. those that significantly differ from deformations computed by the linear formulation.

2.1. Solid elements

The same structure, which may be referred to as a block, with dimensions $10\times10\times1.5$ and clamped over one surface with dimensions 10×1.5 will be discretized with both tetrahedron and hexahedron element. The geometry with kinematic boundary conditions is depicted in Figure 5, left. The material is linear elastic with the following properties: Young's modulus $E=2\cdot10^{11}$ and Poisson ration $\nu=0.3$. The load cases are chosen to be different for the discretization with the tetrahedron element and for the discretization with the hexahedron element. In both cases the force is set to be $F=10^{10}$ in order to produce sufficiently large, geometrically nonlinear deformations. As shown in Figure 5, middle, in the case of discretization with the tetrahedron element, the force acts only at one corner of the structure so as to bend and twist it at the same time. Figure 5, right, shows discretization with the hexahedron element and the load case with a pair of oppositely oriented forces that cause twisting of the considered structure.

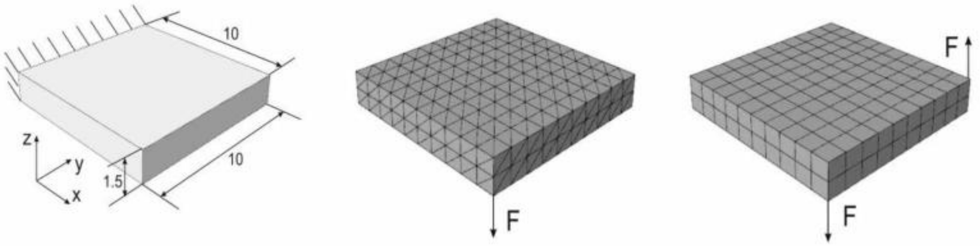

Figure 5. Block structure: geometry and kinematic boundary conditions (left), load cases and discretization with tetrahedrons (middle) and hexahedrons (right)

In order to visualize the deformations and get the feeling for the magnitude of deformation, the unscaled deformations (i.e. scale factor set to 1) are depicted in Figures 6 (the FE model with tetrahedron elements and one force) and 7 (the FE model with hexahedron elements and two forces) together with the undeformed structure. The structure is shown from different perspectives. Obviously, the magnitude of deformation is well beyond the realm of linearity.

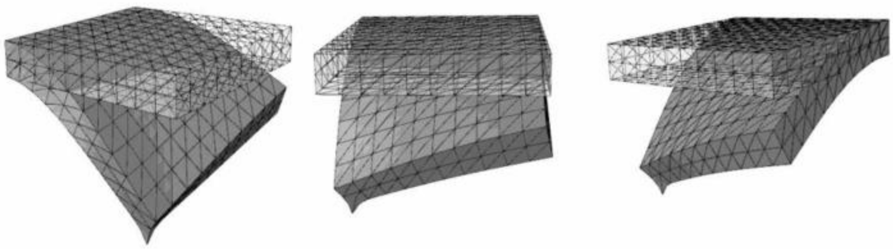

Figure 6. Deformed and undeformed block structure under single force load, discretization with tetrahedron elements, three different perspectives

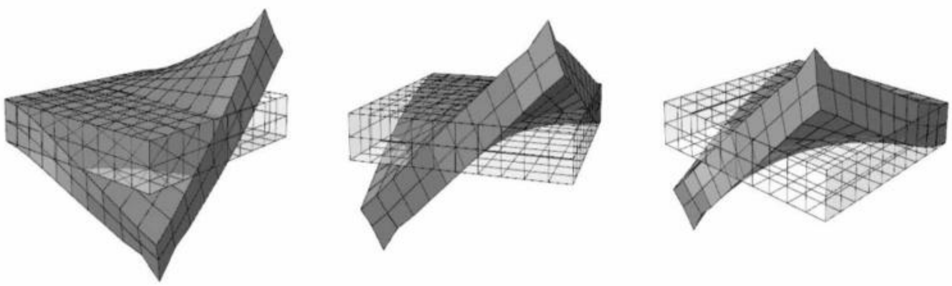

Figure 7. Deformed and undeformed block structure under force couple load, discretization with hexahedron elements, three different perspectives

As a representative point to follow its displacements with the gradually increasing loading, the point at which the force acts in Figure 5, middle, is selected. Its displacements in all three global directions are considered in both cases and compared with the linear and geometrically nonlinear results from Abaqus. The results are given in Figure 8, for the case shown in Figure 5, middle, and Figure 9, for the case shown in Figure 5, right. One may notice that the results for the same global displacements show a similar trend in both considered cases. Comparing the displacements in the same directions in those two cases, one would notice that the major distinction is in the relative difference between linear and geometrically nonlinear results. There is actually a significant difference between the linear and geometrically nonlinear results, and it goes even up to 50%. This was expected and,

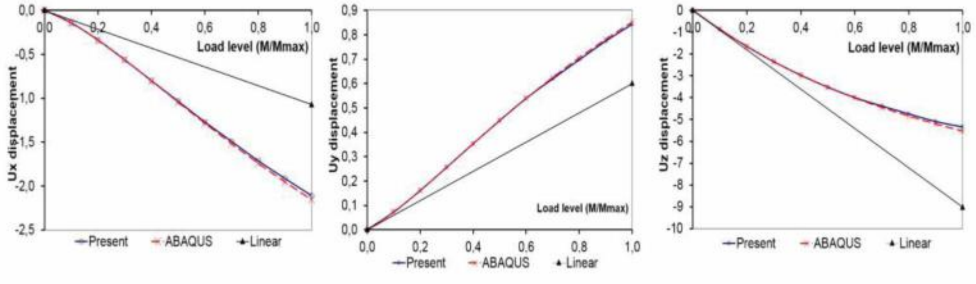

Figure 8. Model with tetrahedron elements – displacements in three global directions

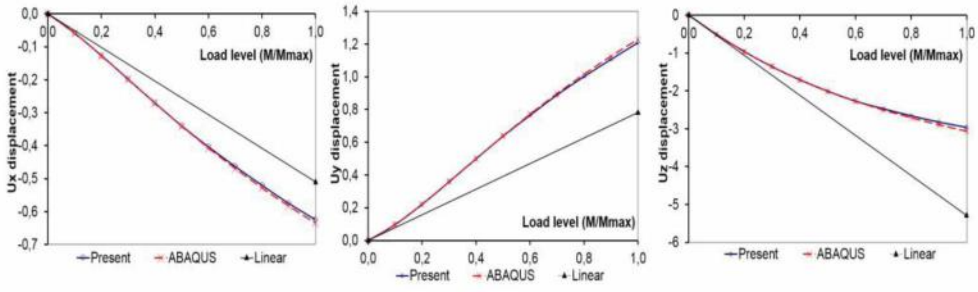

Figure 9. Model with hexahedron elements – displacements in three global directions

in fact, the loading was chosen with this objective. On the other hand, there is also a good agreement between the nonlinear results by Abaqus and present formulation. The difference is observable in the last 30-40% of the loading but stays in the limits of up to 2%, which is an acceptable result for many different applications. In addition, the highest difference is at the full loading, where local element deformations start to kick in and this is an effect not accounted for by the present formulation. As long as this effect is not present, the difference in the results is practically negligible.

2.2. Shell element

The example considered for the shell element is a typical benchmark case used in development of shell elements for geometrically nonlinear analysis. It a straight beam, with one end clamped, while the free end is exposed to a bending moment of such a magnitude that the beam bends into a circle. The moment required to produce such a deformation can be computed analytically assuming beam kinematics and is given as $M = \pi Ebh^3/6l$ (Bathe and Bolourchis, 1979), where E is the Young's modulus, while b, l and h are the width, length and thickness of the beam, respectively. In this case, the Young's modulus is the same as in the previous cases with solid elements, while Poisson ratio is set to zero, so that the shell element reproduces the beam behavior (Poisson effect is neglected over the beam's cross-section). Dimensions of the beam can be seen in Figure 10, left, while Figure 10, right, shows the FE mesh applied.

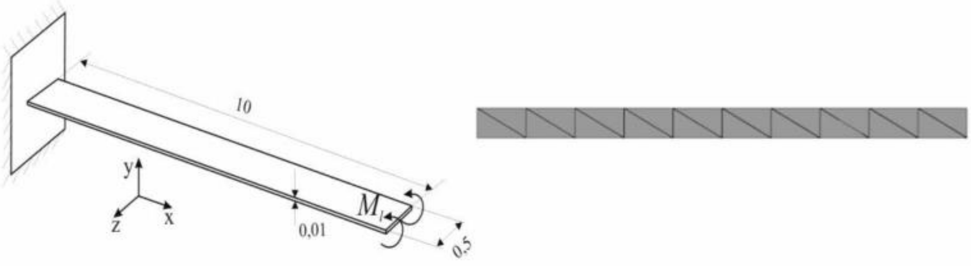

Figure 10. Beam model and discretization with triangular shell elements

Interestingly, Abaqus encounters a problem to complete the computation with its 3-node shell element. The computation runs until approximately 95% of the load, and when this level is reached, a converged solution is not found any more (automatic stepping was used to facilitate the computation). Figure 11 shows the initial and deformed configuration as computed by Abaqus. The relatively coarse mesh is the reason for the faceted deformed geometry and could be one of the reasons for the computational issues encountered by Abaqus.

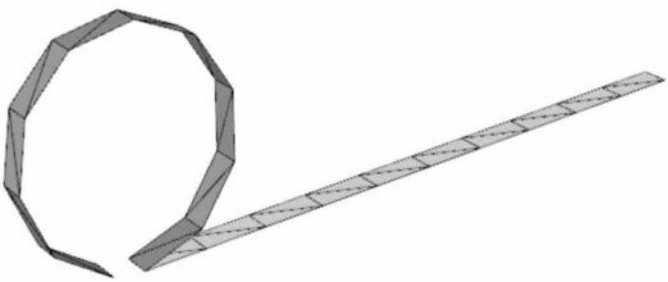

Figure 11. Beam model – deformed (Abaqus) and undeformed configuration

The results for the displacements along the x- and y-axes are given in Figure 12. The diagrams include only nonlinear results as the large difference between those and linear results would make the inclusion of linear results unreasonable. The computation with the proposed corotational formulation proceeds till 100% of the load. One may notice a good agreement of the results up to the load level computed by Abaqus.

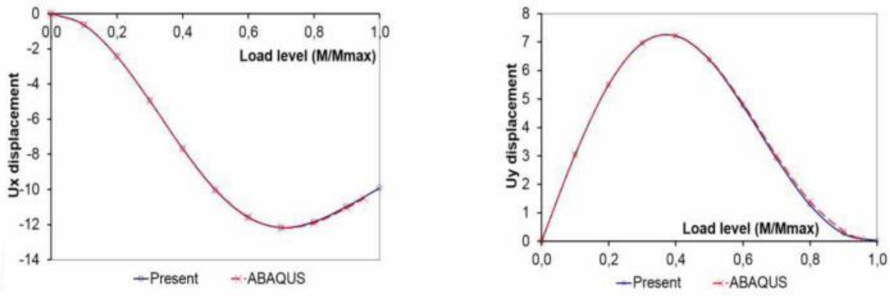

Figure 12. Beam model – free end displacements in the x- and y-directions

4. Conclusions

The paper elaborates a simplified corotational FE formulation in which the element local behavior is described as linear, but the element rigid-body rotation is accounted for. Hence, it neglects the effect of the element pure deformation and element stress state on the element tangential stiffness matrix. This is where the numerical savings are made, thus rendering the formulation very efficient in terms of computational effort and also numerically stable. At the same time, this means that it delivers results that are an approximation compared to the results delivered by the rigorous geometrically nonlinear FE formulations, such as total and updated Lagrange formulation, or the rigorous corotational FE formulation.

The examples were focused on accuracy of predicting the structural displacements, which is equivalent to the accuracy of predicting the deformed structural configuration. It was shown in the considered examples that the discrepancy between the rigorous results and those obtained by the proposed formulation is only a few percent for fairly large deformations. Of course, the achievable accuracy certainly depends on the nature of the deformation and is expected to be better in cases where local rigid-body rotation dominates. Furthermore, this means that the formulation can successfully be used for certain engineering simulations where this level of accuracy is acceptable. For instance, it can be a very attractive alternative for consideration of elastic bodies in Multi-Body System (MBS) simulations, which is currently mainly done based on the modal-superposition technique thus covering only linear deformations with respect to the local frame of the whole structure. The proposed formulation would offer better accuracy and fidelity of the full-scale FE model, while keeping the numerical effort in acceptable limits. Another interesting field of application would be Virtual Reality (VR) where physics-based real-time simulations have always played a challenging task (Marinkovic et al., 2018, Marinkovic & Zehn, 2019, Zehn & Marinkovic , 2019). In this field, the presented formulation can be successfully used for various types of simulators such as surgery (Marinkovic & Zehn, 2018), assembly planning and practicing assembling of various complex products, thus improving the productivity, etc.

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LBWA – Z-MAIRCA MODEL SUPPORTING DECISION MAKING IN THE ARMY

Darko Božanić [1]*, Dragiša Jurišić [2], Dražan Erkić [3]

[1] University of Defence in Belgrade, Military Academy, Belgrade, Serbia
[2] Security Research Centre, Banja Luka, Bosnia and Herzegovina
[3] Police department Zvornik, Ministry of Internal Affairs Republic of Srpska, Bosnia and Herzegovina

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Abstract: *The paper presents a hybrid model LBWA – Z-MAIRCA used to support decision making in the selection of a location of a camp (camp space), which has a role of providing individuals and army units with regular life and operation conditions in the field, i.e. in the conditions outside the barracks. The paper defines and explains the criteria affecting the selection of a camp (camp space), and the LBWA method is used to define the weight coefficients of the criteria. Using the MAIRCA method, which is modified with Z-numbers, it is selected the best alternative. In the final phase of the model development, the sensitivity analysis is performed and the results obtained by the developed model are compared with the results obtained by applying other methods and their various modifications.*

Keywords: *LBWA, MAIRCA, Z-number, fuzzy number, MCDM*

1. Introduction

The army performs numerous different activities. A part of these activities is realized outside the locations of permanent residence (outside the barracks), i.e. in the field. When organizing longer stays in the field, it is necessary to provide basic conditions for life and operation. These conditions are provided by adequate organization of a camp space (camp).

The camp, i.e. the camp space, means organized land space with camp facilities for accommodation and resting of units outside the populated area (Military Lexicon, 1981). It consists of tents, barracks, huts, casemates, sometimes a building or a combination. It is organized in all situations when the need arises (in peace, state of emergency and war) for the realization of trainings, works, combat operations, *etc.*

* Corresponding author.
dbozanic@yahoo.com (D. Božanić), bicbl@yahoo.com (D. Jurišić), drazan.erkic@hotmail.com (D. Erkić)

In the camps, it is necessary to provide space for various activities: accommodation, economic, medical, recreational, technical, storage, sanitary facilities and quarters. (Hristov, 1978).

Considering a series of conditions that a camp space should meet, the selection of the location for the organization of a camp space is an issue ideal for solving by multi-criteria decision-making methods. The literature dealing with this issue usually provides general conditions on which the selection should depend, which further indicates that experience plays a significant role in making such decisions. In order to group experiences and help less experienced decision makers, a model is developed and presented in this paper. The model is based on the experiences of the engineering leaders of the Serbian Army, but it is also applicable to other branches. The experiences of engineering officers are used because the engineering units of the Serbian Army have constant engagements outside the barracks due to the performance of a wide range of operations and are very often in a situation to organize camp spaces for the life and operation of their units for longer periods.

The camp space selection issue by multi-criteria decision-making methods has not been particularly considered in the literature available to the authors. This issue belongs to the group of the location issues, which have been considered in different ways in the literature. Božanić and Pamučar (2010) perform the selection of the bridge crossing location by applying fuzzy logic system. Tavakkoli-Moghaddam et al. (2011) perform plant location selection using the AHP and VIKOR method. Żak and Węglińsk (2014) perform the selection of the logistics center location base applying ELECTRE method. Bagocius et al. (2014) use several methods (SAW, TOPSIS, COPRAS) for selecting a location for a liquefied natural gas terminal in the Eastern Baltic Sea. Tomic et al. (2014) used AHP as a support in making logistic center location decisions. Tuzkaya et al. (2015), by using the ANP-DEMATEL model, select the location for emergency logistics centers. Božanić et al. (2016a) apply a hybrid model, fuzzy AHP – MABAC, for the selection of the location for preparing laying-up positions. Pamučar et al. (2016a) use a fuzzy AHP-TOPSIS model for the selection of a brigade artillery group firing position in a defensive operation. Di Matteo et al. (2016) propose a methodology for the optimization of the location on the territory of emergency operation centers using the AHP-ELECTRE model. Gigović et al. (2017), by applying GIS and the DEMATEL, ANP and MABAC methods, perform the selection of the location for wind farms in Serbia. Milosavljević et al. (2018) determine the potential macro location of the container terminal in Serbia, by applying the TOPSIS, ELECTRE and MABAC methods. Sennaroglu and Celebi (2018) present a location selection problem for a military airport using the AHP, PROMETHEE and VIKOR methods. Božanić et al. (2019b) use the FUCOM-fuzzy MABAC model for the selection of the location for construction of single-span bailey bridge.

As can be obtained from the analyzed literature, the authors use different methods of multi-criteria decision making in their research. In this paper, a hybrid model based on the LBWA (Level Based Weight Assessment) method and the MAIRCA (Multi-Attributive Ideal-Real Comparative Analysis method) modified by Z-numbers (Z-MAIRCA) is applied.

2. LBWA – Z-MAIRCA Model

The LBWA – Z-MAIRCA model consists of four phases. In Figure 1, it is presented the scheme of the model.

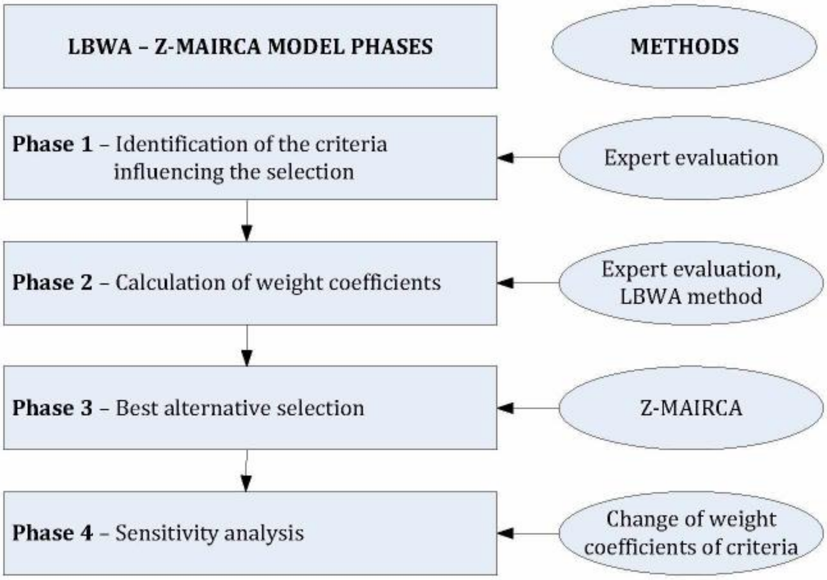

Figure 1. Graphic scheme of the LBWA – Z-MAIRCA model

In the first phase of the model, the criteria on the basis of which the selection is made by expert evaluation are defined. Through the second phase, it is performed the calculation of weight coefficients of the criteria using expert evaluation and the LBWA method. In the third phase, the best alternative is selected using Z-numbers and the fuzzy MAIRCA method. The last phase includes the sensitivity analysis of the developed model.

2.1. The LBWA method

The LBWA method was presented for the first time in the paper by Žižović and Pamučar (2020). The method has a relatively simple mathematical apparatus, and can be used in both individual and group decision making. In the paper by Pamučar et al. (2020), it is presented a fuzzified LBWA method.

At the beginning of the application of the LBWA method, it is defined the set of criteria $S = \{C_1, C_2, \dots, C_n\}$, where n represents the number of criteria influencing the selection. After the set of criteria was defined (S), the method is to be applied. The steps of the LBWA method are presented in the following section (Žižović and Pamučar, 2020).

Step 1. Determining the most significant criterion from the set of defined criteria(S), i.e. the criterion with the highest influence on the decision.

Step 2. Grouping the criteria by significance level. The significance levels are defined as follows:

- *Level* S_1: At the level S_1, the criteria from the set S whose significance is equal to or up to twice as lower from the significance of the criterion defined as the most significant are grouped;
- *Level* S_2: At the level S_2, the criteria from the set S whose significance is exactly twice or up to three times as lower from the significance of the criterion defined as the most significant are grouped;
- ...
- *Level* S_k: At the level S_k, the criteria from the set S whose significance is exactly k times as lower from the significance of the criterion defined as the most significant, i.e. up to $k+1$ times as lower from the significance of the most significant criterion, are grouped.

Applying previously presented rules, a decision maker establishes rough classification of the observed criteria. If the significance of the criterion C_j is denoted by $s(C_j)$, where $j \in \{1,2,\ldots,n\}$, then $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where for every level $i \in \{1,2,\ldots,k\}$, it is true that

$$S_i = \{C_{i_1}, C_{i_2}, \ldots, C_{i_s}\} = \{C_j \in S : i \le s(C_j) < i+1\} \tag{1}$$

Also, for each $p,q \in \{1,2,\ldots,k\}$ such that $p \ne q$ holds $S_p \cap S_q = \varnothing$. Thus, in this way, the partition of the set of criteria S is well defined.

Step 3. Within the formed subsets (levels) of the influence of the criteria, it is performed the comparison of the criteria by their significance. Every criterion $C_{i_p} \in S_i$ in the subset $S_i = \{C_{i_1}, C_{i_2}, \ldots, C_{i_s}\}$ is assigned an integer $I_{i_p} \in \{0,1,\ldots,r\}$ such that the most important criterion C_i is assigned $I_i = 0$, and if C_{i_p} is more significant than C_{i_q}, then $I_p < I_q$, and if C_{i_p} is equivalent to C_{i_q}, then $I_p = I_q$. The maximum value of the scale for the criteria comparison is defined by applying the expression (2)

$$r = \max\{|S_1|, |S_2|, \ldots, |S_k|\} \tag{2}$$

Step 4. Based on the defined maximum value of the scale for the comparison of criteria (*r*), it is defined the elasticity coefficient $r_0 \in N$ (where *N* represents the set of real numbers) which should meet the criteria where $r_0 > r$, $r = \max\{|S_1|, |S_2|, \ldots, |S_k|\}$. The creators of the method recommend to define initial values of the weight coefficients based on the elasticity coefficient $r_0 = r+1$. Considering that the parameter r_0 causes smaller changes of the value of the weight coefficients, taking the other value of the elasticity coefficient is recommended for additional settings of the weight coefficients in accordance with the decision makers' own preferences.

Step 5. The calculation of the influence function of the criteria. The influence function $f: S \rightarrow R$ is defined in the following way. For every criterion $C_{i_p} \in S_i$, the influence function is defined:

$$f(C_{i_p}) = \frac{r_0}{i \cdot r_0 + I_{i_p}} \quad (3)$$

where i represents the number of the level/subset into which the criterion is classified, r_0 represents the elasticity coefficient, while $I_{i_p} \in \{0,1,\ldots,r\}$ represents the value which is assigned to the criterion C_{i_p} within the observed level.

Step 6. The calculation of the optimum values of the weigh coefficients. By applying the expression (4), it is calculated the weight coefficient of the most influential criterion:

$$w_1 = \frac{1}{1 + f(C_2) + \cdots + f(C_n)} \quad (4)$$

The values of the weight coefficients of other criteria are obtained by applying the expression (5):

$$w_j = f(C_j) \cdot w_1 \quad (5)$$

where $j = 2,3,\ldots,n$, and n represents a total number of criteria.

2.2. Z-MAIRCA

A wide range of uncertainties following decision-making processes influences a number of researchers when they select a model of multi-criteria decision-making and opt for various modifications of classic methods (*e.g.* using fuzzy logic, rough numbers, *etc.*). The selection of a location for a camp space is accompanied by uncertainties and inaccuracies, which is why the MAIRCA method, fuzzified with Z-numbers, is selected. The MAIRCA method was first published in the papers written by Pamučar et al. (2014) and Gigović et al. (2016). Since then, it has been applied in its original form (Pamučar et al., 2018; Tešić and Božanić, 2018; Adar and Delice, 2019, 2020; Ayçin and Orçun, 2019; Ayçin, 2020), but also through various modifications in fuzzy and rough environment (Pamučar et al., 2017b; Chatterjee et al., 2018; Badi and Ballem, 2018; Stević, 2018; Božanić et al., 2019a; Arsić et al., 2019; Hashemkhani et al., 2020; Boral et al., 2020).

Given that the Z-numbers are used for the modification, their most basic description is provided below. Z-numbers represent a type of fuzzy numbers, i.e. two fuzzy numbers, which are in a specific relationship. Triangular fuzzy numbers are used in this paper, as in Figure 2.

Triangular fuzzy numbers have the form $\tilde{T} = (t_1, t_2, t_3)$; t_1 - the left distribution of the confidence interval of fuzzy number T, t_2 - fuzzy number membership function has the maximum value - equal to 1, and t_3 - the right distribution of the confidence interval of fuzzy number $\tilde{T}$ (Pamučar et al., 2012).

A Z-number represents an extension of classic fuzzy number and provides wider opportunities for considering additional uncertainties following decision making. The concept of Z-number was proposed by Zadeh (2011). In 2012, Kang et al. (2012a, 2012b) have already shown in detail the application of Z-numbers in uncertain environment. Later, authors consider the application of Z-numbers with different methods of multi-criteria decision making. Sahrom and Dom (2015) present the use of Z-numbers in the hybrid AHP-Z-number-DEA method. Azadeh and Kokabi (2016) use Z-numbers with the DEA method, Azadeh et al. (2013) with the AHP, Yaakob and Gegov (2015) with the TOPSIS method, Aboutorab et al. (2018) with the Best Worst method, Bobar et al. (2020) and Božanić et al. (2020) with the MABAC method. Salari et al. (2014) elaborate a novel earned value management model using a Z-number.

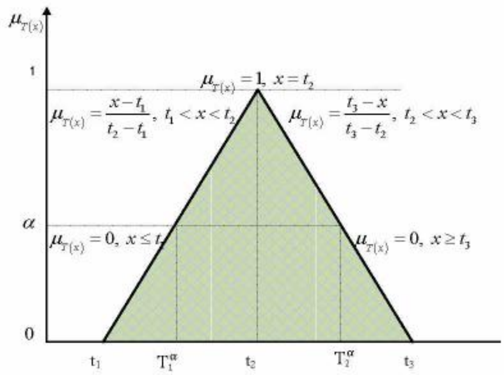

Figure 2. Triangular fuzzy number (Pamučar et al., 2016b)

A Z-number represents an ordered pair of fuzzy numbers that appear as Z = ($\tilde{A}$, $\tilde{B}$) (Zadeh, 2011). The first component, fuzzy number $\tilde{A}$, represents the fuzzy limit of a particular variable *X*, while the second component, fuzzy number $\tilde{B}$, represents the reliability of the first component ($\tilde{A}$). The appearance of the Z-number with triangular fuzzy numbers is shown in Figure 3 (Zadeh, 2011).

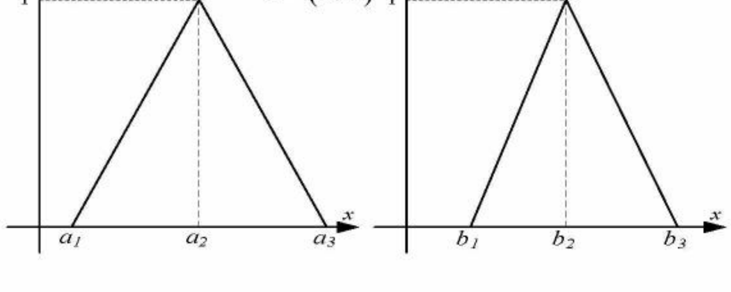

Figure 3. A-Simple Z-number (Kang et al., 2012a)

A general record of triangular Z-numbers can be displayed as

$$\tilde{Z} = \{(a_1, a_2, a_3; w_{\tilde{A}}), (b_1, b_2, b_3; w_{\tilde{B}})\} \tag{6}$$

where the values $w_{\tilde{A}}$ and $w_{\tilde{B}}$ represent weight factors of fuzzy number $\tilde{A}$ referring to $\tilde{B}$, which for the initial Z-number, the majority of authors define as $w_{\tilde{A}} = w_{\tilde{B}} = 1$, $w_{\tilde{A}}, w_{\tilde{B}} \in [0,1]$ ($w_{\tilde{A}}$ is the height of generalized fuzzy number and $0 \le w_{\tilde{A}} \le 1$) (Chutia et al., 2013). The transformation of the Z-number into a classic fuzzy number, with the presented evidence, is shown in Kang et al. (2012b). This transformation consists of three steps:

Convert the second part ($\tilde{B}$) into a crisp number using the centered method (Kang et al., 2012b):

$$\alpha = \frac{a_1 + a_2 + a_3}{3} \tag{7}$$

Add the weight of the second part ($\tilde{B}$) to the first part ($\tilde{A}$). The weighted Z-number can be presented as in Kang et al. (2012b):

$$\tilde{Z}^{\alpha} = \{\langle x, \mu_{\tilde{A}^{\alpha}}(x)\rangle \mid \mu_{\tilde{A}^{\alpha}}(x) = \alpha\mu_{\tilde{A}}(x)\} \tag{8}$$

which can be presented by Figure 4a. This can be written as (Azadeh et al., 2013):

$$\tilde{Z}^{\alpha} = (a_1, a_2, a_3; \alpha) \tag{9}$$

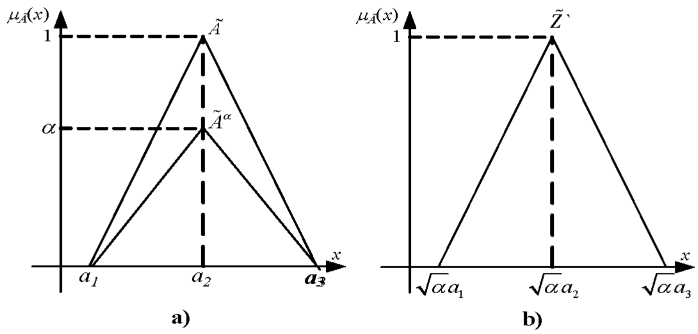

Figure 4. Z-number after multiplying the reliability (a) and the regular fuzzy number transformed from a Z-number (b)

Convert the weighted Z-number into a regular fuzzy number. The regular fuzzy set can be presented as in Kang et al. (2012b)

$$\tilde{Z}' = \left\{\langle x, \mu_{\tilde{Z}'}(x)\rangle \middle| \mu_{\tilde{Z}'}(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right)\right\} \tag{10}$$

$$\tilde{Z}' = \sqrt{\alpha} * \tilde{A} = (\sqrt{\alpha} * a_1, \sqrt{\alpha} * a_2, \sqrt{\alpha} * a_3) \tag{11}$$

and it can be presented as in Figure 4b (Kang et al., 2012b).

The steps of the MAIRCA method modified by Z-numbers are provided as follows:

Step 1. Forming an initial Z decision-making matrix ($\tilde{Z}$) with *m* alternatives and *n* criteria. In this step, decision makers define the value of every alternative by all criteria ($\tilde{a}_{ij}$) and the degree of certainty of the defined value ($\tilde{b}_{ij}$). The arranged pair $[\tilde{a}_{ij}, \tilde{b}_{ij}]$ represents a Z-number, where *i* represents the number of alternatives, $i \in \{1, 2, \ldots m\}$, and *j* the number of criteria, $j \in \{1, 2, \ldots n\}$.

$$\tilde{Z} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \tilde{a}_{11}\tilde{b}_{11} & \tilde{a}_{12}\tilde{b}_{12} & \ldots & \tilde{a}_{1n}\tilde{b}_{1n} \\ \tilde{a}_{21}\tilde{b}_{21} & \tilde{a}_{22}\tilde{b}_{22} & \ldots & \tilde{a}_{2n}\tilde{b}_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \tilde{a}_{m1}\tilde{b}_{m1} & \tilde{a}_{m2}\tilde{b}_{m2} & \ldots & \tilde{a}_{mn}\tilde{b}_{mn} \end{bmatrix} \end{array} \tag{12}$$

The value $\tilde{a}_{ij}$ is defined in accordance with the characteristics of the criteria, while the value $\tilde{b}_{ij}$ is defined by the expressions presented on fuzzy linguistic scale, as in Figure 5.

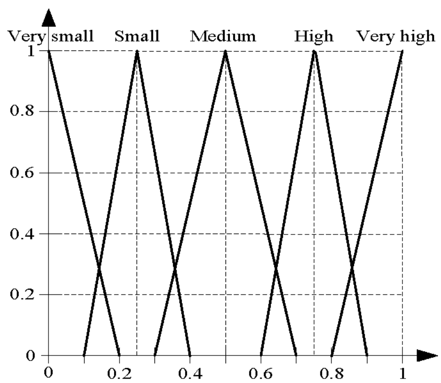

Figure 5. Fuzzy linguistic descriptors for evaluating the degree of conviction of experts (Bobar et al. 2020)

Step 2. Forming an initial decision-making matrix ($\tilde{X}$). The elements of the initial decision-making matrix ($\tilde{X}$) are obtained by converting the elements of the initial Z matrix ($\tilde{Z}$) into the regular fuzzy numbers, by applying the expressions (7)-(11).

$$\tilde{X} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & & & & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & . & . & . & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & . & . & . & \tilde{x}_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \tilde{x}_{m1} & \tilde{x}_{m2} & . & . & . & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{13}$$

Step 3. Normalization of the initial decision-making matrix. The calculation of the elements of normalized matrix depends on the type of criteria. For "benefit" type criteria (bigger criterion value is preferable), this calculation is executed according to the expression:

$$\tilde{n}_{ij} = \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \tag{14}$$

For "cost" type criteria (lower criterion value is preferable), the calculation is executed according to the expression:

$$\tilde{n}_{ij} = \frac{x_{ij} - x_i^+}{x_i^- - x_i^+} \tag{15}$$

The values x_{ij}, x_i^+, x_i^- represent the elements of the initial decision-making matrix ($\tilde{X}$). The values x_i^+, x_i^- are defined as explained bellow:

– $x_i^+ = \max(x_{1r}, x_{2r}, ..., x_{mr})$ – represents maximal values of the right distribution of fuzzy numbers of the observed criteria alternatives;

– $x_i^- = \min(x_{1l}, x_{2l}, ..., x_{ml})$ – represents minimal values of the left distribution of fuzzy numbers of the observed criteria alternatives.

The normalized initial decision-making matrix has the following form:

$$\tilde{N} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} C_1 & C_2 & & & & C_n \end{array} \\ \begin{bmatrix} \tilde{n}_{11} & \tilde{n}_{12} & . & . & . & \tilde{n}_{1n} \\ \tilde{n}_{21} & \tilde{n}_{22} & . & . & . & \tilde{n}_{2n} \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & & & & \cdot \\ \tilde{n}_{m1} & \tilde{n}_{m2} & . & . & . & \tilde{n}_{mn} \end{bmatrix} \end{array} \tag{16}$$

Step 4. Determination of the probability of selection of certain alternatives (P_{A_i}). Decision makers may prefer certain alternatives by assigning different probabilities to the alternatives. In most cases, decision makers are neutral towards the selection of the alternatives. In such case, the preference towards the selection is equal for all the alternatives and it is expressed as follows:

$$P_{A_i} = \frac{1}{m}; \ \sum_{i=1}^{m} P_{A_i} = 1, \ i = 1,2,...,m \tag{17}$$

where *m* represents a total number of alternatives being selected.

Step 5. Forming a theoretical assessment matrix (T_p). In case the condition from Step 4 is met, where the decision maker is neutral in terms of the initial selection of alternatives, so the initial probability (P_{A_i}) of the selection of certain alternatives is the same for all the alternatives, then the theoretical assessment matrix in the form n x 1 is created.

$$T_p = \begin{bmatrix} t_{p1} & t_{p2} & \dots & t_{pn} \end{bmatrix}_{P_{A_i} x W_n} \tag{18}$$

and the matrix elements are calculated as follows:

$$T_p = \begin{bmatrix} P_{A_i} w_1 & P_{A_i} w_2 & \dots & P_{A_i} w_n \end{bmatrix}_{P_{A_i} W_n} \tag{19}$$

where w_n represents the weight coefficient of the criteria.

Step 6. Calculation of real assessment matrix ($\tilde{T}_r$). The calculation of real assessment matrix elements ($\tilde{T}_r$) is performed by applying the expression:

$$\tilde{t}_{rij} = t_{pj} \cdot \tilde{n}_{ij} \tag{20}$$

where t_{pj} represents the elements of the theoretical assessment matrix, and $\tilde{n}_{ij}$ represents the elements of the normalized initial decision-making matrix ($\tilde{N}$). After the calculation, the theoretical assessment matrix is obtained:

$$\tilde{T}_r = \begin{array}{c} \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} \tilde{t}_{r11} & \tilde{t}_{r12} & \dots & \tilde{t}_{r1n} \\ \tilde{t}_{r21} & \tilde{t}_{r22} & \dots & \tilde{t}_{r2n} \\ \dots & \dots & \dots & \dots \\ \tilde{t}_{rm1} & \tilde{t}_{rm2} & \dots & \tilde{t}_{rmn} \end{bmatrix} \end{array} \tag{21}$$

where *n* represents a total number of criteria, and *m* represents a total number of alternatives.

Step 7. Calculation of the gap matrix between theoretical and real weights ($\tilde{G}$):

$$\tilde{g}_{ij} = t_{pj} - \tilde{t}_{rij} \tag{22}$$

After the calculation, it is obtained the total gap matrix ($\tilde{G}$):

$$\tilde{G}=\begin{bmatrix}\tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1n}\\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2n}\\ \cdots & \cdots & \cdots & \cdots\\ \tilde{g}_{m1} & \tilde{g}_{m2} & \cdots & \tilde{g}_{mn}\end{bmatrix} \tag{23}$$

where n represents a total number of criteria, m represents a total number of alternatives being selected, and $\tilde{g}_{ij}$ represents the obtained gap of the alternative *i* by the criterion *j*.

Step 8. Initial ranking of alternatives. For the purpose of ranking alternatives, it is first calculated the values of the criteria functions ($\tilde{Q}_i$) by alternatives. The values of the criteria functions are obtained by summing the gap - the element of the matrix ($\tilde{G}$) by columns:

$$\tilde{Q}_i=\sum_{j=1}^{n}\tilde{g}_{ij},\ i=1,2,...,m \tag{24}$$

where n represents a total number of criteria, m represents a total number of alternatives being selected.

Before defining the initial rank, it is performed defuzzification of the values of the criteria functions ($\tilde{Q}_i$), by applying the expression (Seiford, 1996; Liou and Wang, 1992):

$$q_{ij}=((t_{3ij}-t_{1ij})+(t_{2ij}-t_{1ij}))/3+t_{1ij} \tag{25}$$

$$q_{ij}=\left[\lambda t_{3ij}+t_{2ij}+(1-\lambda)t_{1ij}\right]/2 \tag{26}$$

where λ represents an index of optimism, which can be described as a belief/decision maker's relationship to decision-making risk (Milićević, 2014). The most common optimism index is 0, 0.5 or 1, which corresponds to the pessimistic, average or optimistic view of the decision maker (Milićević, 2014).

Step 9. Final ranking of alternatives. Final rank of alternatives is defined by the application of a dominance index of the first-ranked alternative ($A_{D,1-j}$). It represents the element which defines the value of the first-ranked alternative compared to the remaining alternatives. The dominance index shows the difference between the first-ranked and the other alternatives, and it is defined by the expression:

$$A_{D,1-j}=\left|\frac{|Q_j|-|Q_1|}{|Q_n|}\right|,\ j=2,3,..,m \tag{27}$$

where Q_1 represents the criterion function of the first-ranked alternative, Q_n represents the criterion function of the last-ranked alternative, Q_j represents the criterion function of the alternative being compared with the first-ranked alternative, m represents a total number of alternatives.

For final definition of the first-ranked alternative, it is also necessary to determine a dominance threshold I_D according to the following expression:

$$I_D = \frac{m-1}{m^2} \tag{28}$$

where m represents a total number of alternatives.

If the condition is met where the dominance index $A_{D,1-j}$ is higher or equal to the dominance threshold I_D ($A_{D,1-j} \geq I_D$), then the obtained rank is kept. In case the dominance index $A_{D,1-j}$ is lower than the dominance threshold I_D ($A_{D,1-j} < I_D$), it cannot be certainly concluded that the first-ranked alternative has sufficient advantage compared to the observed alternative.

3. Description of criteria and calculation of weight coefficients of criteria

The selection of a camp space is influenced by a number of criteria. After the analysis of the literature and the survey of experts, seven criteria are defined on which the selection depends.

Criterion 1 (C_1) - **General soil and environmental conditions**. This criterion means the quality of the location where the camp space is planned. The place for the camp space should be clean, dry, drained, slightly sloping, separated from the settlement and away from ponds and swamps at least 2-3 kilometers, in the lee (if the land is exposed to strong winds), out of torrents and floodplains areas (Hristov, 1978). In addition to the above, the camp space should be spacious in order to, under certain conditions, place facilities necessary for the life and operation of the units outside the barracks: residential, economic, medical, recreational, technical, storage, sanitary facilities, *etc.* The criterion is of a linguistic nature.

Criterion 2 (C_2) - **Distance from the road**. In order to ensure uninterrupted life and operation in the field, it is necessary to connect the camp space with local and regional roads (Hristov, 1978). The best variant is that the roads are located right next to the camp space, but very often it will be necessary to build a temporary military road to connect the camp space with the road. The criterion is of a numerical character, where the distance of the camp space from the nearest road is presented in kilometers.

Criterion 3 (C_3) - **Water supply options**. Water supply is a very important component of a camp space. In field conditions, it is necessary to provide sufficient amount of water for normal life and operation of every individual, and thus units, including drinking water, water for cooking food, water for maintaining personal hygiene and cleaning the camp space. The criterion is of a linguistic nature.

Criterion 4 (C_4) - **Scope of works on the arrangement of the camp space**. Regardless of the conditions of the soil on which the organization of the camp space is planned, it is necessary to perform certain works (construction/installation of facilities, construction of temporary roads that connect parts of camp space, *etc.*). The works are carried out in order to arrange the existing land for temporary life and operation of the units. The scope of works depends on a number of factors, such as the type of facilities to be constructed, time planned to be spent by the units in the camp space, the season, and the like (Hristov, 1978). The criterion is of a linguistic nature.

Criterion 5 (C_5) - **Distance from the site where the works are performed**. The main goal of the field conditions is to perform certain works. The site where the unit performs the assigned works should be as close as possible to the camp space. The proximity of the site and the camp space ensures that the people engaged do not waste time traveling to the site and vice versa, that the funds are kept in one place, easier organization of food provision of the unit and the like. The criterion is of a numerical character, where the distance of the camp space from the site is presented in kilometers. In certain cases (*e.g.* construction of a road section), this distance may vary as the work progresses.

Criterion 6 (C_6) - **Direct security of camp space**. Both in peace and during the state of war, the units in the field are obliged to set up direct security of the camp space. The number of persons necessary for the organization of direct security varies and most often depends on the conditions of the land and the layout of the facilities in the camp area. The criterion is of a numerical character, where the minimum number of persons engaged in direct security during one day is defined.

Criterion 7 (C_7) - **Masking conditions**. This criterion exerts its influence on the final decision in situations when the camp space is organized during the implementation of combat operations. The conditions for camouflage include the possibility of hiding or concealing the camp space from enemy reconnaissance (Božanić et al., 2020). Under this criterion, many factors are considered that affect masking, such as the distance from the objects that can be the subject of enemy reconnaissance or action (Hristov, 1978), the possibility of setting up a camp space in the forest, *etc*. The criterion is of a linguistic nature.

All the criteria presented can be divided in two subsets:

- Benefit-type criteria $C^{+} \in \{C_1, C_3, C_7\}$,
- Cost-type criteria $C^{-} \in \{C_2, C_4, C_5, C_6\}$.

The evaluation of the linguistic criteria is performed by applying fuzzy linguistic descriptors, as in Figure 6.

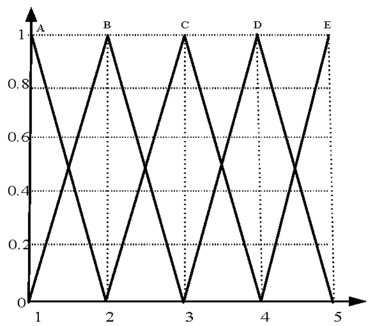

Figure 6. Graphic display of fuzzy linguistic descriptors (Božanić et al. 2016b)

The description of the linguistic criteria is performed by the scale including five fuzzy linguistic descriptors. The marks presented in Figure 6 have the following meanings, depending on the criterion:

- for the criteria C_1, C_3 and C_7: A=very bad (VB), B=bad (B), C=medium (M), D= good (G), E=very good (VG)
- for the criterion C_4: A=very small (VS), B=small (S), C=medium (M), D=large (L), E=very large (VL).

In the second phase of the research, it is performed the calculation of the weight coefficients of the criteria by applying the LBWA method, described in the previous section, on the basis of the input parameters:

- As the most significant criterion it is determined the criterion C_1;
- The criteria are roughly arranged by levels as follows: $S_1=\{C_1,C_7,C_5\}$; $S_2=\{C_3,C_2\}$; $S_3=\{C_6\}$; $S_4=\{C_4\}$;
- Comparing the criteria by levels, the following values are obtained: $S_1: I_1=0$, $I_7=0.8$, $I_5=1.1$; S_2: $I_3=0$, $I_2=2$; S_3: $I_6=0.2$; S_4: $I_4=0.4$.

Applying the expressions (3)-(5), the following weight coefficients of the criteria are obtained:

$$w_j=(0.244, 0.098, 0.122, 0.06, 0.192, 0.08, 0.204).$$

Based on the calculation presented, the conditions for the following phase of the model application, i.e. the selection of the best alternative by the application of the Z-MAIRCA method are created.

4. Testing of the model – selection of the best alternative

In the third phase of the paper, it is performed the testing of the model. Testing is performed with ten alternatives, i.e. potential locations for the organization of a camp space. At the very beginning, it is defined the initial Z decision-making matrix, as in Table 1.

Table 1. Initial Z decision-making matrix

Crit.	C_1		C_2		C_3		C_4		C_5		C_6		C_7	
Alter.	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$
A_1	VB	VS	(3,3.5,4.2)	S	VB	M	VL	M	(3,7,13)	H	(3,5,6)	M	M	M
A_2	B	M	(2,2.9,3.9)	VH	M	M	M	VS	(4.2,9,15)	M	(2,5,7)	VS	M	H
A_3	G	S	(3,3.3,3.6)	VS	B	VS	L	M	(1.5,6,11)	S	(6,8,11)	S	VB	VH
A_4	M	H	(0.5,0.5,0.7)	M	VB	S	VL	S	(1.9,7,12)	VS	(4,4,6)	H	B	VS
A_5	VG	VH	(1.3,1.8,2.2)	S	B	VS	VS	H	(6,12,17)	VH	(12,13,13)	VH	VB	H
A_6	G	M	(4.5,5,5)	H	VG	H	S	VH	(5,11,16)	M	(4,5,5)	VS	G	M
A_7	VB	VS	(2.2,2.7,2.9)	VS	G	VH	L	VS	(2,7,13)	H	(3,5,8)	M	B	S
A_8	M	S	(0.4,0.6,1)	H	G	H	M	H	(1,3,7)	VS	(4,9,14)	S	M	VS
A_9	B	H	(0.9,1.5,1.7)	VH	M	S	S	S	(1.5,3,8.5)	S	(3,7,8)	H	VG	VH
A_{10}	VG	VH	(1.8,2.5,2.8)	M	VG	VH	VS	VH	(6,13,21)	VH	(5,11,14)	VH	G	S

After the definition of the initial Z decision-making matrix, it is performed its quantification, as in Table 2.

Table 2. Quantified initial Z decision-making matrix

Crit.	C_1		C_2			C_7	
Alter.	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	...	$\tilde{A}$	$\tilde{B}$
A_1	(1,1,2)	(0,0,0.2)	(3,3.5,4.2)	(0.1,0.25,0.4)	...	(2,3,4)	(0.3,0.5,0.7)
A_2	(1,2,3)	(0.3,0.5,0.7)	(2,2.9,3.9)	(0.8,1,1)	...	(2,3,4)	(0.55,0.75,0.95)
A_3	(3,4,5)	(0.1,0.25,0.4)	(3,3.3,3.6)	(0,0,0.2)	...	(1,1,2)	(0.8,1,1)
A_4	(2,3,4)	(0.55,0.75,0.95)	(0.5,0.5,0.7)	(0.3,0.5,0.7)	...	(1,2,3)	(0,0,0.2)
A_5	(4,5,5)	(0.8,1,1)	(1.3,1.8,2.2)	(0.1,0.25,0.4)	...	(1,1,2)	(0.55,0.75,0.95)
A_6	(3,4,5)	(0.3,0.5,0.7)	(4.5,5,5)	(0.55,0.75,0.95)	...	(3,4,5)	(0.3,0.5,0.7)
A_7	(1,1,2)	(0,0,0.2)	(2.2,2.7,2.9)	(0,0,0.2)	...	(1,2,3)	(0.1,0.25,0.4)
A_8	(2,3,4)	(0.1,0.25,0.4)	(0.4,0.6,1)	(0.55,0.75,0.95)	...	(2,3,4)	(0,0,0.2)
A_9	(1,2,3)	(0.55,0.75,0.95)	(0.9,1.5,1.7)	(0.8,1,1)	...	(4,5,5)	(0.8,1,1)
A_{10}	(4,5,5)	(0.8,1,1)	(1.8,2.5,2.8)	(0.3,0.5,0.7)	...	(3,4,5)	(0.1,0.25,0.4)

By converting Z-numbers presented in Table 2, it is formed the initial decision-making matrix ($\tilde{X}$), as in Table 3.

Table 3. Initial decision-making matrix

Alter.	C_1	C_2		C_7
A_1	(0.258,0.258,0.516)	(1.5,1.75,2.1)	...	(1.414,2.121,2.828)
A_2	(0.707,1.414,2.121)	(1.932,2.802,3.768)	...	(1.732,2.598,3.464)
A_3	(1.5,2,2.5)	(0.775,0.852,0.93)	...	(0.966,0.966,1.932)
A_4	(1.732,2.598,3.464)	(0.354,0.354,0.495)	...	(0.258,0.516,0.775)
A_5	(3.864,4.83,4.83)	(0.65,0.9,1.1)	...	(0.866,0.866,1.732)
A_6	(2.121,2.828,3.536)	(3.897,4.33,4.33)	...	(2.121,2.828,3.536)
A_7	(0.258,0.258,0.516)	(0.568,0.697,0.749)	...	(0.5,1,1.5)
A_8	(1,1.5,2)	(0.346,0.52,0.866)	...	(0.516,0.775,1.033)
A_9	(0.866,1.732,2.598)	(0.869,1.449,1.642)	...	(3.864,4.83,4.83)
A_{10}	(3.864,4.83,4.83)	(1.273,1.768,1.98)	...	(1.5,2,2.5)

Further, it is performed the normalization of the initial decision-making matrix, as in Table 4.

Table 4. Normalized initial decision-making matrix

Alter.	C_1	C_2		C_7
A_1	(0,0,0.056)	(0.56,0.648,0.71)	...	(0.253,0.407,0.562)
A_2	(0.098,0.253,0.407)	(0.141,0.384,0.602)	...	(0.322,0.512,0.701)
A_3	(0.272,0.381,0.49)	(0.854,0.873,0.893)	...	(0.155,0.155,0.366)
A_4	(0.322,0.512,0.701)	(0.963,0.998,0.998)	...	(0,0.056,0.113)
A_5	(0.789,1,1)	(0.811,0.861,0.924)	...	(0.133,0.133,0.322)
A_6	(0.407,0.562,0.717)	(0,0,0.109)	...	(0.407,0.562,0.717)
A_7	(0,0,0.056)	(0.899,0.912,0.944)	...	(0.053,0.162,0.272)
A_8	(0.162,0.272,0.381)	(0.87,0.957,1)	...	(0.056,0.113,0.169)
A_9	(0.133,0.322,0.512)	(0.675,0.723,0.869)	...	(0.789,1,1)
A_{10}	(0.789,1,1)	(0.59,0.643,0.767)	...	(0.272,0.381,0.49)

Considering that the decision makers did not have different preferences towards the selection of the alternatives, it is calculated that $P_{A_i} = 1/10 = 0.1$. Based on that, it is performed the calculation of the elements of the theoretical assessment matrix provided in Table 5.

Table 5. Theoretical assessment matrix

Alter.	C_1	C_2		C_7
A_{1-10}	(0.024,0.024,0.024)	(0.01,0.01,0.01)	...	(0.02,0.02,0.02)

The elements of the real assessment matrix are presented in Table 6.

Table 6. Real assessment matrix

Alter.	C_1	C_2		C_7
A_1	(0,0,0.001)	(0.005,0.006,0.007)	...	(0.005,0.008,0.011)
A_2	(0.002,0.006,0.01)	(0.001,0.004,0.006)	...	(0.007,0.01,0.014)
A_3	(0.007,0.009,0.012)	(0.008,0.009,0.009)	...	(0.003,0.003,0.007)
A_4	(0.008,0.012,0.017)	(0.009,0.01,0.01)	...	(0,0.001,0.002)
A_5	(0.019,0.024,0.024)	(0.008,0.008,0.009)	...	(0.003,0.003,0.007)
A_6	(0.01,0.014,0.017)	(0,0,0.001)	...	(0.008,0.011,0.015)
A_7	(0,0,0.001)	(0.009,0.009,0.009)	...	(0.001,0.003,0.006)
A_8	(0.004,0.007,0.009)	(0.009,0.009,0.01)	...	(0.001,0.002,0.003)
A_9	(0.003,0.008,0.012)	(0.007,0.007,0.009)	...	(0.016,0.02,0.02)
A_{10}	(0.019,0.024,0.024)	(0.006,0.006,0.008)	...	(0.006,0.008,0.01)

Further, it is performed the calculation of the total gap matrix, as in Table 7.

Table 7. Total gap matrix

Alter.	C_1	C_2		C_7
A_1	(0.023,0.024,0.024)	(0.003,0.003,0.004)	...	(0.009,0.012,0.015)
A_2	(0.014,0.018,0.022)	(0.004,0.006,0.008)	...	(0.006,0.01,0.014)
A_3	(0.012,0.015,0.018)	(0.001,0.001,0.001)	...	(0.013,0.017,0.017)
A_4	(0.007,0.012,0.017)	(0,0,0)	...	(0.018,0.019,0.02)
A_5	(0,0,0.005)	(0.001,0.001,0.002)	...	(0.014,0.018,0.018)
A_6	(0.007,0.011,0.014)	(0.009,0.01,0.01)	...	(0.006,0.009,0.012)
A_7	(0.023,0.024,0.024)	(0.001,0.001,0.001)	...	(0.015,0.017,0.019)
A_8	(0.015,0.018,0.02)	(0,0,0.001)	...	(0.017,0.018,0.019)
A_9	(0.012,0.017,0.021)	(0.001,0.003,0.003)	...	(0,0,0.004)
A_{10}	(0,0,0.005)	(0.002,0.003,0.004)	...	(0.01,0.013,0.015)

In the further process of application of the Z-MAIRCA model, the gap of alternatives is calculated, and the obtained values are defuzzified, on the basis of which the initial rank of the alternatives is defined. Then, the calculation of the dominance index and the definition of the final rank are performed, as in Table 8.

Table 8. Ranking alternatives

Alter.	Alternative gap $\tilde{Q}_i$	Alternative gap Q_i	Initial rank	$A_{D,1-j}$	Final rank
A_1	(0.052,0.064,0.074)	0.0559	10	0.534	10
A_3	(0.042,0.054,0.063)	0.0465	9	0.365	9
A_4	(0.041,0.05,0.058)	0.0437	7	0.316	7
A_5	(0.038,0.05,0.062)	0.042	6	0.285	6
A_6	(0.027,0.041,0.056)	0.0318	3	0.103	3
A_7	(0.041,0.053,0.065)	0.0454	8	0.345	8
A_8	(0.037,0.047,0.058)	0.0402	5	0.252	5
A_9	(0.023,0.034,0.049)	0.0262	2	0.003	1*
A_{10}	(0.022,0.035,0.057)	0.0261	1	0.000	1

In accordance with the obtained dominance threshold ($I_D = 0.09$), it can be noted that the advantage of the initially first-ranked alternative (A_{10}) is not significant enough, compared to the second-ranked alternative (A_9). Accordingly, a decision maker can select any of the two mentioned alternatives as the first-ranked.

5. Sensitivity analysis

An inevitable section of any model is a sensitivity analysis. There are different approaches to sensitivity analysis (Pamučar et al., 2017a). In this paper, a sensitivity analysis is performed by favoring the significance (weight coefficient) of one criterion in every scenario. For the needs of the analysis, seven scenarios are defined, as in Table 9.

Table 9. Sensitivity analysis scenarios

Criteria	S-0	S-1	S-2	S-3	S-4	S5	S6	S7
C_1	0.244	0.4	0.1	0.1	0.1	0.1	0.1	0.1
C_2	0.098	0.1	0.4	0.1	0.1	0.1	0.1	0.1
C_3	0.122	0.1	0.1	0.4	0.1	0.1	0.1	0.1
C_4	0.06	0.1	0.1	0.1	0.4	0.1	0.1	0.1
C_5	0.192	0.1	0.1	0.1	0.1	0.4	0.1	0.1
C_6	0.08	0.1	0.1	0.1	0.1	0.1	0.4	0.1
C_7	0.204	0.1	0.1	0.1	0.1	0.1	0.1	0.4

By applying the Z-MAIRCA model and the defined weight coefficients by scenarios, the ranks of alternatives shown in Table 10 are obtained. The ranks shown indicate the initial rank, and an asterisk next to certain ranks indicates that in the final ranking, the alternatives marked with an asterisk would be ranked as the first ones.

Table 10. Ranks of alternatives by different scenarios

Alternatives	S-0	S-1	S-2	S-3	S-4	S5	S6	S7
A_1	10	10	9	10	10	10	9	7
A_2	4	6	8	5	3*	4	1	3
A_3	9	8	7	9	9	8	8	8
A_4	7	5	5	7	8	7	7	10
A_5	6	2	6	8	6	9	10	9
A_6	3	3	10	3	5	5	2*	2
A_7	8	9	3*	2	4	3	4*	5
A_8	5	7	2*	4	7	2	5	6
A_9	2*	4	1*	6	1	1	3*	1
A_{10}	1	1	4*	1	2*	6	6	4

The obtained ranks, shown in Table 10, indicate that the favoring of certain criteria affects the differences in ranks, which indicates that the developed model is sensitive to changes in the weight coefficients of the criteria. The rank correlation control is performed using the Spearman's coefficient:

$$S = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2 - 1)} \tag{29}$$

where: S - the value of the Spearman's coefficient; D_i - the difference in the rank of the given element in the vector w and the rank of the correspondent element in the reference vector; n - number of ranked elements. The values of the Spearman's coefficient range between -1 and 1, i.e. from the ideal negative to the ideal positive rank correlation.

Table 11 provides the values of the Spearman's coefficient by comparing all the scenarios mutually, based on the initial rank of alternatives.

Table 11. The value of the Spearman's coefficient based on the initial ranks of alternatives

Scenarios	S-0	S-1	S-2	S-3	S-4	S5	S6	S7
S-0	1	0.794	0.285	0.594	0.830	0.606	0.727	0.727
S-1		1	0.091	0.370	0.552	0.055	0.091	0.224
S-2			1	0.000	0.158	0.606	0.048	-0.048
S-3				1	0.685	0.624	0.624	0.564
S-4					1	0.673	0.661	0.770
S-5						1	0.794	0.685
S-6							1	0.830
S-7								1

From Table 11, it can be observed that the rank correlation in most of the cases is very high. However, the most important correlation of ranks is between the scenario S-0 and the others, where a significant deviation from the scenario S-2 is observed. The S-2 scenario has a low correlation with other scenarios as well. This result presents a combination of two factors: the values of the evaluated alternatives by the criterion C_2 and a significant increase in the weight coefficient of the criterion C_2 in the scenario S-2 (by four times). The deviation is also observed in the correlation of the S-1 strategy with almost all other strategies. The analysis of the ranks shows that the most significant part of the non-correlation is the popping up of the alternative A_5 in the second scenario as the second-ranked. Finally, it is pointed out that in all scenarios, the alternatives A_9 or A_{10} are ranked as the first or one of the first-ranked. According to all the above, it can be concluded that the developed model is sufficiently sensitive. Also, the model can tolerate minor errors in defining the weight coefficients of the criteria, i.e. in the evaluation of the alternatives by criteria.

Given the existence of certain minor deviations in the sensitivity analysis, the results obtained by the Z-MAIRCA model are compared with the results obtained by the MABAC and VIKOR methods (classic and modified with Z-numbers - Z-MABAC and Z-VIKOR and fuzzy numbers - f- MABAC and f-VIKOR) and the MAIRCA (classic and modified with fuzzy numbers - f-MAIRCA). In Table 12, the ranks obtained by the above methods are presented.

Table 12. Ranks of alternatives obtained by applying different methods

Alternatives	Z-MAIRCA	Z-VIKOR	Z-MABAC	f-MAIRCA	f-VIKOR	f-MABAC	MAIRCA	VIKOR	MABAC
A_1	10	10	10	10	10	10	10	10	10
A_2	4	8	5	5	9	7	6	8	6
A_3	9	7	8	8	6	8	7	6	7
A_4	7	5	6	6	7	6	5	7	5
A_5	6	3	7	7	4	5	8	5	8
A_6	3	6	3	2	3	2	4	4	4
A_7	8	9	9	9	8	9	9	9	9
A_8	5	4	4	3	5	4	2	3	2
A_9	2	2	1	4	2	3	1	2	1
A_{10}	1	1	2	1	1	1	3	1	3

Figure 7 shows the rank of alternatives using different methods from which the correlation of ranks is more clearly observed.

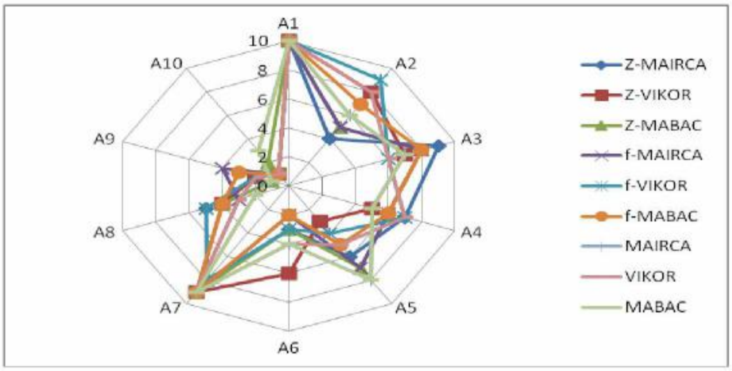

Figure 7. Graphic presentation of the rank of alternatives obtained by applying different methods

From Figure 7 and Table 12, it can be observed a clear dominance of the alternatives A_9 and A_{10}, as well as the rank of the alternatives A_7 and A_1, which are most often ranked as the last ones. Despite the obvious correlation of ranks, in Table 13, the values of the Spearman's correlation coefficient of ranks for different methods and their modifications are provided.

Table 13. Value of the Spearman's coefficient for different methods

Method	Z-MAIRCA	Z-VIKOR	Z-MABAC	f-MAIRCA	f-VIKOR	f-MABAC	MAIRCA	VIKOR	MABAC
Z-MAIRCA	1	0.733	0.952	0.909	0.770	0.903	0.806	0.806	0.806
Z-VIKOR		1	0.770	0.709	0.891	0.855	0.745	0.915	0.745
Z-MABAC			1	0.442	0.309	0.430	0.648	0.867	0.648
f-MAIRCA				1	0.758	0.939	0.867	0.842	0.867
f-VIKOR					1	0.915	0.721	0.952	0.721
f-MABAC						1	0.830	0.927	0.830
MAIRCA							1	0.855	1
VIKOR								1	0.855
MABAC									1

From Table 13, it is clear that there is a high correlation of ranks obtained by different methods and their modifications. It is especially important to point out the high correlation of the ranks of the Z-MAIRCA model with the f-MAIRCA and the classic MAIRCA method. Accordingly, it can be concluded that the developed model provides usable results in conditions of uncertainty. It is also observed that there is an impact of uncertainty treatment on the final ranking of alternatives, and that it can significantly influence the selection, but not to such an extent where the ranks of alternatives are not correlated.

6. Conclusion

The paper explains the phases of development of multi-criteria decision-making model based on the LBWA method and the MAIRCA method modified with Z-numbers. The presented model is successfully applied in the selection of camp space locations. In addition to the description of the model, the paper describes the problem that was being solved, i.e. the selection of a location for a camp space. The highlighted problem belongs to the group of location issues. The analysis of the literature indicates that multi-criteria decision-making methods have a great application in solving this type of problems.

The paper describes in detail the steps of the LBWA method and the MAIRCA method modified with Z-numbers, as well as their previous application in the literature. The paper also presents the basics related to the application of Z-numbers, as a very important way to deal with uncertainty. The model application process itself has followed the definition of the criteria for the selection of the best alternative and the calculation of their weight coefficients using the LBWA method. Seven criteria of different character (benefit and cost-type criteria) are defined, on which the selection of a camp space depends. A part of the criteria, which is of a linguistic nature, clearly indicated the need to apply methods that deal with uncertainty.

The presentation of the model application is performed on ten alternatives. By applying the Z-MAIRCA model, the ranking of alternatives is successfully performed. Finally, sensitivity analysis is done. The results of the sensitivity analysis indicate the possibility of successful application of the model in cases of minor errors in defining the weight coefficients and in evaluating the alternatives according to the criteria.

In the following research, the model presented can be tested in solving other problems as well. On the other hand, it is possible to apply other ways of dealing with uncertainty so as to solve the problem presented in the paper.

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PRIORITIZATION OF ROAD TRANSPORTATION RISKS: AN APPLICATION IN GİRESUN PROVINCE

Salih Memiş [1], Ezgi Demir [2], Çağlar Karamaşa* [3], Selçuk Korucuk [1]

[1] Giresun University, Department of International Trade and Logistics, Giresun, Turkey
[2] Piri Reis University, Department of Management Information Systems, İstanbul, Turkey
[3] Anadolu University, Department of Business Administration, Eskişehir, Turkey

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Abstract: The purpose of this study is to determine and rank the road transportation risk factors that are crucial for effective and economic supply chain management. Road transportation risk factors can be defined as equipment related risks, risk to be lost and disappearance, risks related to delivery and packaging, inadequacy of qualified personnel and technical equipment, risks caused from incompatibility to logistic information system/technology, security risk, compulsory reasons, risks originated from regulations and arrangements, risks related to waiting at customs gate and transport infrastructure based risks. Accordingly, fuzzy PIPRECIA as a multi-criteria ranking method was used to prioritize the risk factors. According to the results, while the transport infrastructure based risks criterion was found as the most important, the risk to be lost and disappearance factor was obtained as the least important one.

Keywords: *Road transportation, road transportation risk factors, PIPRECIA, Fuzzy sets.*

1. Introduction

Goods, money and documents that are subject to commerce are started to circulate in market after globalization happened in 21th century. Companies try to find new methods in order to be competitive and reduce risks in related markets with globalization and the rapid development of information technologies. Circulation of goods is possible with suitable risk management plan under controlled, in time and most economical manner.

* Corresponding author.
salih.memis@giresun.edu.tr (S.Memiş), edemir@pirireis.edu.tr (E.Demir), ckaramasa@anadolu.edu.tr (Ç. Karamaşa), selcuk.korucuk@giresun.edu.tr (S.Korucuk)

International transportation becomes crucial in parallel with the development of international commerce due to consumers' habits in recent years. It is a requirement of transporting related goods and raw materials from one point to another because of rising needs and globalized commerce. Economic growth leads to the increased demand for freight shipment especially. Observed advancements in the communication between transportation and information technologies contribute to the circulation of goods. In this context, local and global commerce can be possible via the assurance of transportation activities.

Each process of international trade contains various risks. Transportation risk can be considered as the most crucial and critical one due to including damages for goods that are subject to international trade. Risks related to transportation activities include not only driver based accidents in a transportation process, but also error based accidents in goods traffic. In other words, transportation risk can be defined as issues such as driver errors, missing and incorrect operations related to goods subject to trade in packaging and loading processes.

It is not possible to develop and generalize international trade without bringing transportation sector based risks that are drivers of commerce and goods circulation under control. Risk and risk management concepts are started to gain importance, while international trade makes progress from exchange periods to virtual worlds. Each step of international trade includes different risks too. Therefore, globalization increased risks in the international trade. Transportation risks in the logistic activities need to be evaluated thoroughly due to having direct impact on the goods subject to trade.

Risks happened in transportation activities can cause loss of property and material damage. Hence, transportation risk can be described as damage risk too. However, issues observed in transportation can cause loss of lives apart from material damage. Additionally, a time concept is handled as an essential risk element because incompatibility in arrangements related to good transport lead to material damage.

Risk management in transportation activities can be differentiated for each mode and include related people identification, determination of danger and related risk, taking a risk control process into account according to the dangers, reviewing process and taking additional precautions for the risk control process.

Road transportation is one of the mostly preferred transportation types due to low cost, delivery time and transport. General transportation and authorization rules are possible for each country. Additional rules can be applied according to the countries involved in a transportation process. That condition creates a risk element as obligation for obeying the rules related to road transportation regulations and arrangements. Accordingly, road transportation risk factors can be stated as equipment related risks, risk to be lost and disappearance, risks related to delivery and packaging, inadequacy of qualified personnel and technical equipment, risks caused from incompatibility to logistic information system/technology, security risk, compulsory reasons, risks related to waiting at customs gate and transport infrastructure based risks (Pezier, 2002; Cavinato, 2004; Tang, 2006; Manuj and Mentzer, 2008; Enyinda et al., 2010; Hoffman et al., 2013; Ho et al., 2015; Kara and Fırat, 2015; Koban and Keser, 2015; Korucuk and Erdal, 2018; Korucuk and Memiş, 2018).

In this way, aforementioned road transportation risk factors are important for all stakeholders and have a direct impact on a business competitive level via cost minimization. In this context, the purpose of this study is to rank the road transportation risk criteria. A case study is made in Girusen province, Turkey. PIPRECIA as a multi-criteria decision-making method is used for prioritization under fuzzy environment in order to better represent decision-makers' judgments.

Other parts of the study are presented as follows: Studies for transportation and related risk factors are explained in the second part. Fuzzy PIPRECIA is introduced in the third section. Case study applied in Giresun province and findings are presented in the fourth part. Conclusions and future suggestions are made in the last section.

2. Literature Review

Transportation and transportation risk factors related studies can be presented as below:

Lazar et al. (2001) made risk evaluation in hazardous waste transportation via geographical information systems. Chen et al. (2003) made overall evaluation related to transportation risks in radioactive substance and waste under normal and accident conditions. Erkut and Ingolfsson (2005) examined transportation risk models in dangerous goods carriage and proposed new ones after a revision process.

Xin et al. (2007) evaluated routing, inventory, planning, management-organization and external factors under logistic risks context. Ghazali (2009) examined the operational risks for highway projects in Malaysia. Risks are defined as wage scales, traffic congestion, road network change and excess load carriage.

Adams (2010) searched a transportation risk based model and proposed a human behaviour based model. Wang (2011) used AHP model for ranking logistical risk factors according to carriage, technology, process, management, decision-making and environment contexts.

Khan (2013) considered the risk factors in employee life cycle and presented various risk analysis methods. Zeng and Song (2015) made fuzzy based risk assessment in order to ensure road safety in project carriage. Govindan and Chaudhuri (2016) applied DEMATEL method for evaluating risk factors in third party logistical service providers. Prakas et al. (2017) proposed supply chain network design structure and model related to supply chain and logistical risks. Furthermore, they observed the efficiency of supply chain risk design in risk evaluation. İzer (2017) investigated new risk reduction technologies for cold chain logistics.

Korucuk and Erdal (2018) ranked logistical risk factors for firms in cold chain transportation and found the most ideal risk management tool. Noriega et al. (2018) examined risk factors related to livestock carriage in Mexico. Korucuk and Memiş (2018) measured the risk factors for the supply chain via AHP and found quality risk as the most essential one. Budzynski et al. (2019) examined tramway transportation risks and made propositions for increasing transportation quality and security.

According to the depth literature review, there is not enough study in order to determine the importance levels for road transportation risk factors and that shows the originality and novelty of this concept. In addition, authors anticipate the contribution of this study to literature from method and application area viewpoint.

3. Methodology

3.1. Fuzzy Pivot Pairwise RElative Criteria Importance Assessment- Fuzzy PIPRECIA Method

The Fuzzy PIPRECIA method was developed by Stević et al. (2018). It consists of 11 steps shown below.

Step 1. Forming the required benchmarking set of criteria and forming a team of decision-makers. Sorting the criteria according to marks from the first to the last, which means they need to be sorted unclassified. Therefore, in this step, their significance is irrelevant.

Step 2. In order to determine the relative importance of criteria, each decision-maker individually evaluates the pre-sorted criteria by starting from the second criterion, Equation (1).

$$\overline{s_j^r} = \begin{cases} > \bar{1} & if \quad C_j > C_{j-1} \\ = \bar{1} & if \quad C_j = C_{j-1} \\ < \bar{1} & if \quad C_j < C_{j-1} \end{cases} \tag{1}$$

$\overline{s_j^r}$ denotes the evaluation of the criteria by a decision-maker r. In order to obtain a matrix $\overline{s_j}$, it is necessary to perform the averaging of matrix $\overline{s_j^r}$ using a geometric mean. Decision-makers evaluate the criteria by applying the linguistic scales developed and defined in Stević et al. (2018).

Step 3. Determining the coefficient $\overline{k_j}$

$$\overline{k_j} = \begin{cases} = \bar{1} & if \quad j = 1 \\ 2 - \overline{s_j} & if \quad j > 1 \end{cases} \tag{2}$$

Step 4. Determining the fuzzy weight $\overline{q_j}$

$$\overline{q_j} = \begin{cases} \underline{= \bar{1}} & if \quad j = 1 \\ \dfrac{q_{j-1}}{\overline{k_j}} & if \quad j > 1 \end{cases} \tag{3}$$

Step 5. Determining the relative weight of the criterion $\overline{w_j}$

$$\overline{w_j} = \frac{\overline{q_j}}{\sum_{j=1}^{n} \overline{q_j}} \tag{4}$$

In the following steps, it is necessary to apply the inverse methodology of the fuzzy PIPRECIA method.

Step 6. Evaluation of the applying scale defined above, but this time starting from a penultimate criterion.

$$\overline{s_j^r}' = \begin{cases} > \bar{1} & if \quad C_j > C_{j+1} \\ = \bar{1} & if \quad C_j = C_{j+1} \\ < \bar{1} & if \quad C_j < C_{j+1} \end{cases} \tag{5}$$

$\overline{s_j^r}'$ denotes the evaluation of the criteria by a decision-maker r.

It is again necessary to average the matrix $\overline{s_j^r}$ by applying a geometric mean.

Step 7. Determining the coefficient $\overline{k_j}'$

$$\overline{k_j}' = \begin{cases} = \bar{1} & if \quad j = n \\ 2 - \overline{s_j}' & if \quad j > n \end{cases} \tag{6}$$

n denotes a total number of criteria. Specifically, in this case, it means that the value of the last criterion is equal to fuzzy number one.

Step 8. Determining the fuzzy weight $\overline{q_j}'$

$$\overline{q_j}' = \begin{cases} = \bar{1} & if \quad j = n \\ \dfrac{\overline{q_{j+1}}'}{\overline{k_j}'} & if \quad j > n \end{cases} \tag{7}$$

Step 9. Determining the relative weight of the criterion $\overline{w_j}'$

$$\overline{w_j}' = \frac{\overline{q_j}'}{\sum_{j=1}^{n} \overline{q_j}'} \tag{8}$$

Step 10. In order to determine the final weights of the criteria, it is first necessary to perform the defuzzification of the fuzzy values $\overline{w_j}$ and $\overline{w_j}'$

$$\overline{w_j}" = \frac{1}{2}(w_j + w_j') . \tag{9}$$

Step 11. Checking the results obtained by applying Spearman and Pearson correlation coefficients.

3.2. The Evaluation of Criteria Using the Fuzzy PIPRECIA Method

In this study, ten criteria are handled for evaluating road transportation risks by eight decision-makers. Criteria related to road transportation risks are presented in Table 1.

Table 1. Criteria related to road transportation risks

Criteria	Mark
Risk to be lost and disappearance	C1
Equipment related risks	C2
Risks related to delivery and packaging	C3
Inadequacy of qualified personnel and technical equipment	C4
Risks caused from incompatibility to logistic information system/technology	C5
Security risk	C6
Compulsory reasons	C7
Risks originated from regulations and arrangements	C8
Risks related to waiting at customs gate	C9
Transport infrastructure based risks	C10

The evaluation of the criteria has been performed using a linguistic scale that involves quantification into fuzzy triangular numbers. Figure 1 and Figure 2 shows the evaluation of the criteria for fuzzy PIPRECIA and inverse fuzzy PIPRECIA by decision-makers and the average values (AV) which are used for further calculation. It is important to note that, compared to the original method developed, the average value (AV) is used here to average decision-makers' preferences (Đalić et al., 2020; Vesković et al., 2020; Tomašević et al., 2020; Stanković et al., 2020), which in this specific case contributed to the more accurate input parameters of the model. Whether a geometric mean or an average value is applied depends directly on a particular case. Both methods of averaging are valid.

PIPR.	C1			C2			C3			C4			C5			C6			C7			C8			C9			C10		
DM1				0.333	0.400	0.500	1.400	1.600	1.650	0.400	0.500	0.667	0.500	0.667	1.000	1.400	1.600	1.650	0.400	0.500	0.667	1.500	1.750	1.800	0.400	0.500	0.667	0.333	0.400	0.500
DM2				0.667	1.000	1.000	1.600	1.900	1.950	0.333	0.400	0.500	0.667	1.000	1.000	1.600	1.900	1.950	0.400	0.500	0.667	1.300	1.450	1.500	0.500	0.667	1.000	1.400	1.600	1.650
DM3				0.400	0.500	0.667	1.400	1.600	1.650	1.400	1.600	1.650	0.500	0.667	1.000	1.400	1.600	1.650	0.333	0.400	0.500	1.500	1.750	1.800	0.500	0.667	1.000	1.500	1.750	1.800
DM4				1.400	1.600	1.650	1.400	1.600	1.650	0.400	0.500	0.667	1.300	1.450	1.500	1.500	1.750	1.800	0.400	0.500	0.667	0.333	0.400	0.500	1.400	1.600	1.650	1.500	1.750	1.800
DM5				1.100	1.150	1.200	0.400	0.500	0.667	0.333	0.400	0.500	1.400	1.600	1.650	1.600	1.900	1.950	0.500	0.667	1.000	1.500	1.750	1.800	1.600	1.900	1.950	1.500	1.750	1.800
DM6				1.100	1.150	1.200	0.500	0.667	1.000	1.300	1.450	1.500	0.286	0.333	0.400	1.600	1.900	1.950	0.400	0.500	0.667	1.500	1.750	1.800	1.600	1.900	1.950	1.400	1.600	1.650
DM7				1.100	1.150	1.200	1.200	1.300	1.350	1.500	1.750	1.800	0.500	0.667	1.000	1.400	1.600	1.650	1.200	1.300	1.350	0.500	0.667	1.000	1.400	1.600	1.650	0.400	0.500	0.667
DM8				1.100	1.150	1.200	1.200	1.300	1.350	0.500	0.667	1.000	1.300	1.450	1.500	1.400	1.600	1.650	0.400	0.500	0.667	1.200	1.300	1.350	1.100	1.150	1.200	0.286	0.333	0.400
AV				0.900	1.013	1.077	1.138	1.308	1.408	0.771	0.908	1.035	0.807	0.979	1.131	1.488	1.731	1.781	0.504	0.608	0.773	1.167	1.352	1.444	1.063	1.248	1.383	1.040	1.210	1.283

Figure 1. Evaluation of criteria by eight DMs for the fuzzy PIPRECIA

PIPR-I	C10			C9			C8			C7			C6			C5			C4			C3			C2			C1		
DM1				1.300	1.450	1.500	1.200	1.300	1.350	0.250	0.286	0.333	1.200	1.300	1.350	0.286	0.333	0.400	1.100	1.150	1.200	1.200	1.300	1.350	0.286	0.333	0.400	1.300	1.450	1.500
DM2				0.286	0.333	0.400	1.100	1.150	1.200	0.333	0.400	0.500	1.200	1.300	1.350	0.222	0.250	0.286	1.000	1.000	1.050	1.300	1.450	1.500	0.222	0.250	0.286	1.000	1.000	1.050
DM3				0.250	0.286	0.333	1.100	1.150	1.200	0.250	0.286	0.333	1.300	1.450	1.500	0.286	0.333	0.400	1.100	1.150	1.200	0.286	0.333	0.400	0.286	0.333	0.400	1.200	1.300	1.350
DM4				0.250	0.286	0.333	0.286	0.333	0.400	1.300	1.450	1.500	1.200	1.300	1.350	0.250	0.286	0.333	0.333	0.400	0.500	1.200	1.300	1.350	0.286	0.333	0.400	0.286	0.333	0.400
DM5				0.250	0.286	0.333	0.222	0.250	0.286	0.250	0.286	0.333	1.100	1.150	1.200	0.222	0.250	0.286	0.286	0.333	0.400	1.300	1.450	1.500	1.200	1.300	1.350	0.500	0.667	1.000
DM6				0.286	0.333	0.400	0.222	0.250	0.286	0.250	0.286	0.333	1.200	1.300	1.350	0.222	0.250	0.286	1.400	1.600	1.650	0.333	0.400	0.500	1.100	1.150	1.200	0.500	0.667	1.000
DM7				1.200	1.300	1.350	0.286	0.333	0.400	1.100	1.150	1.200	0.400	0.500	0.667	0.286	0.333	0.400	1.100	1.150	1.200	0.250	0.286	0.333	0.400	0.500	0.667	0.500	0.667	1.000
DM8				1.400	1.600	1.650	0.500	0.667	1.000	0.400	0.500	0.667	1.200	1.300	1.350	0.286	0.333	0.400	0.333	0.400	0.500	1.100	1.150	1.200	0.400	0.500	0.667	0.500	0.667	1.000
AV				0.653	0.734	0.788	0.614	0.679	0.765	0.517	0.580	0.650	1.100	1.200	1.265	0.257	0.296	0.349	0.832	0.898	0.963	0.871	0.959	1.017	0.522	0.588	0.671	0.723	0.844	1.038

Figure 2. Evaluation of criteria by eight DMs for Inverse fuzzy PIPRECIA

Based on the evaluation of the criteria and their averaging, Equation (1), a matrix sj is formed as in Figure 3.

PIPRECIA	sj		
c1			
c2	0.900	1.013	1.077
c3	1.138	1.308	1.408
c4	0.771	0.908	1.035
c5	0.807	0.979	1.131
c6	1.488	1.731	1.781
c7	0.504	0.608	0.773
c8	1.167	1.352	1.444
c9	1.063	1.248	1.383
c10	1.040	1.210	1.283

Figure 3. Sj form

Applying Equation (2), those values are subtracted from number 2. Following the rules of operations with fuzzy numbers, the kj matrix is obtained as in Figure 4.

kj		
1.000	1.000	1.000
0.923	0.988	1.100
0.592	0.692	0.863
0.965	1.092	1.229
0.869	1.021	1.193
0.219	0.269	0.513
1.227	1.392	1.496
0.556	0.648	0.833
0.617	0.752	0.938
0.717	0.790	0.960

Figure 4. Kj form

Applying Equation (3), the value qj is obtained as in Figure 5.

qj		
1.000	1.000	1.000
0.909	1.013	1.084
1.054	1.464	1.831
0.858	1.341	1.899
0.719	1.314	2.185
1.402	4.888	9.990
0.937	3.513	8.141
1.125	5.421	14.636
1.200	7.209	23.735
1.250	9.130	33.118

Figure 5. Qj form

Applying Equation (4), the relative weights are acquired as in Figure 6.

wj		
0.010	0.028	0.096
0.009	0.028	0.104
0.011	0.040	0.175
0.009	0.037	0.182
0.007	0.036	0.209
0.014	0.135	0.956
0.010	0.097	0.779
0.012	0.149	1.400
0.012	0.199	2.271
0.013	0.252	3.169

Figure 6. Wj form

After that, it is necessary to defuzzify obtained values by using the expression $df_{crisp} = \frac{l+4m+u}{6}$ obtaining the number df_{crisp} 0.036, 0.037, 0.058, 0.056, 0.060, 0.251, 0.196, 0.335, 0.513, 0.698 respectively.

In order to determine the final weights of the criteria, it is necessary to apply Equations (5)–(9) or the methodology of the inverse fuzzy PIPRECIA method. Based on the evaluation by the decision-makers and the application of the average value, the matrix sj' is obtained as in Figure 7.

PIPRECIA-I	sj		
c1	0.723	0.844	1.038
c2	0.522	0.588	0.671
c3	0.871	0.959	1.017
c4	0.832	0.898	0.963
c5	0.257	0.296	0.349
c6	1.100	1.200	1.265
c7	0.517	0.580	0.650
c8	0.614	0.679	0.765
c9	0.653	0.734	0.788
c10			

Figure 7. Sj form

Applying Equation (6), the values of matrix kj' are obtained as in Figure 8. Applying Equation (7), the following values are obtained as in Figure 9.

kj		
0.963	1.156	1.277
1.329	1.413	1.478
0.983	1.041	1.129
1.038	1.102	1.168
1.651	1.704	1.743
0.735	0.800	0.900
1.350	1.420	1.483
1.235	1.321	1.386
1.213	1.266	1.347
1.000	1.000	1.000

Figure 8. Kj form

qj		
0.093	0.165	0.312
0.118	0.191	0.301
0.175	0.269	0.399
0.197	0.280	0.393
0.230	0.309	0.407
0.401	0.527	0.673
0.361	0.421	0.495
0.536	0.598	0.668
0.742	0.790	0.825
1.000	1.000	1.000

Figure 9. Qj form

After that, it is necessary to apply Equation (8) to obtain relative weights for the fuzzy Inverse PIPRECIA method as in Figure 10.

wj		
0.017	0.036	0.081
0.022	0.042	0.078
0.032	0.059	0.104
0.036	0.062	0.102
0.042	0.068	0.106
0.073	0.116	0.175
0.066	0.093	0.128
0.098	0.131	0.173
0.136	0.174	0.214
0.183	0.220	0.260

Figure 10. Wj form

After that, it is necessary to defuzzify obtained values by using the expression $df_{crisp} = \frac{l+4m+u}{6}$ obtaining the number df_{crisp}, 0.040, 0.045, 0.062, 0.064, 0.070, 0.118, 0.094, 0.133, 0.174, 0.220 respectively.

Applying Equation (9), the final weights of road transportation risk criteria and rank of them are obtained as in Figure 11.

	I	II	wj	
C1	0.036	0.040	0.038	**10**
C2	0.037	0.045	0.041	**9**
C3	0.058	0.062	0.060	**8**
C4	0.056	0.064	0.060	**7**
C5	0.060	0.070	0.065	**6**
C6	0.251	0.118	0.185	**4**
C7	0.196	0.094	0.145	**5**
C8	0.335	0.133	0.234	**3**
C9	0.513	0.174	0.343	**2**
C10	0.698	0.220	0.459	**1**

Figure 11. Final weights

It has been shown in Figure 12 the complete previous calculation, and the last column shows the defuzzified values of the relative weights of the criteria in terms of fuzzy PIPRECIA method.

PIPRECIA	sj			kj			qj			wj			Defazi	Rank
c1				1.000	1.000	1.000	1.000	1.000	1.000	0.010	0.028	0.096	0.036	10
c2	0.900	1.013	1.077	0.923	0.988	1.100	0.909	1.013	1.084	0.009	0.028	0.104	0.037	9
c3	1.138	1.308	1.408	0.592	0.692	0.863	1.054	1.464	1.831	0.011	0.040	0.175	0.058	7
c4	0.771	0.908	1.035	0.965	1.092	1.229	0.858	1.341	1.899	0.009	0.037	0.182	0.056	8
c5	0.807	0.979	1.131	0.869	1.021	1.193	0.719	1.314	2.185	0.007	0.036	0.209	0.060	6
c6	1.488	1.731	1.781	0.219	0.269	0.513	1.402	4.888	9.990	0.014	0.135	0.956	0.251	4
c7	0.504	0.608	0.773	1.227	1.392	1.496	0.937	3.513	8.141	0.010	0.097	0.779	0.196	5
c8	1.167	1.352	1.444	0.556	0.648	0.833	1.125	5.421	14.636	0.012	0.149	1.400	0.335	3
c9	1.063	1.248	1.383	0.617	0.752	0.938	1.200	7.209	23.735	0.012	0.199	2.271	0.513	2
c10	1.040	1.210	1.283	0.717	0.790	0.960	1.250	9.130	33.118	0.013	0.252	3.169	0.698	1
							10.452	36.293	97.620				2.241	

Figure 12. Calculation and results obtained by the application of fuzzy PIPRECIA for road transportation risk criteria

Accordingly, calculation and results obtained by the application of inverse fuzzy PIPRECIA for road transportation risk criteria are presented in Figure 13.

PIPRECIA-I	sj			kj			qj			wj			Defazi
c1	0.723	0.844	1.038	0.963	1.156	1.277	0.093	0.165	0.312	0.017	0.036	0.081	0.040
c2	0.522	0.588	0.671	1.329	1.413	1.478	0.118	0.191	0.301	0.022	0.042	0.078	0.045
c3	0.871	0.959	1.017	0.983	1.041	1.129	0.175	0.269	0.399	0.032	0.059	0.104	0.062
c4	0.832	0.898	0.963	1.038	1.102	1.168	0.197	0.280	0.393	0.036	0.062	0.102	0.064
c5	0.257	0.296	0.349	1.651	1.704	1.743	0.230	0.309	0.407	0.042	0.068	0.106	0.070
c6	1.100	1.200	1.265	0.735	0.800	0.900	0.401	0.527	0.673	0.073	0.116	0.175	0.118
c7	0.517	0.580	0.650	1.350	1.420	1.483	0.361	0.421	0.495	0.066	0.093	0.128	0.094
c8	0.614	0.679	0.765	1.235	1.321	1.386	0.536	0.598	0.668	0.098	0.131	0.173	0.133
c9	0.653	0.734	0.788	1.213	1.266	1.347	0.742	0.790	0.825	0.136	0.174	0.214	0.174
c10				1.000	1.000	1.000	1.000	1.000	1.000	0.183	0.220	0.260	0.220
							3.853	4.551	5.472				1.021

Figure 13. Calculation and results obtained by the application of inverse fuzzy PIPRECIA for road transportation risk criteria

Figure 14 shows the final results of the procedure for determining the individual significance of each of the road transportation risk criteria. As explained above, based on the personal preferences of the eight experts, the significance of the observed criteria was obtained using the Fuzzy PIPRECIA method. Then, the defuzzification of the values was carried out to obtain the final weights of all the road transportation risk criteria, and, based on them, we can determine that the most significant criterion is C10 (transport infrastructure based risks) with a weight coefficient of 0.459, followed by the ninth criterion C9 (risks related to waiting at customs gate) with a weight of 0.343. As opposed to that, C1 (risk to be lost and disappearance) was found as the least important criterion with a weight of 0.038.

SCC for the ranks obtained with fuzzy PIPRECIA and Inverse fuzzy PIPRECIA is 0.988, which means that these ranks are nearly to complete correlation. Additionally, Pearson's correlation coefficient has been calculated for the weights of the criteria obtained using these approaches and is 0.956.

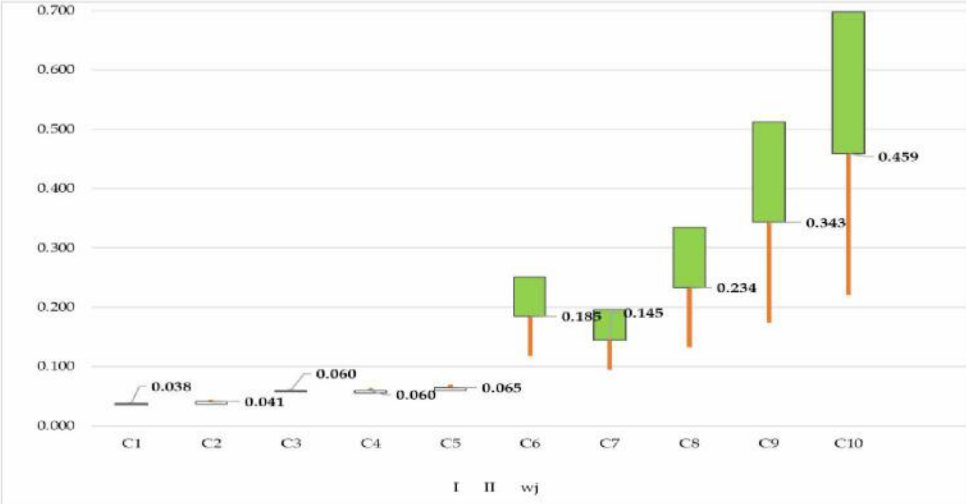

Figure 14. Final values of the road transportation risk criteria obtained using the fuzzy PIPRECIA method

4. Conclusion

The aim of the present study is to determine and rank the road transportation risk factors that are important for effective and economic supply chain management. According to the results of the study transport infrastructure based risks and risks related to waiting at customs gate were obtained as the most important ones. On the other hand, risk to be lost and disappearance and equipment related risks were found as the least important ones. In future studies, transportation risk factors can be enlarged and considered apart from road. Also, criteria can be examined in a large application area. Furthermore, various weighting methods apart from PIPRECIA can be considered in fuzzy, hesitant fuzzy, intuitionistic fuzzy, spherical fuzzy or neutrosophic environments.

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